ORR'S

CIRCLE OF THE SCIENCES.

ORR'S

CIRCLE OF THE SCIENCES:

A SERIES OF TREATISES ON THE PRINCIPLES OF SCIENCE, WITH THEIR APPLICATION TO PRACTICAL PURSUITS.

VOLUME IX.

MECHANICAL PHILOSOPHY:

INCLUDING THE

PROPERTIES OF MATTER, ELEMENTARY STATICS, DYNAMICS, HYDROSTATICS, HYDRODYNAMICS, PNEUMATICS, PRACTICAL MECHANICS, AND THE STEAM-ENGINE—

THE REV. WALTER MITCHELL, M.A.,

Queen's College, Cambridge; late Lecturer on Natural Philosophy at St. Bartholomew's Hospital.

J. R. YOUNG,

Late Professor of Mathematics in Belfast College.

AND

JOHN IMRAY, M.A.,

Civil Engineer and Practical Machinist.

NEW EDITION.

LONDON AND GLASGOW:

RICHARD GRIFFIN AND COMPANY,

PUBLISHERS TO THE UNIVERSITY OF GLASGOW.

1860.

PREFACE.

It may not be out of place, in the closing volume of a work which has entailed on the editor no small anxiety, while it has also yielded him much gratification, if he venture to take a retrospective glance at the course which has been pursued, and the manner in which the promises made in its commencement have been fulfilled.

The Work was undertaken with a zealous desire to supply a series of Elementary Treatises on the various Sciences, trustworthy in their modes of treatment and in their details, and written by authors who were masters of their subjects. Acting on this principle, writers, eminent in science, were induced to lend their aid; and the Treatises produced by them are now submitted, in their completed form, with that confidence which their reputation justifies. That the Editor has been able to carry out the original plan to a successful issue, must be gratifying to those who have so efficiently aided him with their pens; to whom, as well as to that portion of the public who have supported the undertaking throughout its progress, he takes this opportunity of tendering his grateful thanks.

In the course of the undertaking, some of the most abstruse branches of human knowledge have been approached. The "Structure of the Skeleton" has been developed by the great Comparative Anatomist of our day, Professor Owen, with a wonderful power of condensation; and the "Varieties of the Human Race" have been discussed by the equally celebrated Dr. Latham.

The several Treatises on the "Mathematical Sciences" have elicited high praise in quarters well qualified to form an opinion, for the clearness with which the principles have been laid down, and their development carried out.

In Geological Science, it is only necessary to name Professor Ansted as the author of the Treatise; while the monographs on "Mineralogy" and "Crystallography," though sealed books to the many, are recognized by competent judges as masterly instances of the powers of mathematical demonstration in the Rev. Gentleman and learned Professor who have produced them. The collateral subject of Chemistry will,

likewise, furnish acceptable aid to the student in the prosecution of his inquiries in this important field of philosophical research.

In the great branches of Natural Science, embracing Botany and Zoology, the Editor thinks himself fortunate in having secured the assistance of Dr. Smith and Mr. Dallas. The former has furnished a monograph remarkable for its simplicity, its lucid arrangement, and its microscopic illustration. The Natural History of the Animal Kingdom has, in the opinion of some of our most acute critics, happily united the necessary precision requisite for such a monograph, with the popular illustration required by the character of the Work of which it forms a portion.

Nor is it probable that the present Volume on "Mechanical Philosophy" will exhibit any falling off from its predecessors. The Mathematical formulæ of the first half of the Volume will be found to be amply illustrated by the practical knowledge displayed by the Author in that portion of the Volume devoted to Practical Mechanics and the Steam Engine.

It will probably be objected by some, that subjects were originally promised which have not appeared: this is in some sense true. Several treatises on the higher branches of Mathematics were originally contemplated, and would have been supplied; but the readers by whom such subjects are in request were found to be too limited in number to justify their production. Other subjects belonging more to general Technology will be discussed in the CIRCLE OF THE INDUSTRIAL ARTS, a work now in course of Publication under the same superintendence.

It has been objected to this Work that the want of consecutive publication has led to much inconvenience to the Subscribers, and created considerable discontent on their part. This, though much to be regretted, was quite unavoidable; as the labour of producing a weekly sheet, on some of the subjects, was physically impossible, and no mode presented itself of meeting the difficulty but that which was pursued. The Editor hopes, however, to avoid such irregularity in the CIRCLE OF THE INDUSTRIAL ARTS. For the convenience of the subscribers to the present Work, a copious series of Directions to the Binder accompanies the present sheet, which, it is hoped, will obviate any confusion in binding the volumes.

AMEN CORNER, PATERNOSTER ROW.
December 9, 1856.

DIRECTIONS FOR THE BINDER.

The Numbers marked (*) indicate the first half, and those marked (†) the latter half.

VOL. I.—ORGANIC NATURE, Vol. 1.

COMPRISING THE SKELETON AND TEETH, PHYSIOLOGY AND ETHNOGRAPHY.

Contained in Nos. 1, 2, 5, 7, 9, 10, 12, 15, 17, 18, 19, 21, and 25. The Map is intended to face page 305.

VOL. II.—MATHEMATICS.

COMPRISING ARITHMETIC, ALGEBRA WITH SOLUTIONS, PLANE AND PRACTICAL GEOMETRY, TRIGONOMETRY, &c.

Contained in Nos. 3, 6, 11, 14, 26, 16, 22, 29, 30, 32, 33, 35, 37, 38, and †42; solutions to be bound at the end of the Vol. are comprised in Nos. 59, *60, and the last eight pages of 65.

VOL. III.—ORGANIC NATURE, Vol. 2.

COMPRISING BOTANY, AND THE INVERTEBRATE ANIMALS.

Contained in Nos. 39, 41, 43, 45, 48, 51, 46, 47, 49, 50, 53, 54, 56, 57, 58, †60, †62.

VOL. IV.—ELEMENTARY CHEMISTRY.

COMPRISING CHEMISTRY OF THE IMPONDERABLE AGENTS AND INORGANIC BODIES.

Contained in Nos. 55, 61, *62, 63, 64, 66, 67, 68, 70, 72, 75, 78, 82, †88, 90, 92, 95, †97, and four pages of 106 (Title).

VOL. V.—INORGANIC NATURE.

COMPRISING GEOLOGY, CRYSTALLOGRAPHY, MINERALOGY, &c.

Contained in Nos. 8, 13, 20, 23, 24, 27, 31, 34, 36, 40, *42, 44, 52, †77, †80, †83, †84, †86, †93, 96, 98, †100, †102, †103, and 106; Map of Volcanoes to face Title-page.

VOL. VI.—ORGANIC NATURE, Vol. 3.

COMPRISING VERTEBRATE ANIMALS.

Contained in Nos. *65, 69, 71, 74, *77, 79, *80, *83, *84, *86, 87, *88, 91, *93, *97, 99, *100, 101, *102, *103, 104, 105, *106.

VOL. VII.

COMPRISING NAVIGATION, NAUTICAL ASTRONOMY, AND METEOROLOGY.

Contained in Nos. 73, 76, 81, 85, 89, 94, 117, †116, 118, 119, *120, *123, *124, *125, †120, 121, 122, †123, †124, †125, 126, 127, 128, †130, †134 (Title.)

VOL. VIII.—PRACTICAL CHEMISTRY.

COMPRISING ELECTRO-DEPOSITION, PHOTOGRAPHY, CHEMISTRY OF FOOD, AND ARTIFICIAL ILLUMINATION.

Contained in Nos. 107, 108, 109, 110, 111, 112, 113, 114, 115, *116, 129, *130, 131, 132, 133, 134, 135, 136, †141, †142, †143.

VOL. IX.—MECHANICAL PHILOSOPHY.

COMPRISING PROPERTIES OF MATTER, HYDROSTATICS, HYDRODYNAMICS, DYNAMICS, PNEUMATICS, PRACTICAL MECHANICS, AND THE STEAM ENGINE.

Contained in Nos. 4, 28, 145, *146, *149, †146, *147, 148, *150, *151, 137, 138, 139, 140, *141, *142, *143, 144, †147, †149, †150, 151, 152.

The above being the order in which they were published.

Two Sets of Titles accompany each Volume, in order that it may be bound either as a Complete Work or as a Volume of the "CIRCLE," according to the wishes of Subscribers.

☞ Handsome Cloth Cases for binding the Volumes may be had of the Publishers, Price 1s. each.

WM. S. ORR AND CO.,
Amen Corner, Paternoster Row, London.

CONTENTS.

THE PROPERTIES OF MATTER.

STATICS.

DYNAMICS.

SECTION I.—ACCELERATING FORCES.

SECTION II.—MOVING FORCES.

HYDROSTATICS.

HYDRODYNAMICS.

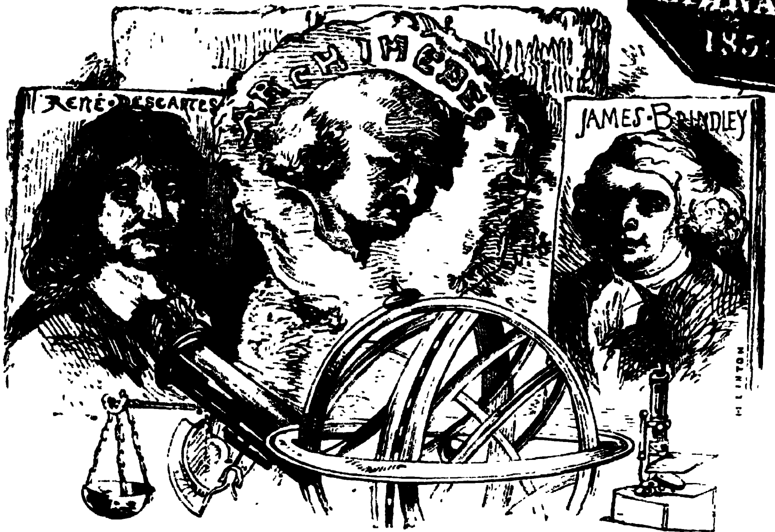
PNEUMATICS.

PRACTICAL MECHANICS.

THE STEAM-ENGINE.

Applications of Steam-Power.

The Communication of Power.



THE PROPERTIES OF MATTER.

Physics.—Natural Philosophy, or, as it is sometimes called, Physics, from a Greek word signifying Nature, embraces within its range all the phenomena and laws of the external world, while the science of Metaphysics investigates those of mind and thought Taking Physics, therefore, in its widest sense, it must necessarily include Natural History and Chemistry, as well as their auxiliary sciences. It has been customary, however, to restrict its application to Mechanical Philosophy, and the laws and properties of heat, light, and electricity.

All bodies are either solid or fluid; and considering gases and vapours as elastic fluids, and mechanical philosophy investigating the laws of rest and motion of bodies when acted on by any forces, the subject may be divided into four divisions—Statics and Hydrostatics, which treat of the equilibrium of solids and fluids; and Dynamics and Hydrodynamics, which treat of their motion. Astronomy, the figure of the earth, and Acoustics, are all branches of mechanics, as they depend upon the laws of equilibrium of either solids or fluids.

Force and Matter.—Force and matter, and their mutual influence on each other, are the great objects of Natural Philosophy. It is clear, therefore, that some knowledge of their nature and properties must form a necessary introduction to the study of Physics; but, at the very commencement of our undertaking, we must confess our ignorance of the essential nature both of matter and force. We have an intuitive conviction of their existence; we know a great deal about them—every fresh accession to our knowledge increases our power over them; the steam-engine and the electric telegraph are results

which have sprung from our acquaintance with their laws; but the more profoundly we investigate their relations, the more we become convinced of our ignorance, and the wide field of discovery which still lies before us. We see a loadstone attract a piece of iron, and we find it attract one end of a magnetic needle and repel the other, and we attribute this attraction and repulsion to the existence of a force in the loadstone, which we call the magnetic force; we see also a stone let fall from the hand descend to the ground, and we ascribe its descent to the attraction of the earth: we call the force which produces this effect gravity. Now, we can investigate the laws of the magnetic forces as well as those of gravitation, and estimate their effects on material bodies; yet, after all, we shall be obliged, with Dr. Young, "to acknowledge our total ignorance of the intimate nature of forces of every description!"

Whenever we see a body in motion, we attribute its motion to the existence of a force. A ship sails by the force of the wind and tide, a boat is propelled by the muscular force of the rower, and a locomotive by the expansion of the steam generated in the boiler; in addition to this, we find that two or more forces which, acting separately on a body, would each cause the body to move, may be so applied as to counteract each other's effects, and keep it at rest. We may therefore define force to be that which either produces, or tends to produce motion.

Having thus arrived at a definition of force, we may be able to obtain one for matter, as matter is that which is either moved, or can be moved by force.

Force and matter may thus be considered correlative terms, and we should be unable to arrive at a knowledge of the existence of the one, without the aid of the other. Every particle of matter in the universe is endowed with distinct properties of force, which fit it in many marvellous ways to act on other particles of matter. How many of these are still hidden from the researches of the human mind, we know not; but this we do know, that all the phenomena of the external world, which we can explain, can be traced to these properties of force to which we can ascribe no other origin, when we contemplate the wisdom of their application, than the will of an almighty and all-wise Creator, unless we would be guilty of the folly of believing that the print which now meets the eye of the reader, owed its existence to no other cause than the attraction of the ink for the paper, occurring by chance, without the intervention of an intelligent agent.

Matter which can be acted on by the force of gravitation is called ponderable, while that which is unaffected by it is termed *imponderable*. The particles of light, caloric or the principle of heat, the electric fluids and ether, are the only known imponderable elements.

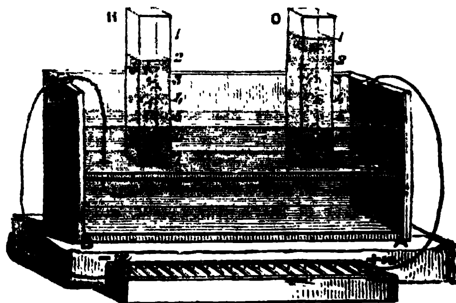
Imponderable Matter.—The prevailing opinion at present is, that heat, light, and electricity are not themselves composed of material particles, but are produced by vibrations of unknown and highly elastic fluids called ethers, which are supposed to fill the whole universe, and penetrate the pores of all solid and fluid bodies. Whether the ether, which is supposed to produce light by its vibrations, be the same as the ether, to which the effects of heat and electricity are attributed, can only at present be matter of conjecture, though the advance of natural science every day leads us to new facts, which seem to refer heat, light, and electricity to the modifications of some common cause.

Ponderable Matter.—All bodies composed of ponderable matter may be divided into simple and compound substances. By simple substances, we mean those which cannot be resolved by the chemist into any simpler elements: thus gold, silver, and

iron are simple substances; brass and steel are compound, brass being composed of copper and zinc, steel of iron and carbon. The copper, zinc, iron, and carbon are all considered elementary substances, since we have not been able to find any simple substances which, combined together, will form them; nor have we been able to resolve them into any other substances than themselves.

The number of elementary, or, perhaps speaking more correctly, undecomposible ponderable substances hitherto discovered amounts to sixty-three; two or three of these are perhaps doubtful, the analysis from which their existence has been assumed requiring further confirmation. These elementary substances may be divided into two classes—metalloids, or non-metallic elements, and metals. There are twelve non-metallic elements—oxygen, fluorine, chlorine, bromine, iodine, selenium, sulphur, phosphorus, boron, carbon, hydrogen, and nitrogen; the rest of the elements are metals. This division is, however, in a considerable degree arbitrary; silicium is sometimes regarded as a non-metallic body, and iodine and bromine as metals.

All ponderable matter hitherto discovered consists either of some one or other of these elementary bodies, or is a compound formed by the combination of two or more of them. Water will afford a familiar instance of a compound substance. It is composed of two elementary bodies, oxygen and hydrogen, combined together in the proportion of one part of hydrogen by weight, to eight parts of oxygen by weight, or two measures of hydrogen to one measure of oxygen, when we take as our measure the cubic space occupied by the gases under the same pressure and at the same temperature. This is proved by analysis, or the decomposition of water into its elements, and by synthesis, or the composition of water by the combination of its elements. The analysis of water is beautifully shown by means of galvanism. If two hollow square glass prisms, closed at O and H, whose bases are each equal to a square inch, and graduated by a scale of inches along one of their edges, so as to show the number of cubic inches they contain, be filled with water, and placed inverted in a vessel A B, also containing water, in such a manner that the prisms may remain full of water; if now the wires of a galvanic battery, terminated each by a piece of platinum foil, be introduced into the vessel, so that the platinum foil forming the positive pole of the battery may be within the prism O, and that forming the negative in H; so soon as the battery is set in action, bubbles of gas will rise, as it were, from the platinum foil, and occupy the upper portion of the prisms. The gas in the upper part of O will be pure oxygen, and that in H pure hydrogen. The gas in H will always occupy twice the space of that in O; but if the gas generated in any given time in O be weighed against that generated in H, though only half in bulk, it will be eight times as heavy. For this beautiful method of analysing water, we are indebted to Nicholson and Carlisle. If now we mix the oxygen in O with the hydrogen in H, both gases will entirely disappear on an electric spark being passed through them, and their place will be occupied by pure water, equal in weight to the sum of the weights of the two



gases; and this proves the composition of water by synthesis, or the combination of its elements. We are thus enabled, by means of electricity, to show the composition of water, both analytically and synthetically. The composition of water can also be determined by making one or other of its elements combine with other elements, which is the chemical mode of analysis.

By methods similar to those employed for the analysis of water, all compound substances hitherto met with have been analysed, and their constituent elements determined; it does not follow, however, that these elements may not hereafter be shown to be either compounds of one another, or composed of elements not yet discovered. For a long time the well-known substances potash and soda were regarded as elementary, until Davy, by means of his powerful galvanic battery, resolved them into combinations of the gas oxygen, with the hitherto unknown metals potassium and sodium—metals so light as to swim on water, and so inflammable as to be set on fire by mere contact with water.

In animal and vegetable bodies, the vital force, whatever it may be, is able, out of the four elements, oxygen, hydrogen, nitrogen, and carbon, to form a number of compounds which seem to defy our powers of enumeration. Liebig tells us that in living bodies, "with carbon and nitrogen, with carbon, hydrogen, and oxygen, with nitrogen and hydrogen, are formed compound atoms, which in their properties are perfectly analogous to chlorine, to oxygen, or to sulphur, or to a metallic body—not in a few isolated points of resemblance, but in all their properties. It is scarcely possible to imagine anything more wonderful than that carbon and nitrogen should form a gaseous compound (cyanogen), in which metals burn with the evolution of light and heat, as in oxygen gas; a compound substance, which in its properties and deportment is a simple substance,—an *element*, the smallest particles of which possess the same form as those of chlorine, bromine, and iodine, since it replaces them in their combinations without any alteration in the crystalline form of the compound."—LIEBIG's *Letters on Chemistry*.

From this we may not unreasonably conclude that it is probable that, as chemistry advances, the number of so called elementary substances may be considerably diminished.

The three Mechanical States of Matter.—All ponderable matter is found in one or other of three states—the solid, liquid, or aëriform. A solid body is composed of particles of matter united together by forces which cause the body to retain its shape unaltered, except the particles are forced asunder or displaced by some degree of violence; a solid body is considered hard or soft, according as it requires a greater or less degree of force to alter its form or displace its particles. The particles which form a liquid body, are united together in such manner as to allow them to move about one another with great freedom and with but little friction; a liquid consequently yields so readily to external force or pressure, that it retains no form of its own, but readily assumes that of any vessel in which it may be placed, without altering its volume. A gas or aëriform fluid is one whose particles mutually repel each other in such a manner that a gas has neither definite form nor volume; its form and volume being only limited by the vessel in which it is inclosed, or by the pressure exerted on it. A quantity of gas, however small, may be made to fill a vessel of any size or shape.

Heat and pressure seem to be chief causes of bodies existing in each of these three states, and it is probable that there is a particular temperature and pressure for every solid body at which it would assume any one of these three states. There is no solid

substance known which may not be rendered liquid, and finally converted into vapour, by the application of a heat sufficiently intense; and we may reasonably conclude that all bodies which are liquid would become solid or freeze, if we could sufficiently reduce their temperature, although some liquids have hitherto resisted the greatest amount of cold we have been able to procure by artificial means. Some vapours or aëriform fluids are readily condensed by cold into the liquid state; others require great intensity of cold, or great pressure, or both combined, to render them liquid; and we may infer, from the gases already condensed by cold and pressure, that all might be, could we only procure the necessary degree of cold and pressure.

Water is known in three states—the solid, the liquid, and the aëriform; and the amount of heat by which it is influenced at the moment determines its existence in one or other of these three conditions. Quicksilver, which like water is liquid at the ordinary temperature of the earth's surface, is known in the solid state during the winter of the arctic regions; and by a temperature such as that at which oil boils it is converted into vapour. Sulphuric acid, that is oil of vitriol, when wholly deprived of water, is known in the solid, the liquid, and the aeriform states, according to the temperature to which it is subjected. Sulphurous acid, the body formed by burning sulphur in the air, is a gas at ordinary temperatures, but on being exposed to cold, that is, being deprived of some portion of its heat (for the idea that cold is a positive quality seems now wholly abandoned), is converted into a liquid.

Chemists term those aëriform bodies which exist in that state at the ordinary temperature of the atmosphere, "gases;" while they restrict the use of the term "vapour" to those which require a higher temperature to retain them in the aëriform state. By this rule, water in the aëriform state is called a vapour, while sulphurous acid, which requires cold to convert it into the liquid state, is called a gas.

Some bodies are known only in the aëriform state, or as gases; that is, the means of converting them into the liquid and solid forms have not yet been discovered. Oxygen gas, or that element of the atmosphere which is essential to the support of ordinary combustion and of animal and vegetable life, is an example. This gas has hitherto resisted all the attempts made by the joint efforts of cold and pressure to condense it into a liquid or a solid. It is not therefore to be concluded that it is impossible for oxygen gas to exist in the liquid or in the solid state; but only that this conversion requires a greater amount of cold, or a greater amount of pressure, or of both conjoined, than science has as yet at its command.

A number of bodies which exist at common temperatures as gases, can be converted into liquids by the joint effect of cold and pressure; a few, like sulphurous acid gas, already mentioned, are changed from the gaseous to the liquid state by cold alone.

Some bodies, as the metal arsenic, by the effect of heat, at least under the ordinary atmospheric pressure, pass at once from the solid to the aëriform state. Carbonic acid, the gas which escapes in the effervescence of soda water, at the freezing point of water (32° Fahr.), is condensed by a considerable pressure into a liquid, and then, being relieved from pressure, part of it passes into a solid by the effect of the cold produced by its own evaporation. Hence, as carbonic acid is not known as a liquid, except under great pressure, it must be regarded as an example of a solid, which passes, under the ordinary pressure of the atmosphere, at once into the aëriform state. Carbonic acid is the only instance yet known of a solidified gas.

From what we have thus incidentally said, it sufficiently appears how much the

pressure of the atmosphere prevailing at any moment must influence the properties of bodies.

How far Bodies can exist in the three Mechanical States.—After the preceding details, we appear to be in a condition to determine how far it is warrantable to assume it to be a law, that all bodies are capable of existing in the solid, the liquid, and the aëriform states. One great limitation to this proposition at once suggests itself, namely, that many compound bodies existing in the solid state are decomposed by heat, before any indication is shown either of fusion or liquefaction. Some of these, however, become fused without decomposition, when, together with a high temperature, a sufficient amount of pressure is applied. Thus limestone, termed by chemists carbonate of lime, and consisting of quick lime and carbonic acid, when exposed in a lime-kiln to heat, gives off carbonic acid gas, and becomes converted into quick lime; but, if a very strong pressure be applied at the same time with the heat, the limestone fuses without losing any part of its volatile ingredient. After the deduction, however, of all the cases of this sort, the number of exceptions occurring in the list of compound bodies is very great; particularly when organic compounds, that is, the animal and vegetable solids, are included.

But let us examine how far it will hold as a law of nature that all simple bodies are capable of existing in the solid, the liquid, and the aëriform states, provided unlimited means were afforded of increasing or diminishing temperature.

Of the sixty-three simple bodies, five exist at the ordinary temperature of the atmosphere, in the aeriform state, namely, oxygen, chlorine, fluorine, nitrogen, and hydrogen; two in the liquid state, the one non-metallic, bromine, and the other metallic quicksilver. The remaining simple bodies exist naturally as solids.

Of the five gaseous simple bodies, none have as yet been converted into the solid state, and only one into the liquid state, namely, chlorine, and this gas, moreover, not when dry, but when combined with moisture, forms crystals.

The two natural liquids, bromine and quicksilver, are known in the solid and the aëriform, as well as in the liquid state, according to the temperature to which they are exposed.

Of the non-metallic simple solid bodies, four are known also in the liquid and aeriform states, namely, iodine, sulphur, phosphorus, and silenium.

The metallic solids are all fusible, or at least all that have been sufficiently examined; the heat required being very various—from below the boiling point, to the highest temperature which the oxy-hydrogen blowpipe can produce.

Thus, of the sixty-three simple bodies in modern chemistry, no more than thirteen are known as existing under any ordinary variation of temperature in the three mechanical states of solid, liquid, and aëriform. Of the remainder, all but eight are known in two of these states, namely, the solid and the liquid; while only one, namely, chlorine, is known in the liquid and in the gaseous states, without being also known in the solid state.

If, then, it be assumed to be a law of nature that all simple bodies are capable of existing in the three mechanical states, solid, liquid, and aëriform, the assumption rests on the not improbable supposition, that the failure to exhibit all these bodies in each of these three states, arises from the inability of the chemists as yet to produce a sufficient degree of heat, or a sufficient degree of cold for the purpose.

Divisibility of Matter.—Whether matter is or is not infinitely divisible, has long been a matter of dispute, and still remains so; indeed, we have not any evidence

for the one opinion or the other which can be deemed strictly conclusive; and perhaps the subject may be one which may for ever elude our powers of analysis. That space may be infinitely divisible, appears to be one of the axiomatic principles of Geometry, and seems, indeed, involved in our definition of a geometrical point, which we define to be "That which has no parts, and which has no magnitude." But we must remember that this geometrical point, as well as the line, which has length without breadth, which we may conceive to be formed by the motion of this point, is only a mental abstraction, and can hardly be said to have a physical existence. The smallest line which we can suppose to exist, can yet be divided geometrically into its thousandth, or millionth part, and that millionth part again into its millionth, and so on for ever; for if we came anywhere to a point where our division could proceed no farther, however minute might be the length of our ultimately indivisible line, we should arrive at a point which had parts and magnitude.

If we conceive matter infinitely divisible, in the same sense that a geometrical solid is infinitely divisible, we should arrive at a physical point, which, like the geometrical one, would be without parts or magnitude; for if possessed of parts or magnitude, it could not have been infinitely divided. Those, therefore, who hold that matter is infinitively divisible, must conceive all matter to be composed of an infinite number of physical points, and maintain the seeming paradox that an infinite number of nothings may make a something. This paradox is not confined to Physics,—it lies at the root of geometry, and infects our highest mathematical analyses, in spite of all the ingenious subtleties which have been invented to get rid of it. If the hypothesis of the infinite divisibility of matter be true, the physical points to which matter must be conceived to be ultimately reduced, must be without volume, and destitute of what we generally suppose to be the ordinary attributes of the ultimate particles of matter—solidity and impenetrability. They are, in fact, mere points, which are centres of force. The ultimate particle of matter which can no longer be divided, is called an atom, a term derived from the Greek language, signifying that which cannot be cut. According to the theory, therefore, of the infinite divisibility of matter, the ultimate atoms are atoms or molecules of force, destitute of any solid nuclei. This is the theory of Boscovich, which has lately been revived, with great force and most powerful reasoning, by Faraday, as the only satisfactory hypothesis for the constitution of matter viewed in relation to the electric and magnetic forces.

That matter is not infinitely divisible, has been held by some of the most distinguished natural philosophers, foremost among whom we must place Newton. They maintain that the ultimate particles of matter are hard and solid, and therefore of a definite mass and volume; though they are so small as to defy all our optical instruments, however great their magnifying power, to enable us to perceive them, even supposing we had mechanical or other means by which we could divide a body until we arrived at its ultimate atoms. These atoms are not only indivisible, but also indestructible and incapable of being worn. The atomic theory of chemistry is supposed to favour the opinion of the finite divisibility of matter. If we take, for instance, a compound substance, such as water, which we have already considered, and which is composed of definite weights of oxygen and hydrogen, it is argued that there must be some point between a distinctly visible particle of water, and its infinite divisibility as a portion of matter, at which the oxygen must be detached from the hydrogen; or that water, by mere mechanical division, would be resolved into its elements, if we could carry our division far enough; but this is mere assumption, for, according to the theory

of Boscovich, if we consider an atom of oxygen as an atom of force, having properties of force different from those of the atom of force constituting the atom of hydrogen, we may conceive the atom of hydrogen to combine with the atom of oxygen, and form a new atom of force—water—whose properties of force may be essentially distinct from those of oxygen and hydrogen. As the atoms of oxygen and hydrogen are mere physical points of force, the atom of water may be the same; and consequently water itself may be capable of infinite divisibility quite as much as either of its constituents.

In addition to these considerations, the atomic theory of chemistry, if we make it to depend upon the fact that the ultimate atoms of matter are indivisible, which it need not, will lead us into no small difficulties, as Dr. Whewell has shown :—"According to the theory," he says, "all salts, compounded of an acid and a base, are analogous in their atomic constitution; and the number of atoms in one such compound being known or assumed, the number of atoms in other salts may be determined. But when we proceed in this course of reasoning to other bodies, as metals, we find ourselves involved in difficulties. The protoxide of iron is a base which, according to all analogy, must consist of one atom of iron, and one of oxygen; but the peroxide of iron is also a base, and it appears, by the analysis of this substance, that it must consist of *two-thirds* of an atom of iron and one atom of oxygen. Here, then, our indivisible atoms must be divisible, even upon chemical grounds. And if we attempt to evade this difficulty by making the peroxide of iron consist of two atoms of iron and three of oxygen, we have to make a corresponding alteration in the theoretical constitution of all bodies analogous to the protoxide: and thus we overturn the very foundation of the theory. Chemical facts, therefore, not only do not prove the atomic theory as a physical truth, but they are not, according to any modification yet devised of the theory, reconcileable with its scheme."

We must remember that each of these atomic theories are mere hypotheses, which may or may not be true. They may afford us most valuable assistance in grouping together physical facts into laws, and thus materially extend our knowledge; but as we have before said of force so we now say of matter—that we are utterly ignorant of its essential constitution. We may still say with Sir I. Newton, that "we do not know what the substance of anything is. All that we see of bodies is their figures and colours—we hear only their sounds—we touch only their outward surfaces,—we smell only their scents, and taste only their savours. We know not their inward substances by any sense or any reflex act."

We have referred to the infinite divisibility of a geometrical straight line, and it is a curious subject to consider how far art is able to carry the actual subdivision of a straight line. The division of a slip of ivory, an inch long, into a hundred equal spaces, each of which is distinctly visible, is a work of no great difficulty. With the fine point of a diamond, applied by means of a very delicate screw, five thousand equi-distant lines can be traced on glass within the space of a quarter of an inch, the effect of the division on the glass being to produce a play of prismatic colours. By similar means, the like division can be produced on a piece of steel. A prize medal was awarded by the jurors of the Great Exhibition to Mr. Nobert, of Prussia, for tracing parallel lines on glass, which were only the one-forty-seventh-thousandth of an inch apart, and required a magnifying power of two thousand to distinguish them.

The ductility of metals exhibits the divisibility of matter in its most practical light. It is difficult, however, when such degrees of extension are described, to convey to the mind an adequate idea of the actual tenuity which is attained; and when the hun-

dredth part of an inch is spoken of, the imagination, unaccustomed to such minutiæ of division, feels totally bewildered.

The following examples will assist us to form an adequate conception of the great tenuity of which some kinds of matter are susceptible. Human hair varies in diameter from the two-hundred-and-fiftieth to the six-hundredth part of an inch. A sheet of common writing paper has a thickness of about the five-hundredth part of an inch. A pound of cotton can be spun into a thread seventy-five miles in length, the diameter of which is the three-hundred-and-fiftieth part of an inch. A pound of wool can be spun into a thread ninety-five miles in length, the diameter of which is the four-hundredth part of an inch. The fibre of the coarsest wool is about the five-hundredth part of an inch—that of the finest being about the fifteen-hundredth part of an inch in diameter. Gold-beater's leaf, which is the pellicle separated from the outer part of the great intestine of the ox, has a thickness of about the three-thousandth part of an inch. The silk line, as spun by the worm, is about the five-thousandth part of an inch thick. A spider's thread is about the thirty-thousandth part of an inch in diameter, so that a single pound of this fine drawn substance would suffice to encircle the globe. A ribbon of gold, six yards long, and an inch and a quarter wide, weighing one-thousand-and-fifty-six grains, is finally extended, by hammering, into two thousand leaves of 3⅝ inches square, or into eighty books, containing each twenty-five leaves. Every leaf weighs rather less than the half of a grain, and its thickness is the one-hundred-and-nine-thousandth part of an inch. In the gilding of buttons, five grains of gold, applied as an amalgam with mercury, are allowed to each gross. In this case, the coating left must be one-hundred-and-ten-thousandth part of an inch in thickness. If a piece of ivory, or a bit of white satin be immersed in a nitro-muriatic solution of gold, and be then plunged into a jar of hydrogen gas, it will become covered with a surface of gold hardly exceeding in thickness the ten-millionth part of an inch. The gilt wire used in embroidery is made by extension over a surface of silver. A silver rod about two feet long and an inch and a half in diameter, is coated with about eight hundred grains of pure gold, although sometimes only one hundred grains of gold is allowed to the pound of silver. The gilded rod, now weighing about twenty pounds, by being drawn through a succession of smaller and smaller holes, is at last stretched to the almost incredible length of two-hundred-and-forty miles, the gold being consequently attenuated three-hundred times, so that each grain covers a surface of nine-thousand-six-hundred square inches. The wire is next flattened, by which a further extension is produced, and its thickness reduced to the four or five-millionth part of an inch. A wire of pure gold can be drawn, the thickness of which shall not exceed the four-thousandth part of an inch. A wire of platinum can be made of still greater fineness. Dr. Wollaston drilled a fine hole through the axis of a cylinder of silver, about the third part of an inch in diameter; through this hole he passed a platinum wire, no more than the thousandth part of an inch in thickness. The silver cylinder was then drawn through a succession of decreasing holes, till its thickness was reduced to the fifteen-hundredth part of an inch; the platinum wire being in the meantime proportionally reduced, became between the four and five-thousandth part of an inch in thickness. By means of nitric acid, the coating of silver was then removed. By carrying the extension of the silver mould still farther, he obtained, on some occasions, platinum wires of no more than the thirty-thousandth part of an inch in thickness. Silver leaf is nearly twice as thick as gold leaf, being the one-hundred-and fifty-thousandth part of an inch in thickness. Copper and tin, forming Dutch leaf, cannot be

beaten thinner than the twenty-thousandth part of an inch. A single grain of blue vitriol,—the sulphate of copper,—gives a fine azure tint to five gallons of water; in this case, the salt must be expanded at least ten millions of times.

The subdivision of odoriferous particles exceeds even these examples. A single grain of musk has been known to perfume a large room for the space of twenty years. Sir John Leslie computes that the musk, in such a case, must have been subdivided three-hundred-and-twenty quadrillions of particles, each of them capable of affecting the olfactory organs. A lump of assifœtidæ, exposed to the air, was found to lose only one grain in seven weeks.

In organic living nature, the infusory animalcules exceed everything which can be conceived of minuteness. The *vibrio undulata* found in duck-weed, is computed to be ten-thousand-million of times less than a hemp-seed.

Having considered the theories of the divisibility of matter into its ultimate particles or atoms, and how far matter can be divided by mechanical and other means, we proceed to the consideration of some other properties common to all ponderable matter.

Gravity.—There is one property common to all ponderable matter—a property not confined to our own globe, but one which astronomers have most triumphantly shown to belong to the matter composing all the heavenly bodies throughout the universe. This property is gravity. We are indebted to Sir I. Newton for the discovery of the law of gravitation—the most general, and, at the same time, one of the most important laws ever revealed to human intellect. It is, that all the heavenly bodies attract one another by a force varying directly as their mass, and inversely as the square of their distance from one another; the mass of a body being considered as the sum of the particles of matter constituting the body, and that this law applies also to every particle of matter in the universe. To estimate this force we must have some measure. Now there are, as we have elsewhere observed, two measures of force; we may estimate, for instance, the earth's attraction for a body by the pressure the body exerts, the measure of which will be its weight, or by the velocity with which it will pass through a given space, which will be measured by the space passed through, in a given time from the commencement of the motion. The first of these measures is found to depend on the mass of a body, and the latter is independent of the mass, being the same for all bodies, however large or heavy they may be, and is therefore generally chosen as the best measure of force, when the motion of bodies is to be estimated. When we say that a force varies directly as the mass of the attracting body, and inversely as the square of its distance from the body attracted, we mean that the force of attraction will be measured by the mass divided by the square of the distance. Thus, if we suppose one body, A, whose mass equals one thousand, or, in other words, a body composed of one thousand particles of matter, and B, another body composed of ten gravitating particles, and conceive these bodies placed at a distance of one mile apart, where they could be influenced by no other matter, the force of A's attraction on B would be represented by one thousand, and of B on A by ten; and if A, by its attraction, drew B through one thousand inches in the first second of time after the bodies' action on one another, B would draw A through ten inches in the same time. If A and B had been ½ a mile apart, A's attraction on B would have been represented by $1000 \div \frac{1}{2^2}$ or $1000 \div \frac{1}{4}$, which $= 4000$, and the distance through which A would have drawn would be 4000 inches; B's attraction on A would have been $10 \div \frac{1}{2^2}$, or 10×4, and A would be drawn through 40

inches. If the distance had been $\frac{1}{3}$ of a mile, A's attraction on B would have been $1000 \div \frac{1}{3^2}$, or $\frac{1}{9}$, which $= 9000$; and B on A, $10 \div \frac{1}{2^2}$, or 90; in this case A would draw B through 9000 inches, while B would draw A through 90 inches, in the first second of time after their action on each other.

It follows, from the law of gravitation, that the smallest stone which falls to the earth attracts the earth, just as truly as the earth attracts the stone; but the mass of the earth is so great, compared with the mass of the stone, that the distance through which the earth is drawn by the stone during its descent is inappreciable to any known measurement, however small. Again, the earth attracts the sun as truly as the sun attracts the earth, but the mass of the sun is so great, as compared with the earth, that in calculation the mass of the earth may be neglected. In the solar system we have planets and comets revolving round the sun, and satellites revolving about planets, in curves which are very nearly such as may be obtained by the section of a cone, and are therefore called conic sections. The reason why the satellites do not fall into their primaries, and the planets and comets into the sun, is because they have at some time received an impulse from some force other than that of gravitation, in a different direction than the line joining the attracting bodies, at the instant of its action. Of the origin of this force we are ignorant, as we are also of its nature. All we know is, that this unknown force, having once acted and then ceased, the law of gravitation, and the three laws of motion, are quite sufficient to account for the motions of the heavenly bodies with the greatest accuracy. We said that the bodies of the solar system moved in curves which were nearly conic sections. Now, though the mass of any one of the planets may be nothing, when compared with the sun, this is by no means the case when compared with one another; and, since all the particles of matter in the solar system attract one another, it follows that one planet must, in some measure, influence the motion of another. If the planets had no influence on each other, they would describe accurately conic sections; but their mutual influence produces a perturbation, which causes each planet to describe an irregular orbit, differing slightly from the regular curve, which it would have described without any disturbing influence. The same applies also to the orbits of the satellites. This perturbation affords the most delicate test of the truth of the law of gravitation and its accuracy. "An idea of the extreme smallness of the perturbations may be learned from the fact, that if we trace on a table six feet in diameter an accurate ellipse, to represent the orbit on which the earth is moving at any instant about the sun, and if we trace by its side the path actually described in a single revolution round the sun, the difference between the original ellipse and the curve actually described is so excessively minute, that the nicest examination with microscopes, continued along the outlines of the two curves, would hardly detect any perceptible interval between them."—HERSCHEL'S *Astronomy*. Yet it was by the elimination of the perturbations produced on Uranus by the known planets, and finding a perturbation still unaccounted for, which led two astronomers, without communication with each other, to calculate the mass and position of an hitherto unknown planet, and to tell the practical astronomer to what point in the heavens to direct his telescope for the discovery of the stranger. By the motion of many of the double stars about one another, and their orbits having been shown to agree with the curves they would describe, if they attracted one another, by the law of gravitation; this wonderful law has been extended from the solar system to the remotest parts of the universe.

As every particle of matter attracts every other, it follows that all masses on the earth's surface must attract one another; but the earth's attraction is so great, on

account of its larger comparative mass, that this attraction is not ordinarily perceptible. It has been shown to exist, however, by the experiment of Mr. Cavendish. Small leaden balls were supported on the ends of a rod, which was suspended at the middle by a slender wire, and when large leaden balls were brought near to them, it was found that the wire was immediately twisted by the motion of the balls. By observations on stars, from stations near the mountain Schehallien, in Scotland, it was proved that the mountain produced a sensible effect in drawing the plumb-line out of the vertical.

Weight—Is the effect which the earth's attraction produces on bodies; and it is a curious fact that, on account of the earth's form not being perfectly spherical, the weight of a body is not the same at the equator and the pole. Thus, a body weighing 194 pounds at the equator, will weigh 195 at the pole, the rate of increase from the equator to the pole taking place according to a geometrical law, depending on the latitude. This increase of weight cannot be determined either by a balance or steelyard, as the weight which is used for a measure is affected proportionally with the body to be weighed. It may be determined by observing the length to which a body may stretch a spiral spring at the equator, and the length to which the same body will stretch the same spring in another latitude. Practically, however, the difference in the force of gravitation is observed by means of pendulums; the length of a pendulum vibrating seconds is greater in a high latitude than in a lower one; and as the application of the pendulum to a clock affords us a ready means of multiplying its variations, we are enabled to distinguish by this means a variation in length, which otherwise would be inappreciable. From experiments made by Newton with hollow pendulums, filled with different kinds of substances, such as gold, silver, lead, glass, sand, common salt, and wood, he found that the attraction of the earth is the same for all substances. Thus we see that under an exhausted receiver, the feather and the guinea fall at the same instant from the top to the bottom—and that consequently a cubic inch of iron weighs more than a cubic inch of wood; not because the earth's attraction is greater for iron than wood, but because the cubic inch of iron contains more gravitating particles than the cubic inch of wood.

Mass.—The mass of a body is the quantity of matter which it contains.

Density.—The term density is used to indicate the quantity of matter contained in a given volume of a mass.

Two bodies are equal in mass if they are of the same weight, however different may be their volumes; but they are equal in density only if the equal volumes of the two bodies have the same weight. A cubic inch of one substance is said to be of twice the density of another, if the cubic inch of the denser body weigh twice as much as the cubic inch of the other.

Specific Gravity.—The weight of a given volume of different substances, compared with the same volume of some standard substance, is called the specific gravity of these substances; the general standard is distilled water, at the ordinary temperature of the air. This is chosen for convenience, as the easiest material which can be readily procured in a state of purity. Reckoning the specific gravity of distilled water as 1·000, the specific gravity of platina is 21·470; gold, 19·260; copper, 8·900; lead, 11·35; cast-iron, 7·248; marble, 2·716; Portland stone, 2·496; beechwood, ·852; cork, ·240; mahogany, 1·063; oak, 1·170.

Since a cubic foot of water weighs nearly 1000 ounces avoirdupois, a cubic foot of platina will weigh about 21470; gold, 19250; iron, 7248; marble, 2716; cork, 240; and oak, 1170 ounces.

There are various methods of finding the specific gravity of bodies, according as they are solid or fluid, or soluble or insoluble, in water. If we could shape a solid body accurately into a cubic inch, we should have no difficulty in finding its specific gravity, by comparing its weight with a cubic inch of water. This is not at all times convenient; we may therefore adopt any of the following methods, as it is a known fact in hydrostatics that any body which sinks in a fluid will just displace a volume of water equal to its bulk. If we weigh the body first in air, and then in water, the difference of its weight in air and its weight in water will be the weight of a volume of water equal in bulk to the body weighed. Considering the specific gravity of water as an unit, the specific gravity of another substance will be its weight, divided by the weight of an equal volume of water. The specific gravity of a substance, therefore, will be its weight in air, divided by the difference of its weight in air and in water. When the substance whose specific gravity is to be determined is in small fragments, it may first be weighed in air, and the weight of the mass of water it displaces may be determined by observing the weight of water the fragments displace when they are placed in a cup of water, previously filled carefully to the brim. There are various other methods of finding specific gravity, and also for ensuring accuracy; but these must be left to be described in their proper place as a branch of hydrostatics.

Cause of Gravity.—Gravesande, in his introduction to "Newton's Philosophy," contends that the cause of gravity is utterly unknown; and that we are to consider it in no other way than as a law of nature, originally and immediately impressed by the Creator, without any dependence on any second law or cause at all. Newton did not profess to give any positive explanation of the force of gravity; but in his Optics he throws out the following query, as he was not sufficiently satisfied with his experiments to hazard anything positive on the subject. He says:—

"Is not this medium (ether) much rarer within the dense bodies of the sun, stars, planets, and comets, than in the empty celestial spaces between them? And in passing from them to greater distances, doth it not grow denser and denser perpetually, and thereby cause the gravity of those great bodies towards one another, and of their parts towards the bodies; every body endeavouring to recede from the denser parts of the medium towards the rarer?

"For if this medium be supposed rarer within the sun's body than at its surface, and rarer there than at the hundredth part of an inch from his body, and rarer there than at the fiftieth part of an inch from his body, and rarer there than at the orb of Saturn; I see no reason why the increase of density should stop anywhere, and not rather be continued through all distances, from the sun to Saturn, and beyond. And though this increase of density may, at great distances, be exceeding slow, yet, if the elastic force of this medium be exceeding great, it may suffice to impel bodies from the denser parts of the medium towards the rarer with all that power which we call gravity. And that the elastic force of this medium is exceeding great, may be gathered from the swiftness of its vibrations. Sounds move about 1140 English feet in a second of time, and in seven or eight minutes of time they move about 100 English miles. Light moves from the sun to us in about seven or eight minutes of time, which distance is about 70,000,000 English miles, supposing the horizontal parallax of the sun to be about twelve seconds, and the vibrations, or pulses of this medium, that they may cause the alternate fits of easy transmission and easy reflexion, must be swifter than light, and, by consequence, above 700,000 times swifter than sounds; and therefore the elastic force of this medium, in proportion to its density, must be above 700,000 multi-

plied by 700,000 (that is, above 490,000,000,000) times greater than the elastic force of the air is in proportion to its density; for the velocities of the pulses of elastic mediums are in a subduplicate ratio of the elasticities and the rarities of the mediums taken together.

"As magnetism is stronger in small loadstones than in great ones, in proportion to their bulk, and gravity is stronger on the surface of small planets than those of great ones, in proportion to their bulk, and small bodies are agitated much more by electric attraction than great ones; so the smallness of the rays of light may contribute very much to the power of the agent by which they are refracted; and if any one should suppose that ether (like our air) may contain particles which endeavour to recede from one another (for I do not know what that ether is), and that its particles are exceedingly smaller than those of air, or even than those of light, the exceeding smallness of such particles may contribute to the greatness of the force by which they recede from one another, and thereby make that medium exceedingly more rare and elastic than air, and, of consequence, exceedingly less able to resist the motions of projectiles, and exceedingly more able to press upon gross bodies, by endeavouring to expand itself."—Newton's *Optics*.

Inertia.—Having spoken of a property or law common to all matter—gravity—by which every particle of matter attracts every other particle, according to a certain mathematical law, we come now to another property of matter to which the name of Inertia is given. The law of inertia is this—that a particle of matter, if it be at rest, possesses no power within itself by which it can put itself in motion, and can only be made to move by some power external to itself; as, for instance, the action of another particle of matter upon it; or, if it be in motion, the particle possesses no property by which it can alter its motion, either in direction or magnitude. What has been said of a particle is also true of a body composed of any number of particles; if the mutual attractions and forces which these particles exert on each other be such that the body is reduced to a state of rest or equilibrium, it cannot put itself in motion, or change its motion if moved. The inertia of a body is its inability to produce any action on itself, so as either to cause its motion, if at rest, or to alter it, if in motion.

Vis Inertia.—Whether inertia is to be regarded as an active force in matter by which it resists motion, or only as we have just described it, the absence of any power in a particle of matter to act upon itself is a matter of dispute among physical philosophers; some contending warmly that it is not a force, and that the term force applied to it can only mislead, while others maintain that it is a force or property of matter quite as truly as gravitation. According to the latter view, we may define the vis inertia, or force of inertia, to be a force or property of matter by which it resists the application of a force,—not by refusing to move, but requiring a greater force to produce in it a given degree of motion in proportion to the mass of the body moved. It is true that we experience an effort in moving any mass—some masses resist our attempts to move them altogether, but then we can trace a great part of this resistance to the action of friction, or some other cause external to the mass we attempt to move; and, by removing these causes, we can move with ease huge masses which would otherwise resist the greatest force we could exert. Thus the slightest effort may cause those large rocks, which are curiously poised, either naturally or artificially, known by the name of rocking-stones, to vibrate backwards and forwards; and we may suppose that by removing all causes of resistance, the body itself would oppose no force of inertia to any motion we could impress on it; yet, if we measure the motion in the body by the

velocity produced by a given force, we should find it to depend on the mass of the body, and that there is, as Dr. Whewell says, "in every case a resistance to motion, which shows itself, not in preventing the motion, but in a reciprocal force, exerted backwards upon the agent by which the motion is produced. And this resistance resides in each portion of matter, for it is increased as we add one portion of matter to another."

The property of matter, which we have described by the term inertia, is the cause of the first law of motion, while the vis inertia is the cause of the third law of motion.—*See Preliminary Treatise.*

Porosity.—If a body be not perfectly solid—that is, if the particles composing it be so united together as to leave spaces between them not filled with matter—the body is said to be porous. Thus, if we suppose a body to be built up of rectangular solid bricks, perfectly glued together, so as to leave no vacuities, we should say it was not porous; while a body composed of a number of spherical balls, fastened together only at their points of contact with each other, would be porous; sponge, charcoal, or loaf-sugar afford good examples of this property. We say, in general, that a stone is porous, and not compact, when it readily absorbs water, or would allow water to pass through it, and in this sense we should say that a glass vessel is not porous. It is reasoned, however, from certain facts, that all bodies are more or less porous; the densest and most compact bodies, such as gold and platina, being really porous, though their pores may be so minute, or the forces of repulsion which their particles exercise may be so great, as to prevent any known ponderable matter from filling the vacuities in the substance. With regard to bodies which are to a certain extent porous, we find they will sometimes suffer one fluid to pass through their pores, while they will retain another; thus a cask which will hold water may suffer oil to ooze out.

The force of gravity acts through the densest substances, of the greatest thickness, without any perceptible diminution of its power, and hence we conclude that it can permeate every substance, without meeting with any resistance. Light passes through many substances of considerable density. Heat and electricity do the same, but they do not travel through all known substances, as the force of gravity does; nor do the same bodies, which may conduct heat or electricity, suffer light to penetrate their substance with the same ease.

From the fact that the densest of bodies, such as gold or platina, and all other metals, expand when heated and contract when cooled, and that this contraction seems to be in proportion to the intensity of the cold produced, we might infer that if we had an unlimited power of increasing the intensity of cold, we might contract any of these metals into a proportionably small space. By hammering or pressure, metals may also, to a certain degree, be compressed in bulk. Now, this contraction or compression could not take place if bodies be composed of hard, unalterable, and incompressible atoms, as Newton supposes them to be, unless we suppose these atoms merely to touch one another at some few points, leaving spaces between them unoccupied with matter. This porosity of matter is not like the porousness of charcoal and stone, which we have already considered—a property which has been proved experimentally, and of which we can obtain a sensible demonstration,—but is a hypothetical property depending upon our hypothesis of the constitution of matter by the union of hard and indestructible atoms. It may be said, however, that the celebrated experiment of the Florentine academicians, who filled a hollow sphere of gold with water, and then forcibly pressed the sphere into another shape, when the water oozed through the sphere, demon-

strated that gold really was porous, and that its pores were larger than the particles of water. Yet perhaps this experiment admits of another explanation. According to the theory of Boscovich, which we have previously explained, gold and the other metals, as well even as water and many other substances, may be regarded as destitute of porosity.

Some curious speculations have been entered into, with regard to the hypothetical porosity of bodies, which we have been considering. Newton's views on this subject have been well popularised by his friend Pemberton, in his "View of Sir I. Newton's Philosophy." Speaking of the great porosity of bodies necessarily required, on the supposition that light consists of material particles, he says that Newton has demonstrated, "how any the least portion of matter may be wrought into a body of any assigned dimensions, how great soever, and yet the pores of that body, none of them greater than any the smallest magnitude proposed at pleasure; notwithstanding which, the parts of the body shall so touch, that the body itself shall be hard and solid. The manner is this:—Suppose the body to be compounded of particles of such figures that when laid together, the pores found between them may be equal in bigness to the particles. How this may be effected, and yet the body be hard and solid, is not difficult to understand; and the pores of such a body may be made of any proposed degree of smallness. But the solid matter of a body so framed will take up only half the space, occupied by the body; and if each constituent particle be composed of other less particles, according to the same rule, the solid parts of such a body will be but a fourth part of its bulk. If every one of these lesser particles again be compounded in the same manner, the solid parts of the whole body shall be but one-eighth of its bulk; and thus, by continuing the composition, the solid parts of the body may be made to bear as small a proportion to the whole magnitude of the body as shall be desired, notwithstanding the body will be, by the contiguity of its parts, capable of being in any degree hard; which shows that this whole globe of earth, nay, all the known bodies in the universe together, as far as we know, may be compounded of no greater a portion of solid matter than might be reduced into a globe of one inch only in diameter, or even less. We see therefore how by this means bodies may easily be made rare enough to transmit light with all that freedom pellucid bodies are found to do, though what is the real structure of bodies we yet know not."

Impenetrability.—Impenetrability is that property of matter which prevents two bodies from occupying the same space at the same time. This is generally said to be a property common to all matter; but it is clear, from what we have said on porosity, that this property can only apply, in this sense, to the ultimate particles, or indivisible atoms of matter; for clearly a mass of gold is penetrable on the atomic hypothesis by matter whose atoms are small enough to enter its pores, though the atom of the gold itself may be impenetrable. This impenetrability, therefore, of matter like the porosity, considered as a general property, must be hypothetical. There are instances, however, of what is called the impenetrability of matter, which are worthy of careful consideration, as making us acquainted with other properties of matter; such experiments as these are generally adduced as proving the impenetrability of matter. If a tumbler be placed with its mouth downwards, and plunged into water or mercury, neither the water nor the mercury can rise within the tumbler, being prevented by what is called the impenetrability of the air. The same thing would occur if our tumbler were filled with steam; but, if the steam were condensed into water, or the air in the tumbler withdrawn, the water or mercury would rush in, and occupy the space contained within the tumbler.

If a mass of gold or platina be immersed in a fluid, the fluid will occupy no part of the gold or platina; but a volume of fluid, equal in volume to the gold or platina, will be displaced by then , and this is taken as a proof of the impenetrability of the gold or platina. On the Newtonian hypothesis of the density of a body being proportional to its mass, and depending on the number of gravitating atoms in a given volume, "platina contains," says Dr. Young, "in a cubic inch, above two-hundred-thousand times as many gravitating atoms as pure hydrogen gas, yet both of these mediums are free from sensible interstices, and appear to be equally continuous; and there may possibly be other substances in nature that contain, in a given space, two-hundred-thousand times as many atoms as platina, although this supposition is not positively probable in all its extent; for the earth is the densest of any of the celestial bodies with which we are fully acquainted, and the earth is only one-fourth as dense as if it were composed entirely of platina; so that we have no reason to believe that there exists in the solar system any considerable quantity of a substance even so dense as platina." The masses of both the platina and hydrogen are porous; but the particles or atoms of the hydrogen must be at a much greater distance apart from one another. The impenetrability of their masses must clearly then be due to some repulsive force exercised in the vacant space between one atom of matter and another, which prevents any other atom of matter from occupying that space, and hence we may have an impenetrability existing even where no matter occupies the impenetrable space. But all this, excepting the fact that one body, whether porous or not, may prevent another from occupying the same space which it does itself, is hypothetical.

When spirits of wine is mixed with water a contraction in bulk takes place. Thus, if a glass tube, closed at one end, be divided into a hundred parts, and half filled with water, and the upper half then filled with coloured spirit, the spirit will float on the water without mixing; if a cork be then placed in the open end of the tube, and the tube be shaken so as to insure the mixture of the liquids, they will contract, and leave about four of the hundreth parts of the tube empty. In this case, the result of the mixture alters the repulsive force of the fluids in such a manner that, though each of the fluids separately could have been compressed so as to lose the hundredth of its bulk only with great difficulty, and by the exercise of a great pressure, yet the repulsive force, by which the atoms resist this pressure, is so diminished in the mixture, that its bulk is contracted into a smaller space than it occupied before the union of the fluids.

Indestructibility.—We can alter the combinations and form of matter, but we can in no way destroy it; and though we may avail ourselves of its properties, in order to obtain an enormous force to do our bidding, and so make ourselves independent of wind and tide, and even anticipate the flight of time, we can create no new property. "One of the most obvious cases," says Sir J. Herschell, "of apparent destruction is, when anything is ground to dust and scattered to the winds. But it is one thing to grind a fabric to powder, and another to annihilate its materials: scattered as they may be, they must fall somewhere, and continue, if only as ingredients of the soil, to perform their humble but useful part in the economy of nature. The destruction produced by fire is more striking. In many cases, as in the burning of a piece of charcoal or a taper, there is no smoke—nothing visibly dissipated and carried away; the burning body wastes and disappears, while nothing seems to be produced but warmth and light, which we are not in the habit of considering as substances; and when all has disappeared, except perhaps some trifling ashes, we naturally enough suppose it is gone, lost,

destroyed. But when the question is examined more exactly, we detect, in the invisible stream of heated air which ascends from the glowing coal or flaming wax, the whole ponderable matter, only united in a new combination with the air, and dissolved in it. Yet, so far from being thereby destroyed, it is only become again what it was before it existed in the form of charcoal or wax,—an active agent in the business of the world, and a main support of vegetable and animal life, and is still susceptible of running again and again the same round, as circumstances may determine; so that, for aught we can see to the contrary, the same identical atom may lie concealed for thousands of centuries in a limestone rock; may at length be quarried, set free in the limekilns, mix with the air, be absorbed from it by plants, and in succession become a part of the frames of myriads of living beings, till some concurrence of events consign it once more to a long repose, which however no way unfits it from again resuming its former activity."

Molecular Forces.—Beside the force of gravitation, which acts at enormous distances from the particle of matter which seems to exert the force of attraction, there are many other forces, both attractive and repulsive, which seem only to come into operation when particles of matter are brought into close approximation with one another. These forces are said to act at insensible distances, while gravity acts at sensible distances, a phrase which only means that these unknown forces act at distances so small from the particles which produce them, that in general we have no means of measuring the distances. To these the name of molecular forces has been applied. The attraction and repulsion that similar or different molecules or particles of matter exercise on one another, which give rise to the solid and fluid states of bodies, determine their hardness or softness, roughness or brittleness, malleability, tenacity, expansibility, elasticity, &c., and the wide range of phenomena, classed under the head of chemical affinity, are attributed to the action of molecular forces.

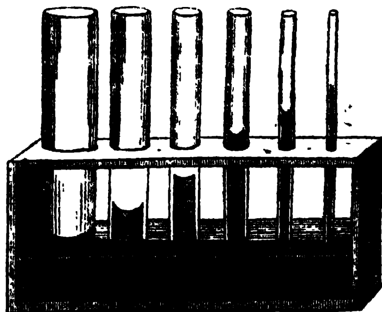
Cohesion.—The term Cohesion is applied to those molecular forces of attraction which hold together the particles of a body so as to form a mass. When this force is strong enough to resist the force of gravitation, the body is a solid; when it is so weak as not to resist the force of gravitation, except in very small masses, but is strong enough to retain the particles of the body within a certain sensible distance of one another, we have a liquid body; but when it is so weak as not to retain the particles within a sensible distance of one another, we have a vapour or gas. The change in the force of cohesion from the solid to the fluid state is so gradual, that in some bodies we find a difficulty whether to consider them solid or imperfectly fluid; and here it may not be amiss to state, that, according to the strict mathematical definition, we are not acquainted with any perfectly rigid solid body, or any perfect fluid. The force of cohesion is manifested more strongly by some bodies than others. A piece of India rubber, cut smoothly and cleanly, and the cut surfaces placed in contact with one another, will be so firmly united by this force, that the united portion will adhere as closely together as any uncut portion of the mass. Plates of glass, which have been smoothly polished and laid on one another, have become so firmly united, that it has been impossible to separate them without breaking. The united portions have been cut, and the edges polished just as if the pieces had been fused together.

That in fluids, when their masses are small, the cohesive force is strong enough to overcome the force of gravity, is manifested by the spherical shape of their drops—by the minute globules into which mercury may be divided. The force of cohesion is not confined to particles of the same substance. Different substances exercise this power on one another; sometimes even more readily than substances of the same kind. Soldering is an

instance in which we avail ourselves of this property, in order to unite metals with one another. Tinning metals is also an application of this property. In order to effect this kind of union we are sometimes obliged to use an acid or a flux, for the purpose of cleaning the surface of the metal to be soldered or tinned. Heat has the power, in most substances, of considerably modifying this force, and sometimes apparently destroying it. At certain temperatures iron and platina possess the property of welding—that is, by pressure or hammering, their surfaces may be brought so within the sphere of the force of cohesion, as to unite several pieces into a body as solid as if it had been cast.

Cohesion takes place also between solids and fluids. If a circular disc of metal, presenting a given area—say four square inches, for instance—be suspended from one end of a balance, so as to have its surface perfectly horizontal, and be accurately counterpoised by weights in a scale-pan at the other extremity of the balance; if a vessel of water, mercury, or any other fluid be placed under the disc of metal, so as to bring the surface of the disc in contact with the fluid, it will be found that a force of attraction will be exercised by the particles on the surface of the disc on those on the surface of the fluid, which will require a weight to be placed in the scale-pan to overcome it, so as to separate the surfaces. That this force is independent, in some measure, of the pressure of the atmosphere on the surface of the disc, is proved by the fact that the weight, which overcomes the cohesive force exercised by the surfaces, varies considerably when discs of different metals of the same weight and superficial surfaces are used. The comparative force of adhesion, in this manner, of gold, silver, and iron for mercury, are as the numbers 2363, 2274, and 610; but it must be observed that little reliance can be placed on these numbers as affording an absolute knowledge of the cohesive force of mercury on these metals, from the difficulty of eliminating the effect produced by the chemical action of the mercury combining with the metals whose surfaces are brought in contact with it, in some measure, and thus introducing into our results a modification due to the adhesion of the particles of mercury to one another.

Capillary Attraction.—If a glass tube be drawn so fine that its internal bore is about equal in fineness to a hair, it is called a capillary tube, from the Latin word *capillus*, a hair. On a tube of this kind being partially immersed in a fluid, the fluid in the bore of the tube in most instances will rise considerably above the surface of the fluid exterior to it. The height to which the fluid rises is dependent on the diameter of the bore of the tube, the height being greater in proportion to the smallness of the bore for the same fluid. We have here a force of attraction, exercised by the particles of the glass for the particles of the fluid, sufficient to overcome the force of gravity, and to cause the fluid to rise in opposition to it. This phenomenon is called Capillary Attraction. The height to which a fluid rises in a tube of the same bore varies for different fluids, and seems to bear no observable ratio to the specific gravity of the fluid employed. If the capillary tube be partially immersed in mercury, an opposite effect is produced, the mercury in the bore of the tube being depressed below the surface of the mercury surrounding the tube.



When the bore of the tube is considerable in size, we do not observe the rise of the fluid; but the surface of the fluid is raised from its horizontal level, where it comes in contact with the interior of the tube, so as to render the surface of the fluid concave instead of perfectly level. This subject has been profoundly and carefully investigated by the French mathematicians.

When two equal plates of glass are united together along one edge, so as to form a small angle with one another, the attraction of the particles of glass for the fluid causes a fluid, into which the plates are partially immersed, to rise in such a manner as to present a curve, which has been proved to be that conic section known by the name of the hyperbola. Capillary attraction is of frequent occurrence in nature, and of the greatest use in the arts. It is by capillary attraction that melted wax, or fat, and the oil rises in the wick of the candle or lamp. By this force the sponge possesses the useful property of absorbing water, and the blotting paper that of ink.

Endosmose and Exosmose.—Some curious phenomena, bearing some analogy to the property of capillary attraction, have been discovered by Dutrochet, which have enabled physiologists to explain some facts which before admitted of no explanation. If two fluids, capable of mixing, be separated from one another by a bladder, or some other porous diaphragm, two currents will be set up, one by the passage of the fluid A through the diaphragm D, to mix with the other fluid B, and the other by the passage of B to A. One of these currents is generally much stronger than the other, so that the bulk of the liquid on one side the diaphragm increases, while the other diminishes. Dutrochet applies the term endosmose (inward impulse) to the stronger current, exosmose (outward impulse) to the weaker. For the same fluids the endosmose or exosmose depends upon the nature of the diaphragm. Endosmose of sulphurous acid of 1·02 sp. gr. takes place to water when bladder is the diaphragm, but from water to the acid when the diaphragm is of baked earthenware not glazed. Through India rubber, which, but for the discovery of these phenomena, we might have considered impervious to water or alcohol, endosmose proceeds from alcohol to water, first slowly, but afterwards quickly, when the India rubber has been acted on by the alcohol; at the same time the alcohol becomes more and more dilute by the action of an opposite stream of water. A bladder tied over a glass filled with alcohol, swells up under water to such an extent, that when the bladder is pricked with a needle the alcohol spirts out in a long stream. In this experiment a little alcohol also passes into the water. Alcohol and water exhibit contrary actions towards India rubber and bladder, because alcohol adheres more strongly to India rubber, water to bladder.



These curious phenomena seem to have no relation to the filtering power of the diaphragm for the fluid which passes through it. We regard India rubber as the best water-tight substance we can procure. A solution of oxalic acid passes through bladder to water at all degrees of concentration and temperature,—the more it is concentrated the faster it travels; on the contrary, it filters through the bladder much more slowly than water (the slowness increasing with the concentration), if the lower surface of the

bladder be placed in contact with a solution of the same strength as that whose rate of filtration is to be determined.

Diffusion of Gases.—Another phenomenon produced by molecular attraction of the particles of matter for one another is that by which a heavy gas, whose surface is in contact with a lighter one, diffuses itself through the lighter gas in opposition to the gravity of its particles. This seems to depend upon some power of attraction which the particles of the gas seem to have for particles or atoms of gas different from themselves. Thus carbonic acid gas, a very heavy gas, diffuses itself through hydrogen, by the attraction which the molecules of hydrogen exercise on those of carbonic acid. This is a wonderful provision of nature; for were it not for the diffusibility of gases we should often be poisoned by an accumulation of carbonic acid gas on the surface of the globe or in our dwellings, just as dogs are poisoned in the Grotto del Cano, where the carbonic acid gas escapes more rapidly from the crevices of the rock than it is diffused through the air.

Catalysis.—This name, which simply means dissolving or decomposing, has been applied by Berzelius to some very obscure phenomena in chemistry, by which a body seems to exercise a force, termed by him the catalytic force, by which it decomposes or causes the combination of substances which come in contact with it without suffering alteration itself. It has been supposed that some instances of catalytic action may be referred to molecular attraction. Thus a piece of very clean platina, carefully cleansed by immersion in acid, and plunged into an explosive mixture of oxygen and hydrogen, causes the gases to explode and combine. Finely divided platina produces the same effect. What is called spongy platina becomes red-hot, and sets fire to a jet of hydrogen gas projected on it in the open air. This action is said to be due to the attraction of the particles of platina; for the particles of oxygen and hydrogen, being strong enough to counteract the mutual repulsion of the particles of the gas, are brought sufficiently close to one another to cause their combination by their mutual affinity. The power freshly-burned charcoal has of absorbing large quantities of different gases, and condensing them within its pores, may be referred perhaps to the same kind of action. Perhaps all solid bodies possess this power of attraction for gases; it is exceedingly difficult to remove from a barometer-tube the film of air which adheres to the surface of its bore, and this can only be done successfully by boiling the mercury in the tube.

Repulsion.—Beside the molecular forces of attraction which we have been considering, particles of matter exercise a repulsive force on one another, when their particles are brought sufficiently near to one another. Heat seems to be the great source of this repulsive power, and some authors have supposed every particle of matter to be surrounded by an atmosphere of heat, to which its repulsive force is to be attributed. Both the attractive and repulsive force may be exercised in the same body at the same time, as we shall see under the next head of our subject.

Elasticity.—The term elasticity is referred to that property of matter which causes a body, whose shape has been altered by the application of some external force, to resume its former shape on the removal of the cause to which its change of form was due. Thus, if a ball of India rubber, glass, or ivory, be dropped upon a hard pavement, it will rebound immediately after it strikes the pavement with considerable force, and if the spot where it strikes be covered with ink or paint, we shall have a manifest proof of the alteration in shape of the ball, produced by its fall on the pavement, by finding a much larger portion of the ball covered with the ink or paint, than could have been

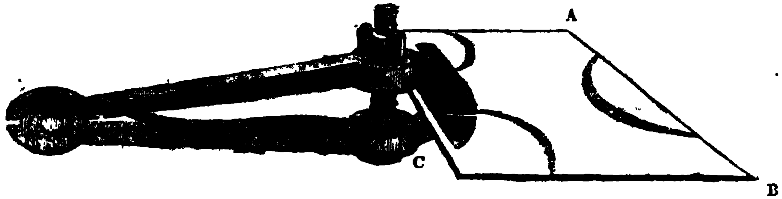
the case if it had retained its spherical shape. The force which produces the rebound is attributed to the elasticity of the ball, and is due to the repulsive force which the particles of the ball exercise on each other, when they are forcibly brought into nearer contact with one another. If this repulsive force be greater than the cohesive force which the particles exercise on one another, or if the force of the blow be sufficiently great to remove the particles beyond the sphere of their cohesive force, our ball will be fractured.

All bodies are more or less elastic, solids as well as fluids, some possessing this property in much greater degree than others. Gases and vapours are called elastic fluids, from the perfectness with which they possess this property, owing to the powerful repulsion their particles exercise on one another,—a repulsion almost without limit. Liquids appear to possess this property in so weak a degree that they are sometimes called inelastic fluids, in contradistinction to vapours and gases; but this inelasticity of fluids is only apparent, and arises from the great force of repulsion which exists between their particles, which causes them to resist any force which tends to bring their particles closer than the sphere of their mutual attraction of cohesion, with such energy that fluids can only be compressed into a smaller bulk by the action of enormous forces. This, however, has been done, and liquids have shown, by the force with which they have recovered their bulk on the removal of the force of compression, that they are highly elastic,—a fact that might have been anticipated by the facility with which liquids transmit the vibrations of a sonorous body, which is made to vibrate when immersed in them. Water conveys sound even more rapidly than air,—sound travelling through air with a velocity of about 1123 feet per second, and through water at the rate of 4708 feet in the same time. According to the experiments of M. Colladon, the sound of a bell was conveyed under water through the Lake of Geneva to the distance of about nine miles. It must be borne in mind that sounds made in the air are heard with difficulty under water, owing to the difficulty with which vibrations are communicated from one medium to another. It is to the elasticity of matter that we are indebted both for the production and communication of sound. If a bell be struck, its shape is altered, the position of its particles are changed, and their elasticity causing the body to resume the shape altered by the blow, a number of vibrations are produced. These vibrations are communicated by the elasticity and vibrations of the air to the exceedingly delicate apparatus of our ear, where their intensities are modified or increased, and communicated to our sense of hearing. That an elastic body is necessary for communicating the vibrations of the bell to our ear is proved by the common experiment of striking a bell under an exhausted receiver, when, if proper precautions be taken to prevent the communication of the vibrations of the bell to the air through the medium of the vibrations of the support of the exhausted receiver, no sound will be heard.

If the vibrations be either fewer or greater than a certain number, in a given time, no sound will be heard, though this limit of the capabilities of hearing varies in individuals; some not being able to hear a sound which is distressingly acute to others.

By the beautiful experiments of Chladni we have been made acquainted with the extraordinary vibrations which are made by a sounding body, and have been enabled to manifest them to the eye. Take a rectangular piece of glass; smooth one of its edges, A B, by grinding it on a stone with a little sand; hold it firmly by means of a common hand-vice, C, placing a little cotton wool between the glass and the vice; then, if the vice be screwed moderately, and the plate be held by the vice, and covered over

with dry sand, upon drawing a fiddle-bow across the edge, A B, a musical note will be produced. As soon as the sound is heard, the particles of sand will be violently agitated, and projected upwards a considerable distance by the vibrations of the glass,



but not equally so over the whole plate. In some parts there will be no apparent agitation, and if the note be sustained a few seconds the sand will be found arranged in beautiful and generally symmetrical figures. If the glass be held by the vice in the same position, the same figure will always appear with the same note; but if, either by different application of the bow, or by altering the part where the bow is applied, a different note is produced, the figure is immediately altered. If now a membranous body, stretched over a hoop, such as a tambourine, be covered with sand, or, what is better, moderately coarse emery, and held near the glass while the plate of glass is vibrating, it will be found that the vibrations of the glass can be conveyed by the air to the tambourine,—the vibrations of the tambourine, though untouched, manifesting themselves by the sound figures formed on its surface by the motion of the particles of sand or emery.

These experiments not only show us how general is the property of elasticity, but they also prove that the particles of a solid body are capable of being moved about one another with considerable force, without destroying the cohesion of the particles, and exceeding the limits of its action, and yet retain the characteristics of a solid body.

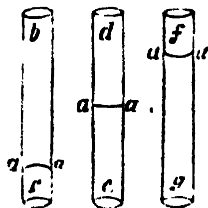
The vibrations thus excited in a solid body, by means of sound, may sometimes exceed the limits of the cohesive force; and a glass tumbler has been broken by the vibrations produced by the voice of a powerful singer. For aught we know to the contrary, the particles of all solid bodies may be in a state of continuous vibration and motion, though we may have no means of rendering their motion visible. Sound curves, which are not manifested on a vibrating glass or metal plate by sand, are brought out by the fine light dust of lycopodium; and we know, from the vibrations of the sounding-board of a musical instrument, that there are many vibrations sufficiently powerful to produce sound, and not move the fine dust of lycopodium. The vibrations which may be produced in a solid body, by the subtile forces of heat, light, and electricity, cannot be exhibited by any such rough expedients as the strewing of sand and lycopodium on their surface, but may be inferred from the beautiful experiments of the polariscope.

If we consider a body as perfectly elastic which acquires, from its elasticity, a force exactly equal and opposite to the force with which it impinges on a resisting body, and if we denominate this perfect elasticity by the number 100, Mr. Hodgkinson has shown that the elasticity of glass will be 94; of hard baked clay, 89; ivory, 81; limestone, 79; hardened steel, 79; cast-iron, 73; bell-metal, 67; cork, 65; elm-wood, across the

fibre, 60; brass, 41; lead, 20; and clay, just malleable by the hand, 17. No solid body is known which is either perfectly elastic, or perfectly inelastic.

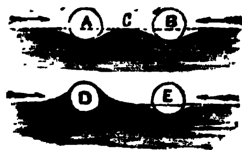
Two metals, neither of which are very elastic, may, by their combination, produce a metal capable of producing musical sounds of great sweetness by its superior elasticity. Such metals are known by the name of bell-metal. It is not necessary that a body should be hard in order to be elastic; India-rubber is an instance of a soft body being highly elastic. This substance also shows that elasticity is affected by heat; a piece of India-rubber losing, to some extent, its elasticity by the application of cold. Vulcanised India rubber is a combination of India rubber and sulphur; more elastic, and preserving its elasticity at a lower temperature than India rubber.

Capillary Repulsion.—If a capillary tube be immersed in mercury, the mercury, instead of rising in the tube, by the attraction of the sides of the tube, will be repelled, and the mercury will not rise to the height of the fluid outside the tube. The same repulsion may be shown to exist in the case where water is used, by greasing the interior of the tube, or dusting it with lycopodium. If the tube be well greased, and dusted with lycopodium, so as to cause the interior surface of the tube to repel the particles of water, the water will rise only a little way up the tube, as in *b c*; and its surface, *a a*, considerably below the surface of the fluid exterior to the tube, will be convex, as in the case of mercury. If the tube be only slightly greased, the water will rise to the level of the external fluid, and its surface be perfectly plain, as in the tube *d e*; and if the tube be perfectly clean, so as to produce the phenomena of capillary attraction, it will rise, as in the tube *f g*, and its surface will be concave.



This repulsion, exercised by the surface of one body on another, is the cause of the globular form of particles of quicksilver on surfaces which the quicksilver is incapable of wetting, and of the similar form of the drops of dew on a leaf which has not been wetted by the water. If drops of water be let fall on a surface of paper which has been greased, or dusted over with lycopodium, a like effect will be produced. A sewing needle, when perfectly dry, may be made to float with ease on the surface of water, which will be visibly repelled from the surface of the needle,—a sufficient volume of air being retained about it, to overcome the force of the gravity of the needle, and prevent it from sinking.

Small bodies floating on water apparently attract and repel one another when they are brought within a certain distance, though this attraction and repulsion is due to the capillary attraction and repulsion of the surface of the bodies and the fluid in which they float, and not exerted by the bodies on each other. Thus, if we grease two cork balls, A and B, and dust them with lycopodium powder, they will, when set upon water, repel the liquid all round, each ball reposing in a hollow space. If brought near to each other, their repulsion, exerted on the water at C, makes a complete depression, and they fall toward one another as though they were attracting each other. It is, however, the lateral pressure of the water beyond, which forces them together. Again, if one of the balls, E, is greased, and dusted with



lycopodium, and the other, D, clean, and therefore capable of being moistened, an elevation will exist all round D, and a depression round E. When placed near together the balls appear to repel each other; the action in this case, as in the former, arising from the figure of the surface of the water.

The dilatability and compressibility of solid or fluid bodies depends upon the action of the repulsive forces which their constituent particles are capable of exerting on one another, and the forces by which they are modified. These forces are shown to differ greatly, according to the composition of the bodies in which they are manifested.

Chemical Affinity.—Besides the molecular forces which we have been considering, there are a vast number which are most important in their effects, and are classed under the general name of Chemical Affinity. Most of the molecular forces which we have hitherto considered are produced by the mutual action of the particles of homogeneous bodies, or bodies composed throughout of the same kind of substance upon one another. The forces of attraction or repulsion manifested by the chemical affinities of substances for one another, are produced by the action of dissimilar substances. We refer the forces which hold a bar of iron together (and by their action cause its rigidity, and the vibrations of its particles, and to which the bar of iron owes most of its characteristic properties) to the forces of attraction and repulsion, which we have already considered; but its rusting when exposed to the air, and its solution in an acid, and the means of obtaining the iron back again from its rust, or from the salt formed by its solution in an acid, we attribute to certain unknown forces and properties of matter, designated by the term chemical affinity, or repulsion.

The consideration of chemical affinity, in all its details, would lead us through the wide domain of the science of chemistry; consequently, we can only give such a view of the subject as may make the term chemical affinity intelligible to such of our readers as are unacquainted with chemistry; referring them for further information to special treatises, but yet indicating the most important properties of matter classed under this name.

We have already mentioned that sixty-three simple substances have been discovered, which have hitherto resisted every attempt to reduce them to simpler elements, and to which we gave the name of simple, or indecomposible elements. All the bodies we meet with in nature are either composed of some one of these simple elements, or else formed by the union of two or more of them. Sometimes this union is merely mechanical: as, for instance, if we were to mix a quantity of sand and sawdust together, and unite them into a mass, by means of glue, or some other substance, the particles of sand, glue, and sawdust would remain unaltered; or the union may be of a different nature, as in the case of iron-rust, to which we have already alluded. Iron is a simple element, and a metal. Oxygen is a simple element, which as yet, in its simple form, has only been obtained as a gas. Now the oxygen is found to unite with iron in certain proportions, and form a new substance—iron-rust—which differs in all its properties from either the iron or the oxygen from which it is formed. By no mechanical means of division whatever can we separate the iron from the oxygen in the most minute particle of the rust which we can procure. Rust is an instance of a chemical combination of iron and oxygen, and is known by the scientific appellation of an oxide of iron. This is not the only oxide of iron; the chemical combination of the oxygen with the iron, to form iron rust, we attribute to certain unknown properties of the particles of matter constituting iron and oxygen, which we call their chemical affinity for one another.

There are two kinds of chemical affinity, differing from one another; one in which substances are capable of combining with one another, one of the substances being in any proportion whatever to the other, and the other in which the substances only unite in certain definite proportions. Solutions of solid substances in fluids are an instance of the former; iron-rust and water are instances of the latter.

Spirits and water are capable of mixing together in any proportions whatever; the same is true for sulphuric acid and water; while some fluids, such as oil and water, cannot be made to unite. Common salt is soluble in water; and if some salt be dissolved in water, the quantity of water may be increased to any amount; and the salt will be combined with every particle of the water; for a given portion of water, however, the converse will not hold good. After a certain amount of salt has been dissolved in the water, the water is incapable of dissolving any more, and the liquid is said to be saturated. The same holds true for many other substances than common salt: sometimes warm water, or a warm liquid, will take up more of a solid than cold, and, in a few instances, the reverse is the case. Some solids which are soluble in one liquid are insoluble in another. Camphor is soluble in spirit, and but slightly soluble in water. If we add water to a solution of camphor in spirit, the camphor will be immediately separated from the mixture in a white powder. We say that this effect is produced by the greater affinity of spirit for water than for camphor. Beside these cases there are innumerable instances in which the chemical elements combine with one another in definite proportions only, by weight, which has led to the important doctrine of chemical equivalents. We have already intimated, that if eight parts of oxygen, by weight, be mixed with one part of hydrogen, by weight, and the mixture be combined, either by passing through it an electric spark, or by any other means of igniting the mixture, an explosion will take place, and a quantity of water, equal in weight to the sum of the weights of the gases, will be formed. If either of the gases be in excess—thus, if there be nine parts of oxygen and two parts of hydrogen—only eight parts of the oxygen will combine with one part of hydrogen, and the remaining parts of oxygen and hydrogen will remain uncombined. It does not follow that these are the only proportions in which these two elements will combine. To take another instance, sixteen parts, by weight, of sulphur combine with eight parts of oxygen to form twenty-four parts, by weight, of hyposulphurous acid. The same weight of sulphur combines with twice the previous weight of oxygen to form sulphurous acid, and with three times the first weight of oxygen to form sulphuric acid. Now what we have said, for the combinations of sulphur and oxygen, is true for the combinations of every other elementary body, when they unite together to form a new chemical compound by the action of their affinity for one another. Each substance has a certain weight for itself: thus, if we take the weight of hydrogen as 1, for one unit, oxygen will be 8 very nearly (it is not exactly 8 according to the analysis of Berzelius, but 8·013, which differs from 8 by so small a fraction, that we may use 8 for it as an approximation), carbon, 6; nitrogen, 14; sulphur, 16; iron, 28; and so on.

Now it has been found, by an immense number of analyses, that these simple elements, when they combine together, always do so according to some simple multiples of their respective proportional number. The new substances, thus formed by the combination of two elementary substances, have, for their equivalent or combining weight, the weight which is the sum of the weights of their constituents; and combine either with other simple elements, or their compounds, generally in simple multiples of their equivalent weights. The elements are generally designated by chemists by the first

letter or two of their names, sometimes using the Latin names of the substances. Thus oxygen is represented by O, hydrogen by H, carbon by C, nitrogen by N, iron by Fe, gold by Au, &c. Now in this way compounds of these substances are easily expressed. water is HO, hypo-sulphurous acid SO, sulphurous acid SO^2, sulphuric acid SO^3, hypo-sulphuric acid S^2O^5; where the figures placed above the symbol of the substance shows the multiple of its equivalent weight, in which it enters into the combination. Thus, in the last instance, twice the equivalent weight of sulphur combines with five times the equivalent weight of oxygen to form the compound called Hypo-sulphuric Acid.

We have frequently mentioned that 8 parts by weight of oxygen combine with 1 part by weight of hydrogen to form water. To form a table of equivalent numbers we say that 1 is the equivalent of hydrogen and 8 of oxygen; we then find by analysis that 16 parts of sulphur by weight combine with 8 parts by weight of oxygen, twice 8, and three times 8 parts of oxygen to form three distinct compounds possessing different properties; we therefore take 16 as the equivalent number for sulphur. In the same way 28 is found to be the equivalent for iron, and the process is continued, till we find the equivalent number for every elementary substance. Instead of commencing with hydrogen for our unit, we might have chosen any other element. Tables have been constructed, in which the equivalent of oxygen is 1, in which it is 100, and in which hydrogen is taken as 1.

Dr. Prout proposed the table in which hydrogen is taken as the unit, because he supposed that all the elements combined in simple multiples of the equivalent of hydrogen. Dr. Thomson followed him, and thus constructed a table in which no fractions entered: thus, if we take 1 for the equivalent of oxygen, 0·125 is the equivalent of hydrogen, 0·75 of carbon, 1·75 of nitrogen, 2 of sulphur, and 3·5 of iron; but if we take 8 for the equivalent of oxygen, 1 is that of hydrogen, 6 of carbon, 14 of nitrogen, 16 of sulphur, and 28 of iron. The more accurate analyses of Berzelius have shown that these latter equivalents are not quite true: thus, according to Berzelius, oxygen is 8·018, carbon 6·125, sulphur 16·120; but for general purposes, where great accuracy is not required, the numbers in Thomson's table will be found very convenient. The equivalent of any body in the oxygen scale may be found approximately from that in the hydrogen scale, by dividing its equivalent in the latter by 8; thus, for hydrogen $\frac{1}{8} = 0{\cdot}125$, for carbon $\frac{6}{8} = {\cdot}75$; and any number in the hydrogen scale from the oxygen, by simply multiplying the latter by 8.

Now the forces which cause the elementary particles of matter and their combinations to unite with one another in these definite proportions, and thus to form compounds possessing distinct properties from the substances which compose them, we call the forces of chemical affinity. The molecular forces, which we have already considered, materially modify these forces. Few of the chemical forces are sufficiently powerful to overcome the cohesion of a solid body, unless one of the combining elements is either in a state of solution, or else in a fluid state. Heat, light, and electricity, possess a considerable power in modifying these chemical affinities of substances. Both heat and electricity possess the power of overcoming these forces, and resolving many of the compound substances into their simple elements. This power of decomposition is also exercised by different substances on one another. Some substances are said to possess a greater affinity for one substance than another. When one substance, on being added to a fluid in which another substance is dissolved, causes the latter to be separated from the fluid and is itself dissolved, the action is called single elective affi-

nity. Sulphuric acid has a single elective affinity for the following substances, in the order in which they stand:—

Baryta.	Lime.
Strontia.	Ammonia.
Potassa.	Magnesia.
Soda.	

Thus, among these substances, the affinity of sulphuric acid for baryta is considered the strongest, and for magnesia the weakest. None of the substances can separate baryta from its solution in sulphuric acid; all can separate magnesia from its solution. Again, soda can separate lime, ammonia, and magnesia, but not potassa, strontia, or baryta, from their solution in sulphuric acid.

Sometimes compositions and decompositions, which cannot be effected by single elective affinity, are produced by a double decomposition, which is called double elective affinity. Thus, if 130 parts of the nitrate of baryta, dissolved in water, be mixed with a solution of 88 parts of sulphate of potash, the nitric acid will leave the baryta to combine with the potash, and form 102 parts of nitrate of potash, which will remain in solution while the sulphuric acid will combine with the baryta, and 116 parts of the sulphate of baryta will be precipitated.

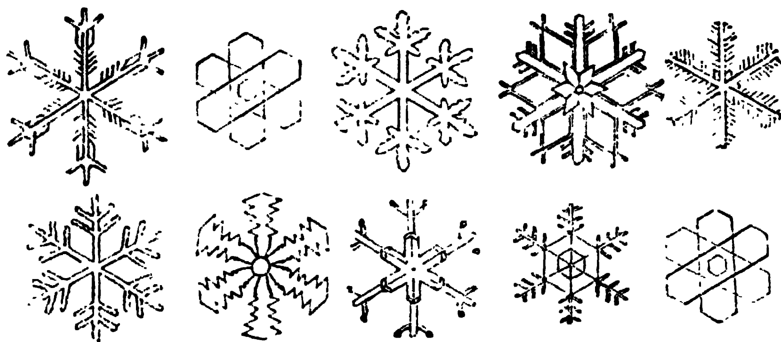
We are not enabled by these decompositions to arrive at a correct knowledge of the real strength of the chemical affinities of substances as compared with one another, as so many unknown disturbing causes may influence our results. Thus, when a stream of hydrogen gas is passed over oxide of iron heated to redness, the oxide is reduced to the metallic state, and water is generated, from which we might infer that hydrogen has a stronger affinity for iron than for oxgen. If, on the contrary, watery vapour is brought into contact with red-hot iron, the vapour is decomposed, and oxygen combines with the iron, which would lead us to conclude that the affinity of iron for oxygen was stronger than that for hydrogen. These inferences are clearly incompatible with one another, since the affinity of oxygen for the elements iron and hydrogen must be either equal or unequal. We are not enabled, therefore, to measure the relative intensity of the chemical affinities of different substances for one another.

Crystalline Force.—Bodies which assume the solid form, are either amorphous, that is, without shape or definite form; crystals, in which case their surfaces are generally plane, and inclined to one another at definite angles; or crystalline, when they are composed of an irregular aggregation of minute crystals. Barley-sugar is an instance of an amorphous body; sugar-candy, of a crystal; and a lump of loaf-sugar, of a crystalline body. Marble is an instance of crystalline form; but, if it be ground into the finest dust to which it can be possibly reduced, the particles, viewed under a powerful microscope, will present well-formed crystals, resembling those of Iceland spar. It is probable that most, if not all, substances would crystallize in passing from the fluid to the solid state, if some disturbing cause did not prevent the particles from arranging themselves symmetrically about one another, under the influence of those molecular forces which cause the body to assume the solid state. Many substances are found as crystals in the mineral kingdom, which we cannot crystallize in the laboratory. Water is frozen readily into ice, which possesses some of the attributes of a crystalline body, both as to cleavage and optical properties, without presenting us with definite forms. The crystals of water cannot be obtained artificially; but in snow they are presented in great variety and most beautiful combinations.

Crystals may be formed in various ways: by sublimation, as in the case of arsenic

and sulphur; by the evaporation or cooling of a fluid in which the solid is dissolved; and by the simple passage from the fluid to the solid state, as in the case of sulphur and most metals.

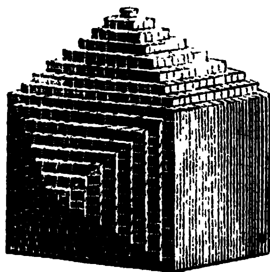
The varieties of crystals are endless; more than seven hundred different crystals



VARIOUS FORMS OF SNOW CRYSTALS.

of one substance—carbonate of lime—have been figured and described. It is found, however, that the most complicated forms may be reduced to comparatively few simple ones, by observing what simple symmetrical solids would be formed by those faces of a crystal which have a symmetrical relation to one another—if those faces were supposed to be continued, so as to form a solid.

These simple symmetrical solids are frequently found perfectly developed among the crystals of those substances from whose more complicated forms they may be derived. The simple forms thus obtained are found to arrange themselves into six distinct groups; the forms in each group being connected with one another by distinct relations and laws. These groups are termed systems of crystallization. It was supposed by Haüy, that there was a primitive form for the elementary atom of every crystallizable substance, and that all the varieties of forms presented by its crystals might be built up by the union of these primitive atoms. Thus the accompanying diagram shows how he conceived the more complicated form of the rhombic dodecahedron, a solid bounded by twelve equal and similar rhombs, might be built up of cubical particles. Haüy's primitive atoms were cubes, octohedra, rhomboids, and square or oblique prisms, according to the system to which the crystals belonged. This hypothesis of Haüy, though it is not now maintained, on account of the many difficulties which a more extended study of crystallography has presented to its reception, has led, however, to the discovery of the geometrical laws upon which the systems of crystallography are founded, and has impressed its nomenclature too firmly on chemical science to be easily eradicated. Dr. Wollaston, instead of supposing crystals to be formed of cubical or other solid particles, bounded



by plane faces, considered all the crystalline forms of the cubical system of crystals to be built up of small spheres, and the forms of the other systems by prolate and oblate spheroids. His hypothesis, however, is not more successful than that of Haüy's, in accounting for the exceedingly complicated arrangements which would have to be made, in order to build up many forms occurring very frequently among crystals.

Cleavage.—A most interesting property of crystals is, that many of them can be split or cleaved in certain definite directions, parallel to certain geometrical solids, with great ease; thus showing that there are certain directions in which the cohesive forces which hold the particles of crystals together act with less intensity than others.

The forms of crystals and their cleavages present us with the best hope, by a careful study of them, to arrive at a knowledge of the laws of molecular force; as these solids may be considered mathematical and geometrical expressions of the laws by which these forces act on the particles of a solid body.

Different substances, which are found to crystallize in forms whose solid angles present the same measurement, are called iso-morphous, that is, of *like shape*—the form of their ultimate atoms, according to the hypotheses of Haüy and Wollaston, being supposed to be identical. When the same substance is found crystallizing in forms belonging to two or three different systems of crystals, it is said to be di-morphous, or tri-morphous—that is, of two or three primitive forms thus the atoms of carbon and sulphur are di-morphous—that is, in one condition they take one primitive form, and in the other another. It is found that this change takes place in a body by an alteration of temperature. It is this fact of dimorphism which presents an almost insuperable objection to the theories of Haüy and Wollaston. It would be wiser, however, to confess our utter ignorance of the form of the ultimate particles of matter, if they have any, and patiently to collect new facts, until some more happy hypothesis may be discovered to enable us to group together a larger number of facts into some law of nature.

It is found that certain substances are capable of replacing one another in certain compounds—such, for instance, as the alums and garnets—without altering their crystalline forms. Substances which thus replace one another, are said to be isomorphous elements.

Strength of Materials.—The strength of a solid is identical with the degree in which it possesses the power of retaining its figure against the tendency of its component parts to obey the influence of gravitation or any similar disturbing force.

The effects of a force acting on a solid body are numerous; the chief being extension, compression, detrusion, flexure, torsion, alteration, and fracture. The power of resisting these effects is not to be determined, in the case of particular substances, otherwise than by experiment. A body is subjected to extension when a weight is suspended below a fixed point; and in this case a body retains its form by its cohesion, assisted by *rigidity*. When a weight is supported on a pillar placed below it, the pillar is compressed, while it plainly resists the effect by a repulsive force, but secondarily, also, by *rigidity*. Detrusion takes place when a transverse force is applied close to a fixed point, in the same manner in which the blades of a pair of scissors act on the pin; and the force which resists this operation is principally the lateral adhesion of the component parts of the substance, aided by a degree of adhesive and repulsive force. Flexure is brought about by the application of three or more forces to different parts of a substance, by which it is bent, some of its parts being extended, others compressed. In torsion, or twisting, the central particles remain in their natural state; while those

which are in opposite parts of the circumference are displaced in opposite directions. When by the operation of any, or of several, of these forces, a permanent change is effected, that kind of alteration has occurred which is termed "settling," or "taking a set." Fracture is the limit of all the before-mentioned effects.—YOUNG.

Two kinds of forces are concerned in all these effects—namely, simple pressure and impulse. The simplest way in which a body can be broken is by tearing it asunder. The cohesive force continues to be increased as long as the tenacity of the substance allows the particles to be separated from each other without a permanent alteration of form; and when this has been produced, the same force, if its action is continued, is generally capable of causing a total separation of continuity.

A body of a pound weight, falling from the height of a yard, will produce the same effect in breaking any substance as a body of three pounds falling from the height of a foot, since their momenta are equal. If the pressure of one hundred pounds break a given substance after extending it through the space of an inch, the same will break it by striking it with the velocity that would be acquired by the fall of a heavy body from the height of half an inch, and a weight of one pound would break it by falling from a height of fifty inches.

The following passage from the work of an eminent philosopher illustrates some of the difficulties in this subject:—"There is a limit beyond which the velocity of a body striking another cannot be increased without overcoming its resilience, and breaking it, however small the bulk of the first body may be; and this limit depends on the inertia of the parts of the second body, which must not be disregarded, when they are impelled with a considerable velocity. For it is demonstrable that there is a certain velocity, dependent on the nature of a substance, with which the effect of any impulse or pressure is transmitted through it; a certain portion of time, which is shorter accordingly as the body is more elastic, being required for the propagation of the force through any part of it; and if the actual velocity of an impulse be in greater proportion to this velocity than the extension or compression of which the substance is capable is to its whole length, it is obvious that a separation must be produced, since no parts can be extended or compressed which are not yet affected by the impulse, and the length of the portion affected at any instant is not sufficient to allow the required extension or compression. Thus if the velocity with which an impression is transmitted by a certain kind of wood be 15,000 feet in a second, and it be susceptible of compression to the $\frac{1}{200}$th of its length, the greatest velocity that it can resist will be 75 feet in a second, which is equal to that of a body falling from a height of about 90 feet. And by a similar comparison we may determine the velocity which will be sufficient to penetrate or break off a substance in any other manner, if we calculate the velocity required to convey the impulse from one part of the substance to the other, and ascertain the degree in which it can have its dimensions altered without fracture.

"It is easy to understand from this statement the different qualities of natural bodies with respect to hardness, softness, toughness, and brittleness. A column of chalk, capable of supporting only a pound, will perhaps be compressed by it only a thousandth part of its length: a column of India rubber, capable of suspending a pound, may be extended to more than twice its length; the India rubber will therefore resist the energy of an impulse incomparably greater than the chalk. A diamond, the hardest substance in nature, may be broken by a moderate blow with a small hammer in the direction of one of its cleavage planes. A weight of 1,000 pounds, moving with a velocity of one foot in a second, and acting on a small surface of a board, may possess sufficient energy

to break or penetrate it. With a velocity of 100 feet in a second, a weight of $\frac{1}{10}$th of a pound will possess the same energy and produce the same effect, if it act on a similar surface; but if the wood be so constituted as to be wholly incapable of resisting a velocity of 100 feet in a second, it may be penetrated by a weight of $\frac{1}{100}$th of a pound as well as by $\frac{1}{10}$th, and by a moderately soft substance as well as by a harder. The whole board, however, if at liberty, would receive a much greater momentum from the impulse of the large weight than from that of the small one, its action being continued for a much longer time. And it is for this reason that a ball shot by a pistol will perforate a sheet of paper standing upright on a table without upsetting it."

Thus, however easy it is to describe in general terms the nature of a solid as that form of matter in which the cohesive force resists gravitation and preserves form, we find it to be a very complex subject when our attention is directed to the relative strength of particular substances, and to the numerous modes in which that strength may be tried. As respects, however, the force of cohesion in solids to resist gravitation, one or two illustrations remain. If a substance be conceived to be raised in the form of a pillar to an unlimited altitude, a height will be at last attained at which its own weight will crush its base. In the same manner, if a pillar of any substance be conceived to be suspended by one extremity, there is a certain length of every substance sufficient by its own weight to snap such a pillar at its upper part. There is a certain height beyond which trees of each particular kind of wood cannot rise, owing to their own weight proving too great for the lower part of the trunk to sustain it without being crushed. In like manner, animals cannot exceed that size the weight of which can be sustained by the strength of their textures. Animals of much greater size can exist in water than on land, since the movements of the parts are sustained by the superior elasticity of the water over air.

We find physiologists laying it down as impossible for human beings to have existed on the earth of very great superior stature to the height of men in general. The argument proceeds on the assumption that the component textures of a human body could not have been much stronger than those of the present race of men. If this be admitted, it follows that, under any considerable increase in the bulk and the weight of the body the textures would have been torn asunder in the ordinary movements of the frame. In correspondence with these views is the speculation dwelt on by some writers on astronomy, that if human beings exist in the planet Jupiter they must be of very diminutive stature, since, owing to the enormous mass of that planet, the force of gravitation at its surface very far exceeds that force at the surface of our earth. On the contrary, that if human beings exist on some of the more recently discovered asteroids, such as Ceres and Vesta, the mass of which is insignificant as compared to the earth, they may be of the most gigantic height, even one hundred feet high, without experiencing any more difficulty in moving about the surface of their planet, than man finds in moving on the earth.

Heat, Light, and Electricity.—The effects of heat, light, and electricity on the properties of matter, are so numerous, and ponderable matter affects these agents in so many ways, that no description of them can be given without discussing the laws of these marvellous agents of the creation, and we must therefore reserve these interesting topics for our separate essays on Heat, Light, and Electricity.



MECHANICAL PHILOSOPHY.

Mechanical Philosophy is that branch of Natural Philosophy which investigates the laws that govern the action of force on matter. By the application of the purely mathematical sciences to a few general results, suggested by experiment and universal experience, we demonstrate these laws in their highest degree of generalization, and under all the various conditions in which force can produce an effect on matter.

As we have previously shown, in our treatise on the Properties of Matter, matter may exist in three states—the solid, liquid, and aëriform. Force we have defined to be that which either produces, or tends to produce, the motion of matter. Force may therefore be considered in relation to its effects on these three conditions of matter.

When forces produce the rest or equilibrium of the body, or system of bodies to which they are applied, we have the three following distinct subjects:—

1. **Statics**,—the rest, or equilibrium of *solid bodies* under the influence of forces.

2. **Hydrostatics**,—the rest, or equilibrium of *fluid bodies* under the influence of forces.

3. **Aerostatics**,—the rest, or equilibrium of *aeriform bodies* under the influence of forces.

When motion is the result of the application of forces to these conditions of matter, we have then these three subjects:—

1. **Dynamics**,—the motion of *solid bodies* produced by the influence of forces.

2. **Hydrodynamics**,—the motion of *fluid bodies* produced by the influence of forces.

3. **Aerodynamics**, or **Pneumatics**,—the motion of *aëriform bodies* produced by the influence of forces.

STATICS.

That branch of mechanical philosophy which treats of the rest, or equilibrium of solid bodies, when under the action of any forces, is called Statics. Statics is derived from a Greek word, στατική, the science of weight.

Material Particle.—The smallest portion into which any solid body can be conceived to be divided by any means whatever, we call a Material Particle. This material particle is an abstract mental conception, altogether independent of any theories of the ultimate division of matter, and does not therefore correspond with the atom discussed in the properties of matter. The material particle is a mere geometrical point, which we conceive destitute of every property save one—that of being set in motion, or brought to rest under the influence of forces which are supposed to act upon it. It is conceived to be destitute of form or sensible magnitude.

Rigid Body.—A rigid body is a collection or assemblage of a number of material particles, held together in an invariable form, by forces of such intensity, that no conceivable force is supposed to be capable of altering the form of the body, or disturbing the position of any of its particles.

This rigid body is also a mental abstraction, having no representative in nature. No material solid, with which we are acquainted, is perfectly rigid, though the unknown molecular forces, which hold the particles of most solid bodies together, are so great, that for many practical purposes we may regard them, within certain limits, as rigid. The various solid bodies which occur in nature, though differing from one another by many properties, seem to possess one property in common—that of being able to transmit any force applied to any one of their particles, unimpaired in intensity, through all the particles of their substance, which are in the same straight line with the direction of the application of the force. Experiment shows that most solid bodies possess this property more or less; it is this which distinguishes, in a great measure, solids from fluids,—any force applied to a particle of a fluid being conveyed through every particle throughout its substance in every direction.

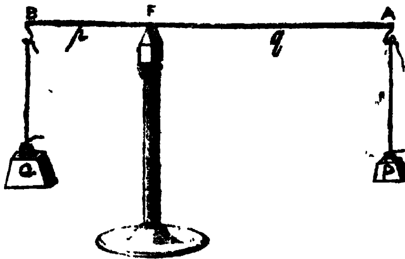
This property, which natural solid bodies possess more or less, we shall assume our rigid body to possess perfectly; and this will enable us to enunciate a principle upon which the whole science of Statics may be said to depend.

Principle of the Transmission of Force.—When a force, acting in combination with others, holds a rigid body in equilibrium, the equilibrium of the body will not be disturbed if we transfer the point of application of the force to any other point whatever in the line in which the force is acting.

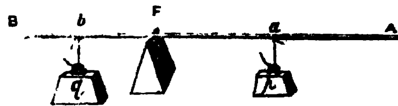
Upon this one assumption, suggested by nature and experiment, together with the abstract idea of the nature of force as capable of producing motion, and the idea already defined of a rigid body, the whole science of Statics can be built up without any further reference to nature or experiment. We must bear in mind, however, when we apply the conclusions to which the science of Statics leads us, that these conclusions will only hold true, for the solid bodies of nature, so far as they possess the properties of perfect rigidity, and the perfect transmission of force. Within these limits we may apply our science to the action of forces on natural bodies with accuracy.

Statics an Abstract Science.—Statics is an abstract mathematical science, and many of its results cannot be directly confirmed by experiment. Thus one of the simplest propositions, which we shall hereafter prove, is, that if two weights, P and Q, be suspended from the extremities of a rigid rod, A B, resting on the sharp edge of a

support at F, the weights P and Q will balance one another, and the rod A B will be in a state of equilibrium, provided A F be as many inches in length as Q is ounces in weight, and F B as many inches in length as P is ounces in weight. This proposition cannot be proved by experiment, since it is only true on the supposition that A B is destitute of weight, and no bar, whose weight is inappreciable, can be found sufficiently rigid to support the weights P and Q however small, and still remain perfectly straight under their pressure, and that of the sharp edge at F. It is true that another proposition of Statics would enable us to take into account the weight of the bar A B.



If the bar A B were uniform throughout in thickness and material, the bar would balance itself on the edge F, provided a weight q, equal the weight of the portion of the bar A F, was suspended from the point b, F b being equal the half of A F; and a weight, p, equal the weight of the portion of the bar F B, suspended from a, F a being equal the half of B F.



Supposing the difficulty of the weight of the bar thus obviated, there would still remain another difficulty in the way of a perfect experimental proof of our simple proposition. According to theory, the slightest additional weight, added to either of the weights P or Q, ought to destroy the equilibrium of the bar A B, and alter its perfectly horizontal position. But this supposes that the edge of the support F will exercise no friction on the surface of the rod A B. By a proper choice of materials, for the rod and the support, and also of the angle of the edge of the support, this friction may be considerably diminished, but it can never be absolutely destroyed. If we could succeed in fulfilling all the conditions we have indicated as necessary for the experimental proof of the very simple proposition of our science, which we have enunciated, we should be able to construct a perfect balance. That these difficulties are real, and not merely imaginary, is proved by the fact that a balance, sufficiently sensitive and accurate to supply the wants of the modern analytical chemist, is a very expensive instrument, and requires great skill and accuracy, as well as scientific knowledge, for its construction.

Though we cannot appeal to experiment for a rigid proof of our propositions, if we make the necessary allowances for the difference between the bodies on which our experiments are conducted, and the imaginary bodies which our science supposes perfectly rigid, and so forth, careful experiments will serve not only to give us clearer views of our science, but also to confirm our confidence in our abstract reasonings, by the approximate coincidence of our experimental results with our theoretical conclusions.

Thus, if in the experiment before alluded to, we make the necessary allowance for the weight of the bar, by applying the small weights p and q to the points a and b, our experiments will approximate to the theoretical proposition in proportion as the friction of F on the bar is diminished, and the bar A B is perfectly straight and accurately divided at the points a and b.

Rest or Equilibrium.—Having defined a material particle and a rigid solid body, and their theoretical properties and condition, we must next consider what we

mean by their rest or motion. We say that a body or a particle is in motion when we perceive the body or particle to change its position, and that it is at rest when it does not change its position. This clearly implies, however, that we have some means of measuring this alteration or change of position, by reference to other bodies which we conceive to be at rest. Hence it happens that what we suppose to be rest and motion, are generally only apparent rest and motion, and that there is no subject on which our senses more frequently lead us to form erroneous conclusions than when we attempt to discriminate the real or absolute rest and motion of objects from their apparent rest or motion. When we see a body change its position relatively to other bodies, we cannot tell whether the body which seems to move really does so or not, till we know whether the bodies relatively to which it appears to change its position, are themselves in a state of rest, or whether they are all moving together. It may so happen that the bodies may be moving, though we are ourselves unconscious of their motion, and in this case the body which apparently moves may be really at rest. Instances of this kind are of daily occurrence. A person on board ship, where his own body, as well as all the surrounding parts of the ship are apparently at rest, sees all the objects which the ship passes, carried, as it were, past him with great velocity; and were he not conscious, from other considerations, that the ship on which he stands was really in motion, he would have some difficulty in conceiving that the apparent motion of the stationary objects the ship passes by, was not real instead of imaginary. This is strikingly illustrated by looking out of a railway carriage, when moving quickly: the objects, such as trees and houses, near the carriage, are apparently carried by the window with great rapidity, while those at a greater distance seem to pass more slowly. If, while waiting at a station, a train passes slowly by, it is often very difficult to tell whether the train in which we are seated is at rest or in motion, unless we correct our impression by looking out of the opposite window, where we can compare the position of the train with the fixed objects of the station.

Apparent Rest or Motion of Heavenly Bodies.—A popular knowledge of astronomy is now so common, that we have no difficulty in persuading persons that the apparent motion of the sun and stars in the heavens is not real. Yet this is a fact which can only be demonstrated by a long course of intricate reasoning, founded on an immense number of careful observations.

To an ordinary observer, the vault of heaven, studded with stars, appears to revolve round an imaginary axis, while the earth seems perfectly at rest; each of the fixed stars describes a circle which brings it back to the position in which it was first observed in an invariable interval of about 23 hours 56 minutes. What are called the fixed stars seem all to move together with the vault of heaven, and never to change their relative positions with respect to one another. The planets and comets, as well as the sun and moon, have an apparent motion among the fixed stars. In the case of the planets, comets, and moon, this apparent motion is exceedingly complex. Sometimes a planet will appear to move forward with great rapidity among the stars of a constellation; then it will come, as it were, to a stand still, remain some time at rest, and then move backward. Ancient astronomers were well acquainted with the phenomena of real and apparent motion, yet after centuries of painful research, and the invention of most complicated motions, to account for the apparent paths of the planets on the vault of heaven, without avail, it was reserved for the genius of Copernicus and Kepler to resolve these motions into simpler ones, on the simple hypothesis that all the planets were moving round the sun in orbits nearly elliptical, and never at rest.

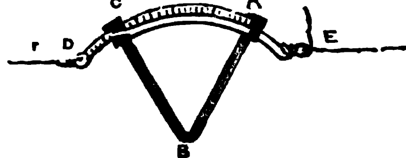
Astronomy teaches us that every object on the earth's surface which appears to us at rest, is really subject, along with the earth on which it is placed, to two motions; one by which it is carried round the earth's axis by its diurnal rotation, and another by which it describes the earth's orbit round the sun in a year.

From these considerations it appears that there can be no absolute rest for any particle of matter by which we are surrounded, that perpetual motion is the real condition of all material objects, and that when we apply the reasonings and principles of the science of statics to nature, it is the apparent and not real rest of bodies to which we must have regard.

In the application of the principles and conclusions of statics to nature, we may consider the earth as absolutely fixed and at rest, and neglect its real motion without introducing any sensible error into our experiments. According to the law of inertia, considered in the "Properties of Matter," all the bodies on the earth will partake of the earth's motion, and we know of no force which will deprive them of this motion; if, therefore, any forces produce the relative motion or rest of a terrestrial body, compared with bodies fixed with regard to the earth, that relative rest or motion will be the same whether the earth be really at rest or in motion. A familiar instance may make this assertion clear. If a ship under the influence of steam and tide be moving rapidly through smooth water, all mechanical powers, such as blocks and pullies, the windlass, and all the parts of the most complicated steam-engine, will act under precisely the same circumstances, exert the same forces, be brought to a state of apparent rest or motion compared with the ship as a fixed object, whether the ship be at rest or in motion. A game of billiards, which requires considerable practical skill in mechanical science, may be played in the cabin of a ship without the players being aware whether the ship is at rest or in motion. Though, with respect to the earth, the balls, which were apparently at rest while the ship was moving, would be in reality not at rest, but moving with the ship.

Measure of Force—Dynamometers.—We regard all forces as equal which produce the same mechanical effects, and in statics we consider forces as equal, which produce the same effects, when applied in a similar manner, in bringing a body into a state of equilibrium. Instruments, by means of which these effects are estimated, are called dynamometers, or force measurers.

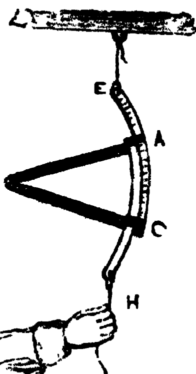
A very simple dynamometer consists of a thin flat bar of steel, A B C, bent into an angle at B, and properly tempered, so that if any force be applied at A and C, to bring the extremities nearer together, the force of elasticity of the steel will cause the extremities, A and C, to resume their position as soon as the forces are removed. The greater the forces applied



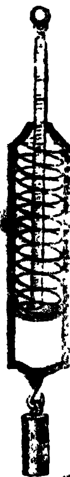
to A and C, the nearer these extremities of the bar will be brought to one another. To measure this effect, a circular arc of metal, A C D, is fixed perpendicularly to the surface of the steel bar at A, passing freely through an opening in the other extremity of the bar at C. Another arc, C A E, is similarly fixed at C, passing through an opening at A. Rings are fixed at D and E for the convenient application of forces whose effects are to be estimated; and one of the arcs, A D, is graduated by a number of equal divisions.

If the extremity E of the dynamometer be fixed to a beam, and a weight, P, hung

from the ring at D, the graduated arc will show the statical effects of the weight P in bringing the dynamometer to a state of rest, with the extremities of the bent bar nearer to each other than they were before the application of the weight P. If, instead of applying a weight P, we pull with our hand a string attached to D, till we bring the graduated part of the arc at A to the same division, to which the weight P brought it, we produce with our hand a similar statical effect to that produced by the weight. We therefore conclude that the forces exerted by the weight and by the hand are equal to one another, though they are evidently forces of a very different character, the force of the weight being produced by the earth's attraction for the particles of matter of which it is composed, and the force exerted by the arm being derived from the contractions of the muscular fibres of the muscles of the arm, due to some force which we call the muscular force, of the nature of which we are perfectly ignorant.



There are many other dynamometers besides that just described, such as one in which the statical effects of a force causing a spiral spring of wire contained in a metal cylinder to be compressed, is measured on a graduated rod, as in the accompanying figure. The various kinds of balances and steel yards, which will be described hereafter, are all dynamometers.



Unit of Force.—The experiment on the dynamometer, just described, would lead us to infer, what is found to be true in practice, that weight is the most convenient measure of force which we can adopt in statics. We say that a force of ten or twelve pounds is exerted on a body, if the force produces the same statical effect on the body which a weight of ten or twelve pounds applied in a similar way would produce. The unit of force generally adopted in this country is that called a pound weight. But here we may be met with the inquiry, what is a pound weight? This is a very important question, and has been determined by legislative enactment, after careful deliberation, and the report of a scientific commission appointed for the consideration of the difficult subject of standard weights and measures.

Pounds Troy and Avoirdupois.—By the Act of Parliament (5th George IV., c. 74, passed in the year 1824, sections 4 and 5,) it is enacted—"That from and after the first day of May, 1825, the standard brass weight of one pound troy weight, made in the year of 1758, now in the custody of the clerk of the House of Commons, shall be, and the same is hereby declared to be, the original and genuine standard of measure of weight, and that such brass weight shall be, and is hereby declared to be, the original and genuine standard measure of weight, and that such brass weight shall be and is hereby denominated the Imperial Standard Troy Pound, and shall be, and is hereby declared to be, the unit or only standard measure of weight from which all other weights shall be derived, computed, and ascertained, and that one twelfth part of the said troy pound shall be an ounce, and that one twentieth

part of such ounce shall be a pennyweight, and that one twenty-fourth part of such pennyweight shall be a grain; so that 5760 such grains shall be a troy pound, and that 7000 such grains shall be, and they are hereby declared to be, a pound avoirdupois, and that one sixteenth part of the said pound avoirdupois shall be an ounce avoirdupois, and that one sixteenth part of such ounce shall be a dram.

"And whereas it is expedient that the said standard troy pound, if lost, destroyed, defaced, or otherwise injured, should be restored of the same weight by reference to some invariable natural standard; and whereas it has been ascertained by the commissioners appointed by his Majesty to inquire into the subject of weights and measures, that a cubic inch of distilled water weighed in air by brass weights at the temperature of 62 degrees of Fahrenheit's thermometer, the barometer being at 30 inches, is equal to 252 grains, and 456 thousandth parts of a grain, of which, as aforesaid, the Imperial Standard Troy Pound contains 5760: Be it therefore enacted, that if at any time hereafter the said Imperial Troy Pound shall be lost, or shall in any manner be destroyed, defaced, or otherwise injured, it shall and may be restored by making, under the direction of the Lord High Treasurer, or the Commissioners of His Majesty's Treasury of the United Kingdom of Great Britain and Ireland, or any three of them for the time being, a new standard troy pound, bearing the same proportion to the weight of a cubic inch of distilled water, as the said standard pound hereby established bears to such cubic inch of water."

From this Act of Parliament it appears that the troy pound is a certain arbitrary piece of brass which, weighed in air at a temperature of 62° Fahrenheit, the mercury in the barometer standing at a height of 30 inches, is equal to the weight of 22 cubic inches, and 815 thousand parts of a cubic inch of distilled water. Distilled water is therefore the natural standard to which the unit of weight is ultimately referred. This standard, however, is referred to a cubic inch, or a volume of water an inch in height, an inch in breadth, and an inch in depth. We must, therefore, have some standard measure of length.

Unit of Length.—The Act of Parliament just cited has also determined the standard unit of the measure of length. By the first section of that act, it is enacted —"That from and after the 1st day of May, 1825, the straight line, or distance between the centres of the two points in the gold studs in the straight brass rod, now in the custody of the clerk of the House of Commons, whereon the words and figures, 'Standard yard 1760' are engraved, shall be, and the same is hereby declared to be, the original and genuine standard of that measure of length or lineal extension called a yard, and that the same straight line or distance between the centres of the said two points in the said gold studs in the said brass rod, the brass being at the temperature of 62 degrees by Fahrenheit's thermometer, shall be and is hereby denominated the 'Imperial Standard Yard,' and shall be, and is hereby declared to be, the unit or only standard measure of extension, wherefrom or whereby all other measures of extension whatsoever, whether the same be lineal, superficial, or solid, shall be derived, computed, or ascertained; and that all measures of length shall be taken in parts or multiples, or certain proportions of the said standard yard; and that one third part of the said standard yard shall be a foot, and the twelfth part of such foot shall be an inch, and that the pole or perch in length shall contain 5 such yards and a half, the furlong 220 such yards, and the mile 1760 such yards.

"And whereas the said standard yard, if lost, destroyed, defaced, or otherwise injured, should be restored of the same length by reference to some invariable natural

standard; and whereas it ha been ascertained, by the commissioners appointed by his Majesty to inquire into the subject of weights and measures, that the said yard hereby declared to be the Imperial Standard Yard, when compared with a pendulum vibrating seconds of mean time in the latitude of London, in a vacuum at the level of the sea, is in the proportion of 36 to 39 inches, and 1393 ten thousandths part of an inch: Be it therefore enacted and declared, That if at any time hereafter the said imperial standard yard shall be in any manner destroyed, defaced, or otherwise injured, it shall and may be restored by making, under the direction of the Lord High Treasurer, or the Commissioners of His Majesty's Treasury of the United Kingdom of Great Britain and Ireland, or any three of them, for the time being, a new standard yard bearing the same proportion to such pendulum as aforesaid, as the said imperial standard yard bears to such pendulum."

Unit of Time.—Time is the ultimate natural standard to which all measures of length, weight, and capacity are referred in this kingdom. Time certainly is one of the most convenient and perhaps the only natural standard to which we can have recourse to fix with scientific accuracy our units of measurement. The value of time in this respect depends upon the fact deduced by astronomers, from observations collected and registered for many centuries, that the mean length of day and night is invariable; in other words, that the earth always completes its rotation on its axis in the same period of time. This portion of time is obtained by observing the period which elapses between the passage of a fixed star over a certain imaginary line in the heavens, called the meridian of the place of observation, and its next appearance on this line, and is called a sidereal day. The sidereal day is divided into twenty-four hours, each hour into sixty minutes, and each minute into sixty seconds. A clock is an instrument for measuring time, and is set in motion by a weight; its motion is regulated by the vibrations of a pendulum, and the number of its vibrations are registered by means of a series of wheels, and indicated on the graduated face of the clock by the motion of hands or pointers. The length of the pendulum is so regulated that its time of vibration may be as nearly as possible one second, and these vibrations being registered by the clock, if the clock indicates that 24 hours, or $24 \times 60 \times 60$, or 86400 seconds have elapsed between one transit of the star over the meridian and another, the clock is said to be correct, and will afford us an accurate measure of sidereal time. Such a clock is called a sidereal clock, and is one of the most valuable and useful instruments in an observatory. Sidereal time is not, however, the time used for the ordinary transactions of life. The true solar day is the interval which elapses between one passage of the sun over the meridian and another, as shown by a dial or other means of astronomical observation; this day is not invariable in length, but changes from day to day, being sometimes longer and sometimes shorter. To avoid this inconvenience, a mean solar day is chosen by the supposed revolution of a fictitious sun, which shall be invariable in length; and this time is divided into 24 hours, and these hours into minutes and seconds, as in the case of the sidereal day. This mean solar day is the time used for the ordinary or civil reckoning of time. What is called the equation of time, is an astronomical calculation, which shows the difference between the time shown by the dial or the true solar time, and that indicated by the ordinary clocks, which show the mean solar or civil measure of time. The sidereal day is 23 hours, 56 minutes, 4 seconds, and 9 hundredths of a second of a mean solar day, and the mean solar day is 24 hours, 3 minutes, 56 seconds, and 55 hundredth parts of a second of a sidereal day. The pendulum vibrating seconds, from which, by Act of Parliament, the standard measure of length,

if lost, is to be obtained, is to be a pendulum vibrating a second of mean solar time under the conditions specified in the act. So great, however, are the scientific and mechanical difficulties to be overcome in determining accurately the length of this second's pendulum, that though the standard yard was so injured by the fire when the houses of Parliament were burnt down in 1834, and the standard pound troy altogether lost, no attempt has been made to restore the lost standards. The last scientific commission seems to have considered the attempt to do so altogether hopeless. Fortunately, the Royal Astronomical Society had a very beautiful scale constructed about the year 1832, and three feet of this scale were compared many hundreds of times with the Parliamentary standard, and this scale must now be considered the scientific English standard, and the best evidence of the parliamentary standard which exists. There is also great doubt as to the accuracy of Captain Kater's determination of the length of the second's pendulum which is adopted by the Act of Parliament; so that if the standard were to be restored according to that act, it would probably differ from the lost standard (Bailey's Report on a Standard Scale, Astronom. Soc. Mem., vol. ix.) Two Acts of Parliament, 5 and 6 Wm. IV., c. 63, and 16 Vic., c. 29, June, 1853, have been enacted since the fire, on the subject of weights and measures; but neither of them notice the loss of the standards, speaking of them as if they were still in existence.

French Standards.—The French standards are derived from actual measurements of the earth's surface. From these measurements the length of a line drawn from the pole of the earth to the equator is deduced. The ten-millionth part of this line is called a metre; and this is the French standard of length. The deca-metre is 10 metres, the hecto-metre is 100 metres, the kilo-metre 1000 metres, and the myriametre is 10,000; while the deci-metre is the 10th part of a metre, the centi-metre the 100th part of a metre, and the milli-metre the 1000th part of a metre—Greek prefixes being used for the multiples of the metre, and Latin for its parts. The gramme, which is the standard of weight, is derived from the standard of length, and is equal to the weight of a cubic centimetre of distilled water, weighed at the freezing point. The same prefixes are used before the gramme, which are added to the metre to express its multiples and parts. Thus a kilo-gramme is a 1000 grammes, and a milli-gramme the 1000th part of a gramme.

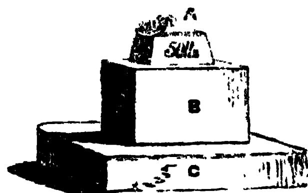
The metre is equal to 3·280899 English feet, and the kilo-gramme is equal to 3·204597 pounds avoirdupois.

Pressure and Tension.—When a material body is in a state of equilibrium under the influence of forces, the forces applied to the body may either have a tendency to press the particles of the body or to crush it, in which case the forces are called pressures, or they may have a tendency to separate the particles of the body or tear it, in which case they are denominated tensions. A weight placed on a body exerts a pressure on it. Two men pulling a rope, or weights suspended from a rope, exert a tension on the rope throughout its substance; and if one part of the rope be weaker than another, and the weight or force be sufficiently great, the rope will break or be torn asunder at that part. The tie-beams of a roof, which prevent the weight of the roof from thrusting the walls of a building out of the perpendicular, are under tension, while the walls support the pressure of the roof. It is of great importance, practically, to distinguish between pressure and tension; for some substances will bear a large amount of pressure without injury, but will be torn asunder by a far less amount of tension.

Flexible Cords.—In theroretical statics, as we conceive our solid bodies to be

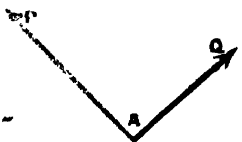
perfectly rigid, so we conceive the strings which support our weights to be perfectly flexible, and at the same time perfectly inextensible. It is needless to remark that no such cords or strings are to be found in nature; but these hypothetical bodies enable us to divest our problems of many difficulties, and to arrive at conclusions which may afterwards be used in practice with great accuracy, when experiment has enabled us to determine the want of flexibility in the material we use. For the same reason, friction, or the resistance which surfaces not perfectly smooth oppose to the motion of a body over them, is at first neglected in our problems.

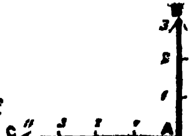
Action and Reaction.—It is an axiom of statics—that is, it is a self-evident truth, or one which admits of no other proof than universal experience—that whatever force one rigid body exerts upon another rigid body, the latter opposes that force by an equal force, which is called its reaction. Thus if a beam of wood, B, standing upright on a floor, C, supports on one of its extremities a 50 lb. weight, A, the weight A will exert a pressure of 50 lbs., acting downwards, on the beam, and the beam will convey this pressure to the floor. But the rigidity of the beam opposes a force to the weight which prevents the weight from falling, or crushing the beam; and thus a reaction equal to 50 lbs. is exerted upwards by the beam upon the weight. Again, the beam presses on the floor with its own weight, in addition to that of the 50 lb. weight; and if the floor be strong enough, and of a material sufficiently rigid to prevent B sinking into it, the floor will sustain the pressure of A and B, which acts downwards; but it will re-act upwards on B with a force equal to the pressure B exerts upon it.



Equilibrium of a Material Particle.—If a material particle be acted upon by two forces which are equal to one another, but acting in opposite directions in the same straight line, they will neutralise one another, and the particle will be at rest. This is self-evident, and depends upon our fundamental idea of the equality of forces. Supposing, however, that the two forces do not act on the particle in the same straight line, but in the direction of straight lines inclined to one another at some angle, in what direction must a force be applied to the particle, and of what magnitude must it be, to neutralise the effect of these two forces, both in the case where the two forces are equal, and also where they are unequal? This is one of the most important, and indeed the fundamental proposition of statics. Before we can discover it we must adopt some means of representing forces and their directions.

Geometrical Representation of Forces.—If we represent the material point by a geometrical point A, we may draw a line, A P, to represent the direction in which a force, say of P pounds, is acting on the particle, and A Q to represent the direction in which a force of Q pounds is acting on A; then if we take A P, P inches in length, and A Q, Q inches in length, the lines A P and A Q will represent the forces P and Q acting on A, both in magnitude and direction. Thus if two forces, one of 4 lbs. and another of 3 lbs., are supposed to act on a particle at right angles, or perpendicular to one another, we should represent them by the lines A B, 3 inches in length, and A C, 4 inches in length, drawn perpendicularly to one another.



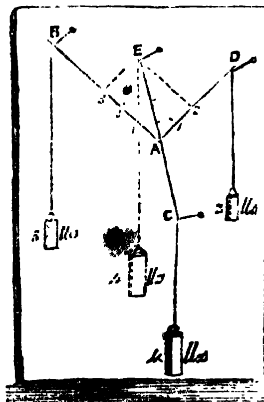


It is not necessary to use an inch as the representative of a pound, or the unit of weight. Any other convenient measure, such as the eighth or tenth of an inch, may be used, if we keep to the same scale throughout. It is usual to indicate the direction in which the force acts by the addition of an arrow-head to the line which represents it.

Resultant.—If two forces, P and Q, represented in magnitude and direction by the lines A P and A Q, act upon the point A, so as to cause it to move, it must begin to move in some direction. Let A R represent that direction. Now we can conceive that some force, S, could be applied to A in the opposite direction to A R, in which A would begin to move, which would stop its motion and keep it at rest. Let A S represent this force in magnitude and direction. Then the particle A would be kept at rest by the three forces, P, Q, and S, acting in the directions A P, A Q, and A S. If, now, the line S A be produced to R, and A R be taken equal to A S, A R will represent in magnitude and direction a force, R, equal to S, which, if P and Q were removed, would counteract the effect of the force represented by A S on A. A R, therefore, represents in magnitude and direction a force which would have the same effect on A, when it acted upon it by itself, that the forces represented by A P and A Q would have if they both acted on A.

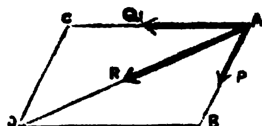
The force R, represented by A R, is called the *resultant* of the forces P and Q, represented by A P and A Q; and P and Q are called the *components* of R.

Thus, for example, if three perfectly smooth pegs, B, C, and D, be inserted in a board standing in a vertical position, and three strings be attached to a point, or simply knotted together at A, and weights of 2 lb., 3 lb., and 4 lb. fixed to their extremities, as in the figure, the weights will balance each other; and, neglecting the friction on the pegs B, C, and D, and supposing the strings perfectly flexible, the whole will come to rest in the position represented by the diagram. The point or knot A will be kept at rest by the tensions of the weights—that is, by a force of 3 lbs. acting in the direction of A B, 2 lbs. in the direction of A D, and 4 lbs. in that of A C. Let another peg, E, be inserted in the board, somewhere in the straight line C A produced, attach another string to A, and pass it over A E, with a weight of 4 lbs. at its other extremity, care being taken to support this latter weight so as not to allow it to act on A till we require it. If, now, the weights of 2 lb. and 3 lb. be supported so as to take off their tensions from A at the same instant that the second 4 lb. weight is allowed to exert its tension on A in the direction A E, the equilibrium of the point A will not be disturbed, since the tension of 4 lbs. acting in the direction A E will exactly balance the tension of the 4 lb. acting in the direction A C. The tension of 4 lb. acting in the direction A E produces, therefore, the same effect on the point A that the joint tensions of 3 lb. acting in the direction A B, and of 2 lb. in the direction A D, both together have upon it. The 4 lb. tension, acting in the direction A E, is called the *resultant* of the 3 lb. tension in the direction A B, and the 2 lb. tension in that of A D.



PROPOSITION I.

Parallelogram of Forces.—The proposition which enables us to represent the resultant of any two forces which act upon a material particle in magnitude and direction, when the magnitude and direction of the two forces are given, is called the parallelogram of forces, and is as follows:—



Let P and Q be two forces acting upon material point A.

Let the line AB represent the force P in magnitude and direction, and AC the force Q in magnitude and direction.

Through B draw BD parallel to AC and through C, CD parallel to AB.

Let D be the point where the lines BD and CD meet.

Then by construction the figure ABDC is a parallelogram.

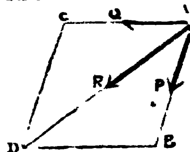
Join AD, AD is the diagonal of the parallelogram.

This diagonal AD will represent the resultant R of the two forces P and Q acting in the directions AB and AC, in magnitude and direction.

The parallelogram of forces may therefore be thus enunciated. *If two forces acting upon a material particle be represented in magnitude and direction by two adjacent sides of a parallelogram, the diagonal of the parallelogram, drawn through the point where these sides meet, will represent their resultant both in magnitude and direction.*

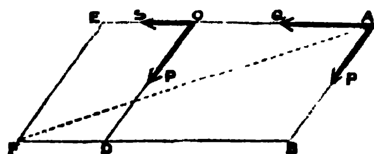
We shall first show that this proposition is true for the *direction* of the resultant, and then that it is also true for its *magnitude*.

1st. To prove that the parallelogram of forces is true for the *direction* of the resultant.



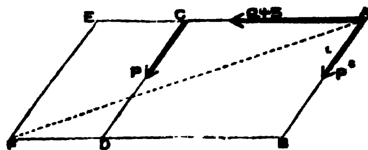
When the forces P and Q are equal, the direction of the resultant will manifestly bisect the angle BAC, since no reason can be alleged why the resultant force R should incline more to one force, P, than to the other, Q. Since ABCD is a parallelogram whose sides AC and AB are equal, its diagonal, AD, will bisect the angle BAC, and therefore AD will represent the *direction* of the resultant when P and Q are equal.

Let us now assume the proposition to be true for two unequal forces P and Q, and also for two unequal forces P and S, we can then prove that it will be true for the forces P and Q + S.



Draw a parallelogram A B D C, having one side A B proportional to the force P and the adjacent side AC to that of Q, produce AC to E and make CE in the same proportion to S that the other lines bear to P and Q. Complete the parallelogram E D.

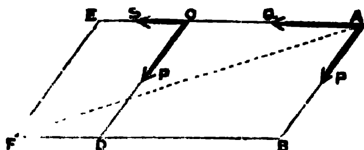
According to our assumption the resultant of P and Q will act in the direction AD, and that of P and S in the direction CF. Now if all the points in the two parallelograms be supposed to be rigidly connected with one another, a force may be transferred from any point to another, provided the latter be in the same straight line in which the force is acting, without disturbing the equilibrium.

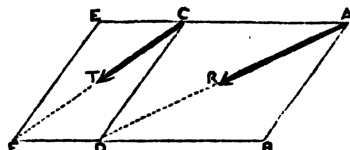


Hence the force S acting at C may be transferred to A, and we shall then have a

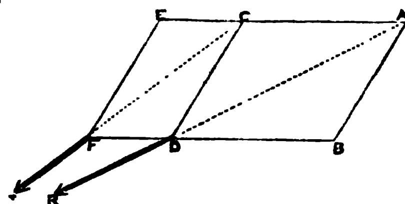
force Q + S acting on A in the direction A C, a force P acting on A in the direction A B, and a force P acting on C in the direction C D.

If the proposition be true for P and Q + S their resultant will act in the direction A F, and these forces may be transferred from A to F. What we have therefore to show is, that the forces P and Q acting at A, and P and S acting at C, may be so transferred, by the principle of transmission of force, without altering the conditions of equilibrium that we may have the forces Q + S and P acting at F and P at C.



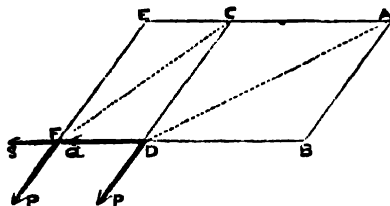


By our assumption P and Q acting at A may be replaced by their resultant R acting in the diagonal A D of the parallelogram C B, and the forces P and S by their resultant T acting in the diagonal C F of the parallelogram E D.

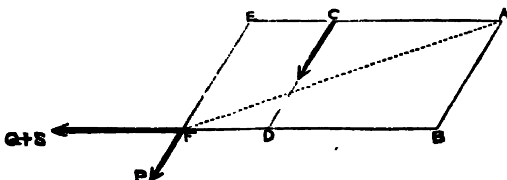


Now, according to the principle of the transmission of force, the force R may be transferred from A to D in the direction A D, and the force T from C to F in the direction C F, as represented in the accompanying diagram.

Upon the same principle that we replaced the forces by their resultants we may replace these resultants, again by the forces from which they were obtained; and we shall then have R replaced by the force P acting in the direction of C D produced and Q in that of D F, and T replaced by P acting in E F produced, and S acting in D F produced.



Lastly, the force Q may be transferred from D to F in the direction of D F produced, and the force P from D to C in the direction C D; so that we ultimately have the forces Q + S and P acting at F, and P at C, without having altered the conditions of equilibrium of any of the points of the parallelograms. Hence, if our proposition be true for two forces P and Q, and also for P and S, as regards the *direction* of the resultant, it is also true for the forces P and Q + S.

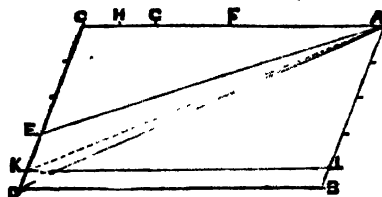


Now, we have shown that it is true for two equal forces, hence it is true for P and P, and also for P and P, it must therefore be true for P and P + P, or for P and 2 P. Being true for P and 2 P, and also for P and P, it is true for P and P + 2 P, or for P and 3 P, and so on; it may be shown to be true for P and *m*P, where *m* represents any whole number.

By similar reasoning the proposition may be extended to the forces nP and mP, where m and n represents any whole numbers whatever.

Hence the proposition is true, as to the *direction* of the result, for any two forces which are commensurable; or in other words, for any two forces which have a common measure, or can be expressed in terms of a common unit.

The proposition can be extended to the case of incommensurable forces, or forces which have no common measure.



Let the lines A B and A C represent two incommensurable forces, in magnitude and direction, of which A C is the greater. Complete the parallelogram A B D C, by drawing B D parallel to A C, and C D parallel to A B. If the resultant does not act in the direction of the diagonal A D of the parallelogram A B D C, let A E represent the direction in which it does act.

Divide the line A C in two equal parts in the point F; similarly divide F C in two equal parts in G, and G C in H; continue this subdivision until a part, such as H C, is obtained, which is less than D E.

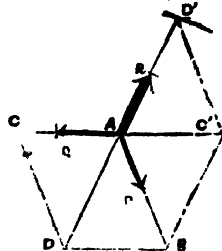
A C may, therefore, be divided into a number of equal parts, each of which are equal to H C.

Set off distances each equal to H C along the line C D, commencing from the point C; then one of these divisions, such as K, must fall between E and D, since A C and C D have no common measure, and H C is less than E D.

From A B cut off A L = C K, and join K L and A K.

A C and A L will therefore have a common measure H C, and will consequently represent two commensurable forces; and A K, the diagonal of the parallelogram A L K C, will, by what we have previously proved, represent the direction of their resultant. The resultant A K, therefore, of the forces A L and A C, is further from A C than A E, the resultant of the forces A C and A B; but this cannot be true, since A B, being greater than A L, A E ought to be further from A C, or nearer to A B than A K. Consequently, the supposition that the resultant of A C and A B acts in the *direction* of the line A E, leads to an absurdity; and similarly it may be shown, that if it be supposed to act in any other direction than the line A D, the diagonal of the parallelogram A B D C, we shall be led to a like absurd conclusion.

Hence we infer, that if two forces, acting on a point, whether commensurable or incommensurable, be represented in magnitude and direction by two adjacent sides of a parallelogram, the diagonal of the parallelogram will represent the *direction* in which the resultant of these forces will act. We have next to show, that this diagonal will also represent the *magnitude* as well as the *direction* of the resultant.



Let A B, one side of the parallelogram A B C D, represent a force P, in magnitude and direction, and the side A C, another force Q, in magnitude and direction, these two forces both acting on the point A.

Let R represent the force which is the resultant of the forces P and Q; this force will act, according to what we have already proved, in the *direction* A D, A D being the diagonal of the parallelogram A B C D.

Produce A D to D′, and take A D′ in the same proportion to R that A C bears to Q, or A B to P.

Draw D′ C′ parallel to A B, and B C′ parallel to A D′, meeting in C′, and join A C′, A B C′ D′ will be a parallelogram, A C′ its diagonal, and B C′ will be = A D′.

Since R is the resultant of the forces P and Q, a force R acting on A in the direction A D′, in the same straight line, but in the opposite direction to that in which the resultant of P and Q acts, will keep the point A at rest, when acted on by the forces P and Q.

Hence the forces P Q and R, represented in magnitude and direction by A B, A C and A D′ acting on A, will keep it at rest.

Any one of these three forces will be equal in magnitude to the resultant of the other two, but it will act in an opposite direction to it in the same straight line.

Now A C′, the diagonal of the parallelogram A D′ C′ B is the direction in which the resultant of the forces R and P represented by A D′ and A B acts. Therefore A C′ and A C must be in the same straight line, and since A C is parallel to B D, A C′ will also be parallel to B D, and since B C′ was drawn parallel to A D, A C′ B D must be a parallelogram, and B C′ will = A D.

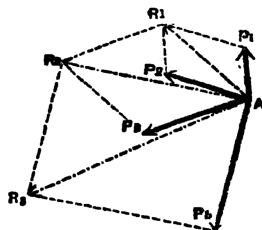
But B C′ = A D′. Therefore A D = A D′, and since A D′ represents R in *magnitude*, A D will represent R both in *magnitude and direction*.

We are indebted to M. Duchayla, a French mathematician, for this very simple and beautiful demonstration of the parallelogram of forces. It may be proved by other methods, but they either require a knowledge of the higher branches of mathematical analysis, or else assume the principles of Dynamics.

Some writers first demonstrate the properties of the lever, and from these deduce the parallelogram of forces.

PROPOSITION II.

To find the Resultant of any number of Forces acting on a Material Point in the same Plane.



Let P_1 P_2 P_3 and P_4, four forces acting on a point A in the same plane, be represented in *magnitude and direction* by the lines A P_1, A P_2, A P_3 and A P_4.

Draw P_1 R_1 parallel to A P_2, and P_2 R_1 parallel to A P_1, meeting in R_1; join A R_1. Then A R_1 the diagonal of the parallelogram A P_1 R_1 P_2 will by Prop. I. represent the resultant of the forces P_1 and P_2 acting on A in the directions A P_1 and A P_2.

Let R_1 be this force. Then R_1 alone acting on A in the direction A R_1 will produce on A the same effect as the two forces P_1 and P_2 acting together, in the directions A P_1 and A P_2.

Consequently the two forces P_1 and P_2 acting on A in the directions A P_1 and A P_2 may be replaced by a single force R_1 acting in the direction A R_1.

Again, draw R_1 R_2 parallel to A P_3 and P_3 R_2 parallel to A R_1 meeting in R_2; join A R_2. Then by Prop. I. a force R_2 represented in magnitude and direction by A R_2 the diagonal of the parallelogram A R_1 R_2 P_3, will have the same effect on A as the two forces R_1 and P_3 acting in the direction A R_1 and A P_3.

But R_1 acting on A in the directions A R_1 produces on A the same effect as P_1 and P_2 acting in the directions A P_1 and A P_2.

Hence the force R_2 acting on A in the direction A R_2 produces the same effect as the three forces P_1 P_2 and P_3 acting in the directions A P_1, A P_2 and A P_3.

Lastly, draw P_4 R_3 parallel to A R_2 and R_2 R_3 parallel to A P_4, meeting in R_3. Join A R_3. Then by Prop. I. a force R_3, represented in magnitude and direction by A R_3, will have the same effect on A as the two forces R_2 and P_4 have, acting in the directions A R_2 and AP_4.

But R_2, acting on A in direction A R_2, produces the same effect as P_1 P_2 and P_3 acting in the directions A P_1, A P_2, and A P_3.

Consequently a force R_3, acting in the direction A R_3, produces on A the same effect as the four forces P_1, P_2, P_3 and P_4 acting in the directions A P_1, A P_2, A P_3 and A P_4; or, in other words, A R_3 represents the resultant of these forces *in magnitude and direction.*

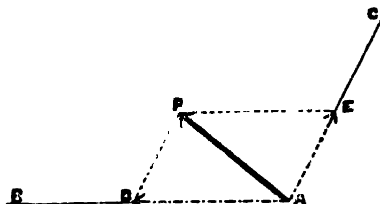
The same method may be extended to any number of forces, and affords an easy geometrical construction for finding the single resultant of any number of forces acting upon a material particle.

PROPOSITION III.

Resolution of Forces.

By means of the parallelogram of forces we can generally replace a single force by two others acting in any directions we please in the same plane; this is called *resolving* a force, and the forces by which it is replaced are termed its *resolved* parts.

Thus if a force P, acting on a point A, be represented in magnitude and direction by the straight line A P; and A B, A C drawn through A be the arbitrary directions in which we wish to resolve the force P.



Through P draw P D parallel to A C meeting A B in D, and also P E parallel to A D meeting A C in E.

Then, by Prop. I., A E and A D will represent two forces in magnitude acting in the directions A C and A B, which produce on A the same effect as the simple force P acting in the direction A P, and therefore may replace that force without altering the conditions of equilibrium.

A E represents the *resolved* part of P along A C, and A D its *resolved* part in the direction A B.

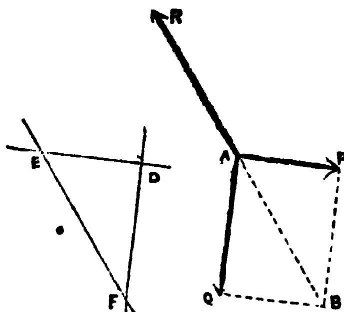
In resolving forces, it is generally found more convenient to choose the direction A C perpendicular to A B.

PROPOSITION IV.

Triangle of Forces.

If a material point be kept in equilibrium by the action of three forces acting upon it, in the same plane, the sides of any triangle drawn parallel to the directions of these three forces will be proportional to them; and conversely, three forces, acting on a material particle, will keep it at rest, if these forces be proportional to the sides of a triangle formed by drawing lines parallel to their directions.

Let P and Q, two forces acting on A, be represented in magnitude and direction by the lines A P and A Q.



Through P draw P B parallel to A Q, and through Q, Q B parallel to A P meeting in B; join A B.

Produce A B to R, and make A R equal to A B.

Then, Prop. I., a force R acting on A, represented in magnitude and direction by A R, will counteract the forces P and Q acting in the directions A P and A Q.

And A will be kept in equilibrium by the action of the three forces P, Q and R acting in the directions A P, A Q, and A R.

Take any point E; through E draw E D parallel to A P, and E F parallel to A R.

In E D take any point D, and through D draw D F parallel to A Q, meeting E F in F.

Then by construction the triangle E D F will be equiangular to the triangles A P B or A Q B.

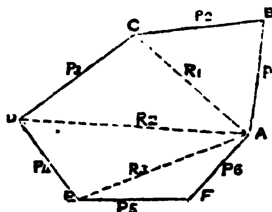
And, by Euc. B. VI., Prop. 4, E D : E F : F D : : A P : A B : B P
: : P : R : Q.

PROPOSITION V.

Polygon of Forces.

If a particle be acted on by any number of forces which are represented in magnitude, or are proportional to the sides of a polygon, it will be at rest, provided each force acts in a direction parallel to the side of the polygon to which it is proportional.

Let A B C D E F be a polygon, whose side, A B, is proportional to the force P_1, B C to P_2, C D to P_3, D E to P_4, E F to P_5, and F A to P_6.



Join A C, A D, and A E.

Let A C = R_1, A D = R_2, and A E = R_3.

Then, by Prop. IV., R_1 will represent the resultant of the forces P_1 and P_2 in magnitude.

R_2 the resultant of R_1 and P_3, or of P_1, P_2, and P_3.

And R_3 the resultant of R_2 and P_4, or of P_1, P_2, P_3, and P_4.

But, by Prop. IV., since R_3, P_5, and P_6 are sides of the triangle A E F, R_3 will represent the resultant of P_5 and P_6.

Hence P_5 and P_6 will counteract the effect of the forces P_1, P_2, P_3, and P_4, provided the conditions of Prop. IV. be fulfilled; *i. e.*, provided these forces act upon a particle in directions parallel to the sides of the polygon.

Since, in Prop. IV., any side of the triangle may be considered as the resultant of the other two, P_6 may be regarded as the resultant of R_3 and P_5, or of P_1, P_2, P_3, P_4, and P_5. Any one side, therefore, of the polygon may be taken as the resultant of all the others.

In the same manner the proposition may be extended to a polygon of seven, eight, or more sides.

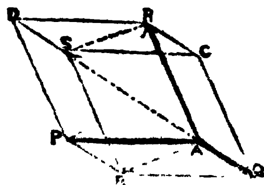
The polygon of forces need not have all its sides in the same plane.

PROPOSITION VI.

Parallelopiped of Forces.

If three forces, whose directions are not in the same plane, act upon a point, and if they be represented in magnitude and direction by three adjacent edges of a parallelopiped, which meet in the point on which the forces act, the resultant of the forces will be represented in magnitude and direction by the diagonal drawn from this point to the opposite solid angle of the parallelopiped.

Let A be the point, acted on by three forces, represented in magnitude and direction by A P, A Q, and A R, which are not in the same plane.



Then, Euc., B. xi. Prop. 2, A Q and A P are in the same plane; complete the parallelogram P A Q B in this plane, by drawing P B parallel to A Q, and Q B parallel to A P.

Similarly in the plane in which A R and A Q lie complete the parallelogram A R C Q.

And in the plane in which A R and A P lie, complete the parallelogram A R D P.

Also complete the parallelograms Q C B S in the plane Q C B, and the parallelogram P B S D in the plane D P B.

We shall then have constructed a parallelopiped, three of whose adjacent edges meeting at A are A P, A Q, and A R.

Join A B, A S, and R S. A B will be the diagonal of the parallelogram A P B Q, and A S the diagonal of the parallelogram A R S B.

Hence A B will represent the resultant of the forces represented by A P and A Q in magnitude and direction.

And A S the resultant of the forces represented by A R and A B.

Therefore A S represents the resultants of the forces represented by A R, A P, and A Q, in magnitude and direction.

By the aid of this proposition, any single force acting on a particle may be resolved into three other forces not acting in the same plane. When a force is so resolved, it is generally found convenient to resolve it into three forces acting at right angles or perpendicularly to one another.

PROPOSITION VII.

Condition of equilibrium when a material particle is acted on by any number of forces whose directions are all in the same straight line.

The condition of equilibrium in this case must evidently be, that the sum of all the forces acting on the particle in one direction must be equal to the sum of all the forces acting in the opposite direction.

It is usual to consider the forces acting in one direction as positive, and those in the opposite as negative. The condition of equilibrium, then, may be thus stated:—the algebraical sum of all the forces is zero, or nothing.

Thus, if A be acted on by the positive forces, 4 lbs., 3 lbs., and 5 lbs., in one direction, and the negative forces, 6 lbs., 5 lbs., and 1 lb., in the opposite.



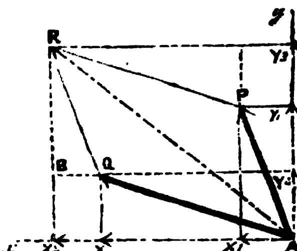
The algebraical sum of these forces will be $4 + 3 + 5 - 6 - 5 - 1 = 12 - 12 = 0$; and A will be at rest under the influence of these forces.

PROPOSITION VIII.

If two straight lines be drawn at right angles to one another through any point, and if two forces act on this point in any direction whatever in the same plane in which these lines are drawn, then the algebraical sum of the resolved parts of the two forces in the direction of each of the lines will be equal to the resolved part of their resultant in the same direction.

Let $A\,x$ and $A\,y$ be the straight lines drawn through the point A perpendicular to one another.

Lines so drawn are called *rectangular axis.*



Let two forces, P and Q, acting on A, be represented in magnitude and direction by A P and A Q.

Draw P R parallel to A Q, and Q R to A P, meeting in R, and join A R

Then, Prop. I., A R = R represents the resultant of P and Q in magnitude and direction.

Through the points P, Q, and R, draw $P\,X_1$, $Q\,X_2$, and $R\,X_3$, perpendicular to $A\,x$, and through the same points $P\,Y_1$, $Q\,Y_2$, and $R\,Y_3$, perpendicular to $A\,y$.

By Prop. III., $A\,X_1$ and $A\,Y_1$ will be the resolved parts of the force P in the directions $A\,x$ and $A\,y$.

Similarly, $A\,X_2$ and $A\,Y_2$ will be the resolved parts of Q; and $A\,X_3$ and $A\,Y_3$ the resolved parts of R in the same directions.

Let $A\,X_1 = X_1$, $A\,X_2 = X_2$, $A\,X_3 = X_3$, $A\,Y_1 = Y_1$, $A\,Y_2 = Y_2$, and $A\,Y_3 = Y_3$.

Produce the line $Y_2\,Q$ to meet the line $R\,X_3$ in the point B.

Since A P R Q is a parallelogram, A P = Q R; and also because B Q is parallel to $A\,x$, and A P to R Q, the angle R Q B = angle $P\,A\,X_1$.

But the angles R B Q and $P\,X_1\,A$ are right angles by construction.

Hence, the triangles R Q B and $P\,A\,X_1$ have the side A P = R Q, and the angles $P\,A\,X_1 = R\,Q\,B$, and $P\,X_1\,A = R\,Q\,B$.

Therefore, Euc. B. I., Prop. 26., $A\,X_1 = B\,Q$, and $P\,X_1 = R\,B$.

And $X_1 + X_2 = A\,X_1 + A\,X_2 = B\,Q + A\,X_2 = X_3\,X_2 + A\,X_2 = A\,X_3 = X_3$.

Since $B\,Q = X_3\,X_2$, because by construction of figure $B\,Q\,X_2\,X_3$ is a parallelogram.

And again, $Y_1 + Y_2 = A\,Y_1 + A\,Y_2 = P\,X_1 + B\,X_3 = R\,B + B\,X_3 = R\,X_3 = A\,Y_3 = Y_3$.

Since $A\,Y_1 = P\,X_1$, $A\,Y_2 = B\,X_3$, and $R\,X_3 = A\,Y_3$, because $A\,X_1\,P\,Y_1$, $A\,X_3\,B\,Y_2$, and $A\,X_3\,R\,Y_3$, are parallelograms by construction of figure.

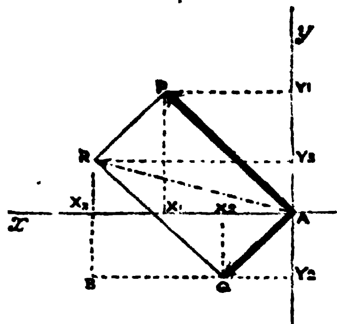
If, therefore, $X_1\,Y_1$ be the resolved parts of a force, P, along the rectangular axes, $A\,x$ and $A\,y$, $X_2\,Y_2$ the resolved parts of Q, and $X_3\,Y_3$ the resolved parts of R, the resultant of P and Q in the direction of the same axes,

$$X_3 = X_1 + X_2, \text{ and } Y_3 = Y_1 + Y_2.$$

The above figure has been so drawn that the forces P and Q both fell within the axes $A\,x$ and $A\,y$; this is not always the case; it may sometimes happen, as in the annexed figure, that one of the axes may fall between the two forces; in this case it will be seen that the resolved part of the force Q, along the axis $A\,y$, will fall on the opposite side of A, from the resolved forces of P and R along the same line.

The resolved forces which lie on one side of A are called positive, and those on the opposite side negative.

Consequently, the resolved parts of P will be X_1 and Y_1, of Q, X_2 and $-Y_2$, and of R, X_3 and Y_3. The same construction being made as in the last case, since A P R Q is a parallelogram, A P = R Q, and since A X_1 is parallel to B Q, and A P to Q R, the angle P A X_1 = angle R Q B, also the angles at B and X_1 are right angles.



Hence, in triangles A P X_1 and Q B R (Euc., B. I., Prop. 26.), R B = P X_1, and B Q = A X_1.

$X_1 + X_2 = A X_1 + A X_2 = B Q + Q Y_2 = B Y_2 = X_3 A = X_3$.

$Y_1 - Y_2 = A Y_1 - A Y_2 = P X_1 - X_3 B = R B - X_3 B = R X_3 = A Y_3 = Y_3$.

And in this case $X_3 = X_1 + X_2$, and $Y_3 = Y_1 - Y_2$.

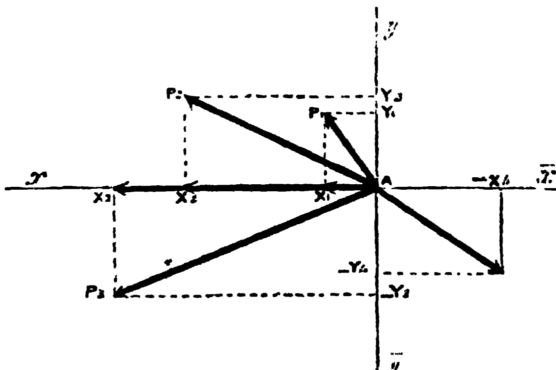
A similar construction and demonstration will apply to every other position in which the rectangular axes may be placed with respect to the forces, and we may say, generally, that the algebraical sum of the resolved parts of any two forces acting upon a material point in the directions of any two rectangular axes passing through that point, will be equal to the resolved parts of their resultant along the same axes.

PROPOSITION IX.

To find the conditions of equilibrium of any number of forces acting upon a material point, the directions of the forces being all in the same plane, but not in the same straight line.

Let P_1, P_2, P_3, and P_4, be four forces acting upon A, represented in magnitude and direction by the lines A P_1, A P_2, A P_3, and A P_4.

Through A draw the rectangular axes A x and A y.



Through the points P_1, P_2, P_3, and P_4, draw $P_1 Y_1$, $P_2 Y_2$, $P_3 Y_3$, and $P_4 Y_4$, perpendicular to A y, and $P_1 X_1$, $P_2 X_2$, $P_3 X_3$, and $P_4 X_4$, perpendicular to A x.

Then, Prop. III., A X_1, A Y_1 = X_1 and Y_1, are the resolved parts of P_1 in the direction of the axes A x and A y.

A X_2 and A Y_2 = X_2 and Y_2 those of P_2, A X_3 and − A Y_3 = X_3 and − Y_3 those of P_3 and − A X_4 and − A Y_4 − X_4 and − Y_4 those of P_4 along the same axes.

Let R_1 be the resultant of the forces P_1 and P_2.

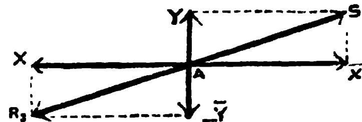
R_2 the resultant of the forces R_1 and P_2, or of P_1, P_2, and P_3.

And R_3 the resultant of the forces R_2 and P_3, or of P_1, P_2, P_3, and P_4.

Then, Prop. VIII., the resolved part of R_1 along the axis of $Ax = X_1 + X_2$, that of $R_2 = X_1 + X_2 + X_3$, and that of $R_3 = X_1 + X_2 + X_3 - X_4$.

Similarly the resolved part of R_1 along the axis $Ay = Y_1 + Y_2$, that of $R_2 = Y_1 + Y_2 + Y_3$, and that of $R_3 = Y_1 + Y_2 - Y_3 - Y_4$.

Let $X = X_1 + X_2 + X_3 - X_4$, the algebraical sum of the resolved parts of the four forces along the axis Ax, and $-Y = Y_1 + Y_2 - Y_3 - Y_4$, the algebraical sum of the resolved parts of these forces along the axis Ay.



Take $AX = X$ along the axis AX, and $AY = -Y$ along the axis AY.

Through X draw XR_3 perpendicular to AX, and through $-Y$, $-YR_3$ perpendicular to AY meeting in the point R_3. Join AR_3.

Then, Prop. III., AR_3 will represent the resultant of the forces P_1, P_2, P_3, and P_4, in magnitude and direction.

This single force AR_3, acting alone, will produce on A the same effect as the forces P_1, P_2, P_3, and P_4.

Produce AR_3 to AS_1, make $AS = AR_3$.

Then, Prop. VII., A force represented in magnitude and direction by AS will exactly counteract the effect of the force represented by AR_1.

Consequently, a force represented in magnitude and direction by AS, will keep the point A in equilibrium when acted on by the forces P_1, P_2, P_3, and P_4.

From S let fall $S\bar{X}$ perpendicular to AX, and SY perpendicular to AY.

Then, Prop. III., $A\bar{X}$ and AY are the resolved parts of AS along the rectangular axes. Now in the triangles AXR_3 and $AS\bar{X}$, the angle XAR_3 = angle $SA\bar{X}$, and he angles at X and $\bar{X}$ are right angles, and also $AR_3 = AS$.

Therefore $A\bar{X} = AX = X$, and $S\bar{X} = XR_3$.

And $AXR_3\bar{Y}$ and $A\bar{X}SY$ are parallelograms.

Consequently, $S\bar{X} = AY$, and $XR_3 = A\bar{Y}$.

Therefore $AY = A\bar{Y} = -Y$.

The resolved parts of AS are therefore $-X$ and Y.

But $X = X_1 + X_2 + X_3 - X_4$, and $-Y = Y_1 + Y_2 - Y_3 - Y_4$.

Therefore $X_1 + X_2 + X_3 - X_4 - X = 0$, and $Y_1 + Y_2 - Y_3 - Y_4 + Y = 0$.

Or if five forces represented in magnitude and direction by AP_1, AP_2, AP_3, AP_4, and AS keep a point in equilibrium, the algebraical sum of the resolved parts of these forces along the rectangular axes passing through this point, will each be equal to zero.

The same demonstration may be applied to any number of forces. It may also be observed that the position of the rectangular axes is perfectly arbitrary, provided only that they are in the same plane in which the forces are supposed to act.

The conditions of equilibrium when all the forces are not in the same plane, involve the discussion of solid geometry. We shall reserve the consideration of this subject to a more advanced portion of our treatise.

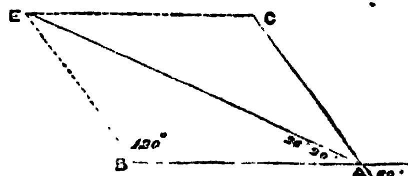
The student will observe that all the conditions of equilibrium, for a material point which we have considered, are geometrical deductions from the parallelogram of forces, and involve no new mechanical principles; and that the parallelogram of forces depends upon one mechanical principle, that of the transmission of a force from any one point to another rigidly connected with it, and in the direction of its action, without altering its effect.

We subjoin two or three examples, to show the method of applying the principles we have determined. Mechanical problems may be solved in two ways; graphically, *i. e.*, by accurately drawing the geometrical figures by mathematical instruments and scales, and obtaining a result, which can be measured by these instruments; or trigonometrically, *i. e.*, by computing the relations of the figures according to the principles of trigonometry, and thus arriving by means of trigonometrical tables at a more accurate result than the former will afford.

We would recommend the student to use both of these methods, as he will by this means obtain a clearer view of the subject than by accustoming himself only to one of them.

PROBLEM I.—*Two forces, represented by 12lbs. and 15lbs., are inclined to each other at an angle of 60°; required the magnitude of the resultant; and its inclination to the greater.*

1st Graphical solution.



Draw a line A B.

Make A B equal to 15 parts of any convenient scale, as the 8th or the 10th of an inch.

Draw A C inclined to A B at an angle of 60°.

Take A C equal to 12 parts of the same scale, that A B is equal to 15 parts.

Through C draw C E parallel to A B, and through B, B E parallel to A C, meeting in the point E.

Join A E.

Then, Prop. I., A E will represent the resultant of the forces represented by A C and A B in magnitude and direction; and if A E be measured by the scale used in drawing A B and A C, it will be found to be $23\frac{4}{10}$ parts of that scale, and the angle E A B will measure very nearly 26° 20′.

The resulting force will therefore be represented by $23\frac{4}{10}$ lbs., and its inclination to the greater force will be 26° 20′ nearly.

The process might have been shortened by the use of Prop. IV., in which case we should have drawn A B = 15 parts of the scale, B E making an angle 180° — 60° or 120° with A B, and B E = 12 parts of the scale, and lastly joined A and E.

2nd Trignometrical solution.

$$AE^2 = AB^2 + BE^2 - 2\,AB \cdot BE \cos. 120°.$$
$$= AB^2 + BE^2 + 2\,AB \cdot BE \cos. 60°.$$
$$= 12^2 + 15^2 + 2 \times 12 \times 15 \times \cdot 5.$$
$$= 144 + 225 + 180 = 549.$$

Hence, $AE = \sqrt{549} = 23\cdot 43.$

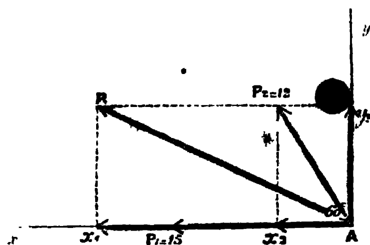
Again, $\frac{\sin.\ EAB}{\sin.\ EBA} = \frac{EB}{EA}$, or $\frac{\sin.\ EAB}{\sin.\ 120°} = \frac{12}{23\cdot 43}$.

Hence, $\sin.\ EAB = \frac{12}{23\cdot 43} \sin.\ 120° = \frac{12}{23\cdot 43} \sin.\ 60°.$

And $\text{Log. sin. } EAB = \text{Log. } 12 + \text{Log. sin. } 60° - \text{Log. } 23\cdot 43.$
$$= 1\cdot 0791812 + 9\cdot 9375306 - 1\cdot 3697723.$$
$$= 9\cdot 6469395.$$
$$= \text{Log. sin. } 26° 20' \text{ nearly.}$$

Angle $EAB = 26° 20'$.

When the resultant of more than two forces is to be found, it will be more convenient to resolve the forces as in Prop. IX. We shall therefore work out the above example by that method.



Draw two straight lines, Ax and Ay at right angles to each other.

Take $AP_1 = 15 =$ force P_1 along Ax.

Draw $AP_2 = 12 =$ force P_2, making an angle 60° with AP_1.

Through P_2 draw $P_2\,y_2$ perpendicular to Ay, and $P_2\,x_2$ perpendicular to Ax.

Then Ax_2, Ay_2 are the resolved parts of P_2 along Ax and Ay; AP_1 is the resolved part of P, along the axis Ax, and its resolved part along Ay is zero.

Produce AP_1 to x', and make $P_1\,x' = Ax_2$.

Then $Ax' = AP_1 + Ax_2$ is the resolved part of the resultant of P_1 and P_2 along Ax and Ay_2 is the resolved part along Ay.

Through x' draw x' R perpendicular to Ax, and produce $y_2 P_2$ to meet x' R in R. Join A R.

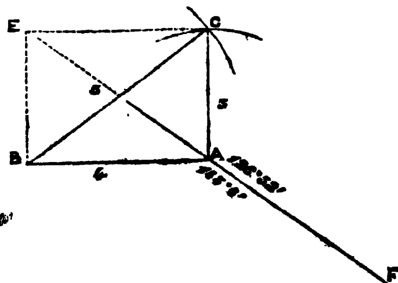
Then A R represents the resultant of P_1 and P_2 in magnitude and direction, $Ay_2 = AP_2 \cos. 60° = 12 \times \cdot5 = 6$, and $Ay_2 = P_2X_2 = AP_2 \sin. 60° = 12 \times \cdot866 = 10\cdot392$.

But $AR^2 = Ay_2{}^2 + Ry_2{}^2 = Ay_2{}^2 + Ax_1{}^2 = (10\cdot392)^2 + (15+6)^2 = (10.392)^2 + (21)^2. = 107\cdot993664 + 441 = 548.993664$.

Therefore $AR = \sqrt{548.993664} = 23\cdot430$.

Again, $\cos. RAx^1 = \frac{Ax^1}{AR} = \frac{21}{23\cdot43} = \cdot89628 = \cos. 26° 20'$ nearly. And therefore angle $RAx' = 26° 20'$. And this is the inclination of the resultant to the greater force.

PROBLEM II.—*Three forces which are to each other as 3, 4, and 5, act upon a point, and keep it at rest; required the angles at which these forces are inclined to each other.*



1st. Graphical solution.

Draw a line $AB = 4$ parts of some scale, with A as a centre and radius A C $= 3$ parts of this scale; describe an arc.

Also, with B as centre, and radius B C $= 5$ parts of the scale; describe another arc.

Let C be the point where the two arcs intersect.

Join A C and B C. Through C draw C E parallel to A B and through B, B E parallel to A C, meeting in E. Join A E and produce A E to F, making $AF = AE$. Then the forces which are to each other as 3, 4 and 5, acting upon the point A, and keeping it at rest, will be represented in magnitude and direction by the lines A C, A B, and A F; and the angles being measured by a protractor, or any other means used for measuring

angles, it will be found that the angle B A C = 90°, the angle B A F = 143° 8′, and the angle C A F = 126° 52′, nearly.

2nd. Trigonometrical solution.

Since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

The forces represented by 3 and 4 must be at right angles to each other, and their resultant 5, will be the diagonal of the rectangular parallelogram whose sides are as 3 and 4.

Let a be the angle between the resultant and the force represented by 3.

Then $\tan. a = \frac{4}{3} = 1 \cdot 33333 = \tan. 53° 8′$, nearly.

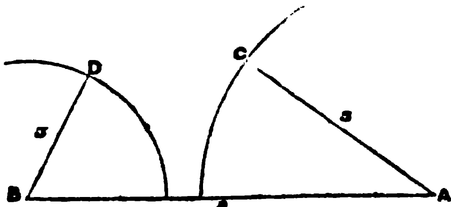
Hence, by a reference to the diagram it will be readily seen that the angle between the forces 3 and 5 is $180° - a = 180° - 53° 8′ = 126° 52′$, while that between the forces 4 and 5 is $90° + a = 90° + 53° 8′ = 143° 8′$.

Problem III.—*Three forces represented by the numbers 3, 5, and 9, cannot under any circumstances produce equilibrium on a point.*

This is evidently true, since by Prop. IV., three forces can only produce equilibrium on a point, when a triangle can be described whose sides are respectively proportional to the magnitudes of the forces. Now, the sides of a triangle never can bear to each other the proportion of the numbers 3, 5, and 9, since in every triangle the sum of any two sides must always be greater than the third, and 3 + 5 = 8, a number less than 9.

If we attempt to solve the proplem graphically, we shall soon perceive its impossibility.

Thus, if we draw a line A B = 9 parts of any scale, and with A as a centre and radius A C = 5 parts of the scale, and with B as a centre and radius BD = 3 parts of the scale, we describe two circles; they evidently will not cut each other in any point, and consequently we cannot construct a triangle whose sides are to each other as the numbers 3, 5, and 9.



We add a few more problems for the practice of the student, taken from the Cambridge Examination Papers and other sources.

Three forces acting in the same plane keep a point at rest; the angles between the directions of the forces are 135°, 120°, and 105°; compare their magnitudes.

Four forces represented by 1, 2, 3 and 4, act in the same plane on a point. The directions of the first and third are at right angles to each other; and so are the directions of the second and fourth; and the second is inclined at an angle of 60° to the first. Find the magnitude and direction of the resultant.

A circular hoop is supported in a horizontal position, and three weights of 4, 5 and 6 lbs. respectively are suspended over its circumference by three strings knotted together at the centre of the hoop. Neglecting the friction of the edge of the hoop, find the angles between the strings when there is equilibrium.

Three equal forces, each equivalent to 6 lbs., act on a point, the first two are inclined to each other at an angle of 75°, and the third is inclined at an angle of 15° to the first. Find the magnitude and direction of the resultant.

The resultant of two forces is 50 lbs., and the angles which it makes with their directions is 20° and 30°; find the component forces.

A boat, fastened to a fixed point by a rope, is acted on at the same time by the wind and the current. Suppose that the wind was S E, the direction of the current S., and the direction of the boat from the fixed point S 20° W., and also that the pressure on the point was 150 lbs., it is required to find the forces of the wind and the current.

In pulling a weight along the ground by a rope, inclined to the horizon at an angle of 45°, I exerted a power of 40 lbs.; required the force with which I dragged the body horizontally.

Four forces are in the same plane, which are to each other as 6, 8, 10, and 12, act upon a given point, and are inclined to a given line at the angles 20°, 40°, 80°, and 150° respectively; find the magnitude and direction of a fifth force which shall balance the others.

Equilibrium of a Rigid Body.—Having considered the conditions of equilibrium of a material particle acted on by any number of forces acting in the same plane, we next proceed to consider the conditions of equilibrium of a rigid body under the same circumstances. Here it may be as well to repeat what has been stated before, that by the term *rigid body*, we understand a body composed of material particles held together by unknown molecular forces of such intensity that the body cannot be altered in shape, or its particles in any way displaced by any forces which can act upon it. This rigid body also possesses the property, that if any force be applied to it, its particles will transmit that force, unimpaired in intensity, to any point in the body, which lies in the line of direction in which the force is acting; consequently, the effect of a force on a rigid body will be the same if we transfer its point of application from any one point in the rigid body to any other, provided these two points are in the line in which the force is supposed to act.

Unless it is otherwise stated, this *hypothetical rigid body* is also considered as being destitute of weight.

If two or more forces acting on a rigid body are applied to the same point of the body, the conditions of equilibrium will be the same as those for a material point under the same circumstances, for the same force which, when applied to the material point, would counteract the effect of these forces, would also keep the body in equilibrium when applied to the point in the body upon which the forces act.

PROPOSITION X.

To find the magnitude and direction of the resultant of two forces acting on different points of a rigid body, the directions of the forces being in the same plane, but not parallel to each other.

Let a force P, represented in magnitude and direction by the line A P, act upon the point A of a rigid body, and another force Q, represented by B Q, act upon the point B of the same body. B Q and A P both being the same plane.

Join A B. Produce P A and Q B to meet in the point C.

Along C B take $CQ' = BQ$, and along C A, $CP' = AP$.

Draw Q′ R′ parallel to C P′, and P′ R′ parallel to C Q′, meeting in R′.

Join C R′, and produce C R to R,′ cutting A B in D.

Make D R = C R′. Then D R will represent the resultant of the forces P and Q in magnitude and direction.

By the principle of the transmission of forces, the points A B and C being supposed to be rigidly connected with each other.

The point of application of the force P may be transferred from A to C, and the force Q from B to C.

Then the forces P and Q acting at C may be replaced by the single force represented in magnitude and direction by C R′, Prop. 1., and this force may be transferred from C to D, and be represented in magnitude and direction by D R.

This construction will enable us to represent graphically the resultant of any two forces acting on a rigid body in the same plane, but in directions not parallel to one another.

For the purpose of calculation, it will be convenient, however, to determine the geometrical relation of the point D to the forces P and Q, and their directions.

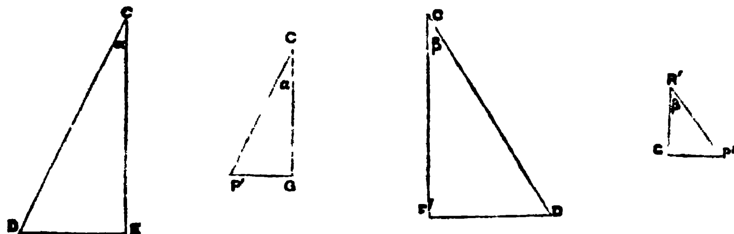
Through D draw D E perpendicular to C A, and D F perpendicular to C B; also through P′ draw P′ G perpendicular to C D.

Let α represent the angle A C D, and β the angle B C D.

Then since by construction P′ R′ is parallel to C Q′, therefore (Euc. B. I., Prop. 27,) angle C R′ P′ = angle Q′ C R′ = β.

The angles at E, F, and G, are right angles by construction.

Hence the figure will afford us two pairs of similar triangles.



Since in the triangles C D E, C G P′ the angles D C E and P′ C G are both = α, and those at E and G′ are right angles (as will probably be more readily perceived in the annexed figures than in the more complicated figure), it follows that the triangles C D E and C P′ G are equiangular triangles and similar to one another, and therefore (Euc. B. VI., Prop. 4,)

$$C D : D E :: C P' : G P'$$

$$\text{or, } \frac{C D}{D E} = \frac{C P'}{G P'}$$

Similarly, since the angles F C D and G R′ P′ are both equal to β, and the angles at F and G right angles, the triangles C F D and R′ G P′ are similar triangles, and

$$DF : CD :: GP' : P'R'$$

$$\text{or, } \frac{DF}{CD} = \frac{GP'}{P'R'}$$

Multiplying equals together, we have

$$\frac{CD}{DE} \cdot \frac{DF}{CD} = \frac{CP'}{GP'} \cdot \frac{GP'}{P'R'}$$

$$\text{and therefore } \frac{DF}{DE} = \frac{CP'}{P'R'}$$

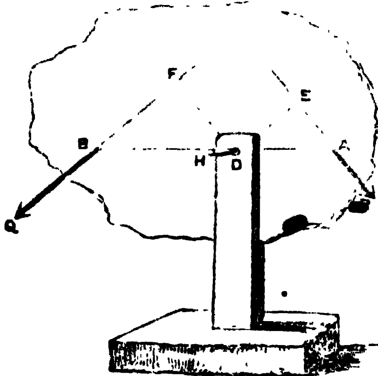
But C P′ R′ Q′ is a parallelogram, and therefore P′ R′ = C Q′.

$$\text{Hence } \frac{DF}{DE} = \frac{CP'}{CQ'} = \frac{P}{Q}$$

$$\text{or, } \frac{\text{The perpendicular from the point D on the direction of the force Q}}{\text{The perpendicular from the point D on the direction of the force P}} = \frac{P}{Q}$$

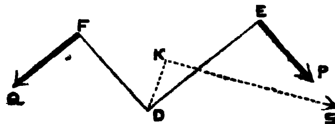
Consequently, the perpendicular from the point D on the direction of the force Q multiplied by Q, equals the perpendicular from the point D on the direction of the force P multiplied by P.

Moment of Forces and Fulcrum.—If we suppose an axis, H D, passing through the plane in which the forces act, fixed with perfect rigidity, and perpendicular to that plane at the point D, and then placed on a stand, as in the accompanying figure, so as to allow the plane to move with perfect freedom about the axis H D, then, since the resultant of the forces P and Q acting at A and B passes through the point D, their joint effect will be counteracted by the reaction of the stand on the axis H D. Neglecting the weight of the plane and the friction of the axis, the plane will be in a state of equilibrium, under the influence of the two forces P and Q, acting at A and B in the directions A P and B Q, together with the reaction of the stand acting on the plane at the point D in a direction opposite to the resultant of the forces P and Q. If the rigid body be reduced to the rigid line A B, A B is called a lever, and the point D its fulcrum.



Since the conditions of equilibrium, or that the resultant of the forces B and Q should pass through D, are as above stated, that

The perpendicular from D *on the direction of the force* Q *multiplied by* Q, *equals the perpendicular from* D *on the direction of the force* P, *multiplied by* P; it follows that the force P may be replaced by a force S without altering these conditions, provided that the perpendicular from D, D K on the direction K S of the force S multiplied by S, be equal to the perpendicular from D on the direction of the force P multiplied by P.



The writers on Mechanics have applied the term moment or momentum to this pro-

duct of the linear units in the length of the perpendicular from a given point on the direction of a force, by the units of force in the given force.

Thus, if the forces P, Q and S be represented in magnitude and direction by E P, F Q, and S K, and D E, D F and D K be the perpendiculars on those directions.

P E × D E, Q F × D F, and S K × D K will be the respective momenta or moments of the forces P Q and S, about the point D.

These products are not only termed the moments of the forces with respect to the point D, but also the moments of the forces with respect to axis H D, perpendicular to the plane in which they act.

The student must be careful not to confound these *statical moments, or momenta*, with the momenta of dynamics; the momentum of dynamics being the product of the mass of a body by its velocity.

If the force P alone were to act on the plane at the point A, it would evidently twist the plane about the fixed axis H D, and Q acting alone at B, would twist the plane about H D in a direction opposite to that in which P would do so. The tendency of P therefore to twist the plane about the axis H D, or point D, is counteracted by the tendency of Q to twist the plane in the opposite direction in the case where equilibrium exists.

We have seen, however, that the tendency of Q to twist the body about the axis H D, will be counteracted by any other force whose statical moment, with respect to H D or D, is equal that of the force P.

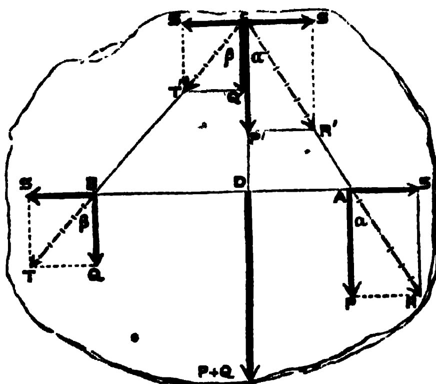
Hence we may consider the statical moment of a force about a given point or axis, as a measure of its tendency to twist the plane to which it is applied about this point or axis.

Positive and Negative Moments.—The moments of those forces whose tendency is to twist a body about the axis or fulcrum, in the direction in which the hands of a watch move, are termed positive; those whose tendency is in the opposite direction, negative.

It will readily be seen, that if the directions of the forces P and Q in Proposition X. had been parallel to one another, the demonstration there used could not be applied; as, in that case, the directions of the forces could not be produced till they met.

PROPOSITION XI.

To find the magnitude and direction of the resultant of two forces acting on different points of a rigid body, the directions of the forces being in the same plane, but parallel to each other.



Let P and Q be the two forces, A and B the two points in the plane.

The force P being represented in magnitude and direction by the line A P, and Q by the line B Q.

Then A P and B Q are parallel to each other.

Join A B.

The conditions of equilibrium will not be altered if any arbitrary force S be applied to A in the direction A S, and to B in the opposite direction B S.

Let these equal and opposite forces be represented in magnitude and direction by A S and B S.

Consequently A S = B S.

Complete the parallelogram S B Q T and the parallelogram S A P R.

Join A R and B T.

Then A R = R represents the resultant of the forces P and S in magnitude and direction, and may be substituted for these forces.

Also B T = T represents the resultant of Q and S in magnitude and direction, and may be substituted for them.

Produce A R and B T to meet in the point C.

Through C draw C D parallel to A P or B Q, meeting A B in D.

The force R acting at A in direction A R may be transferred from R to C in the line R A C, and represented in magnitude and direction by C R′, C R′ being = A R.

Similarly T may be transferred from B to C, and represented in magnitude and direction by C T′, C T′ being equal to B T.

Through C draw the line S C S parallel to A B.

Also through T′ draw T′ Q′ parallel to B A, meeting C D in Q′, and T′ S parallel to D C, meeting S C in S.

Similarly through R′ draw R′ P′ parallel to A B, meeting C D in P′, and R′ S paralle to D C, meeting S C in S.

The parallelogram S C Q′ T′ is similar and equal in all respects to the parallelogram S B Q T, and the parallelogram C P′ R′ S to the parallelogram A P R S.

Hence, Prop. III., the force R acting at C, in the direction C R′, may be replaced by the forces S and P, represented in magnitude and direction by C S and C P′; also the force T acting at C in direction C T′, may be replaced by the forces S and Q′ acting in the directions C S and C Q′.

The equal and opposite forces S and S, acting in the line S C S, will destroy each other.

And the parallel forces P and Q acting at A and B are replaced by the forces P and Q acting at C in the direction C D, and these forces again may be transferred from C to D.

Hence the resultant of the forces P and Q acting at A and B, will be a force P + Q acting at D in a direction parallel to the directions of P and Q.

To determine the position of the point D.

Since P′ R′ is parallel to A D the triangles C P′ R′, C D A are equiangular, and therefore, Euc. B. VI. P. 4—

$$CD : AD :: CP' : R'P', \text{ or } \frac{CD}{AD} = \frac{CP'}{R'P'}$$

Similarly because Q′ T′ is parallel to D B, the triangles C T′ Q′, C B D are equiangular, and

$$BD : CD :: T'Q' : CQ', \text{ or } \frac{BD}{CD} = \frac{T'Q'}{CQ'}$$

Hence $\frac{CD}{AD}, \frac{BD}{CD} = \frac{CP'}{R'P'}, \frac{T'Q'}{CQ'}$.

But $R'P' = T'Q'$, therefore $\frac{BD}{AD} = \frac{CP'}{CQ'} = \frac{P}{Q}$

And $\frac{BD}{AD} + 1 = \frac{P}{Q} + 1$, therefore $\frac{BD + AD}{AD} = \frac{P + Q}{Q}$

But $BD + AD = AB$. Hence $\frac{AB}{AD} = \frac{P+Q}{Q}$

And $AD = \frac{Q}{P+Q} AB$, which determines the point D.

Or, in other words, the distance of the fulcrum from the point of application of one of the forces P, is equal to the other force Q multiplied by the distance between the two forces, and divided by the sum of the two forces.

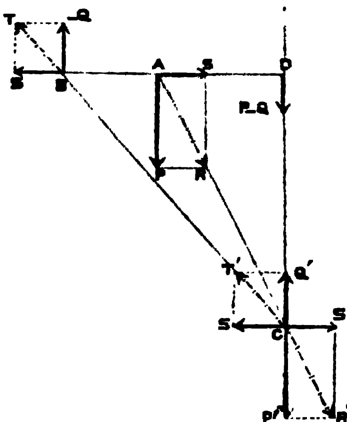
When A B is perpendicular to A P or B Q we have since—

$$\frac{BD}{AD} = \frac{P}{Q}$$

$$Q \times BD = P \times AD;$$

but in this case $Q \times BD$ is the moment of Q with respect to D, and $P \times AD$ the moment of P with respect to D. Hence in this case, as in the last proposition, the moments of P and Q with respect to D are equal to each other.

In the case we have just considered, we supposed the two forces P and Q to be acting in the same direction; when they act in the opposite directions, our construction will be modified as represented by the accompanying diagram.



To A and B, as before, apply two equal and opposite forces represented in magnitude and direction by A S and B S.

Complete the parallelograms A P R S and B Q T S.

Join B T and A R, and produce them till they meet at C.

Through C draw C D parallel to A P or B Q, meeting B A produced in D.

The resultant of the forces S and Q acting at B will be represented in magnitude and direction by B T, and this force may be transferred from B to C, and represented by $CT' = BT$.

The resultant of the forces P and S acting at A will also be represented by A R, and this may be transferred from A to C, and represented by $CR' = AR$.

Through C draw S C S parallel to B A, through T', T' Q' parallel to A B, T' S parallel to D C, and through R', R' P' parallel to A B, meeting D C produced in P', and R' S parallel to D C.

The force C T' may be replaced by the forces represented by C Q' and C S, and the force C R' by the forces represented by C S and C P'.

But $CS = BS$, and $CS = AS$, and $AB = BS$ by construction.

Hence the two equal forces acting on C in opposite directions in the line S C S destroy each other, and the forces P acting at A, and Q acting at B, are replaced by the two forces C Q' and C P' acting on C in opposite directions.

But $CQ' = BQ = Q$, and $CP' = AP = P$.

Hence these two forces may be represented by a single force $P - Q$ acting at C in the direction C P', and this force may be transferred from C to D in the line C D.

The resultant of the two parallel forces, P and Q, acting in opposite directions at the points A and B, will be a force $P - Q$ acting at D in B A produced.

To determine the position of the point D.

Since A P is parallel to D C, the angle A R S = angle A C D, and the angles at C and D are common to the two triangles A R S, A C D.

Therefore the triangles A R S, A C D are equiangular and similar.

And A S : S R : : A D : D C, or $\frac{AS}{SR} = \frac{AD}{DC}$.

Also because B Q is parallel to C D, the angle T B Q = angle B C D.

Again, T Q is parallel to B D, therefore the angle Q T B = angle D B C.

And the two triangles T Q B, B D C are equiangular and similar.

And B Q : Q T : : D C : B D, or $\frac{BQ}{QT} = \frac{DC}{BD}$.

Multiplying the equal fractions together, we have $\frac{AS}{SR} \cdot \frac{BQ}{QT} = \frac{AD}{DC} \cdot \frac{DC}{BD}$.

But Q T = B S = A S, and S R = A P.

Hence $\frac{BQ}{AP} = \frac{AD}{BD}$, or $\frac{Q}{P} = \frac{AD}{BD}$.

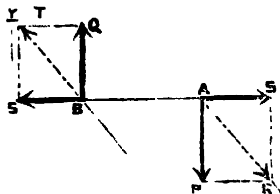
Now B D = A B + A D, therefore $\frac{Q}{P} = \frac{AD}{AB + AD}$.

And Q. A B + Q. A D = P. A D, or Q. A B = (P − Q) A D.

Therefore $AD = \frac{Q.\,AB}{P - Q}$.

This result might have been obtained from the previous case by substituting − Q for Q, which would give P − Q for the resultant, but the expression for A D would be $-\frac{Q.\,AB}{P - Q}$, the negative sign signifying that the point D in this case would lie on a different side of A from what it would when the force Q was positive, or when both forces acted in the same direction.

In the last case, that is, when P and Q act in opposite directions, if P and Q are equal, we cannot represent their resultant by any single force.



For if we pursue the mode of construction adopted before, we shall find the line B T in this case parallel to the line A R; and consequently these lines, if produced, will never meet.

This admits of an easy proof, because B Q is parallel and equal to A P, and B S and A S are equal to each other, and are in the same line.

Therefore the parallelogram A P R S is similar and equal to the parallelogram B Q T S, and the angle P A R = the angle T B Q.

Also because A P is parallel to B Q, the angle P A B = angle Q B A.

And by addition, angle P A R + angle P A B = angle T B Q + angle Q B A.

Or the angle R A B = the angle T B A.

And, by Euc., B. I., p. 27, R A must be parallel to T B.

Our method in this case fails to discover a point D anywhere in the line A B produced, to which a single resultant can be applied.

This is indicated by the formulæ which we have already obtained of P − Q for the magnitude of the resultant, and $\frac{Q.\,AB}{P - Q}$ for A D, the distance of its point of application

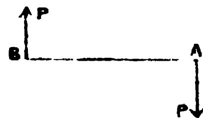
from A; since when $Q = P$, they give us $P - P$ or 0 for the magnitude of the resultant, and $\frac{P.AB}{P-P}$ or $\frac{P.AB}{0}$ the algebraical sign of infinity for A D; a result which shows that our problem in this case is impossible.

Couple.—When two equal and opposite parallel forces act at different points of a rigid body in the same plane, their effect, as we have seen, cannot be counteracted by any single force applied to the body, and their tendency will evidently be to twist the body in the direction of the plane in which they act. Thus P acting on A in the direction A P, and P acting on B in the direction B P, will twist the body round in the direction B A P or A B P.



The term couple is applied to such a system of forces.

Arm of a Couple.—The perpendicular distance between the directions in which the forces producing a couple act, is called the arm of that couple. Thus if the line A B is perpendicular to A P and B P, which represent the directions in which the forces producing a couple act, A B is the arm of that couple.

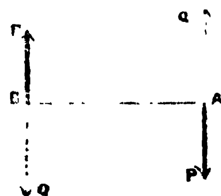


Moment of a Couple.—The product of the arm of a couple and one of the forces producing it, is called the moment of that couple.

Thus if P be the force acting in the direction A P or B P, then $P \times AB$ is the *moment* of the couple, produced by the couple whose *arm* is A B.

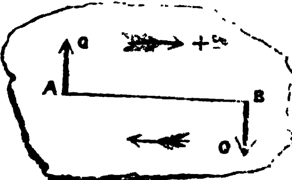
This moment is a measure of the tendency of the couple to twist the body on which it acts, and it is customary to indicate a couple by its moment.

Equilibrium of a Couple.—Though no single force can be found which can counteract the effect of a couple on a rigid body, yet a couple may be found to neutralise the influence of another. Thus if a force Q, equal and opposite to the force P represented by A Q be applied to A in the direction A Q of the line P A produced, and a similar and equal force at B in the line B P produced to Q.



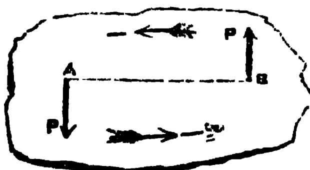
Then the force Q at A being equal and opposite to that of P at A, but in the same straight line, will neutralise it.

Similarly the forces P and Q acting at B will destroy each other, and the body will be in a state of equilibrium under the influence of two couples whose moments are $P \times AB$ and $Q \times AB$, but which tend to twist the body in opposite directions.



A couple whose tendency is to twist the body in the direction in which the hands of a watch move is called a positive couple, such as $Q \times AB$ in the accompanying diagram; while the couple which would cause the body to move in the opposite direction, such as $P \times AB$, is called a negative couple.

It is also convenient to designate a couple by its moments; thus, when we speak of the couple $P \times AB$, we mean the couple whose moment is $P \times AB$, A B representing its arm, and P one of the equal forces acting at its extremity.



It is more convenient generally to represent the product of P and A B by the symbol P · A B, instead of P × A B.

Axis of a Couple.—The *axis* of a couple is a straight line, which is supposed to be drawn perpendicular to its plane, and proportional in length to its *moment*.

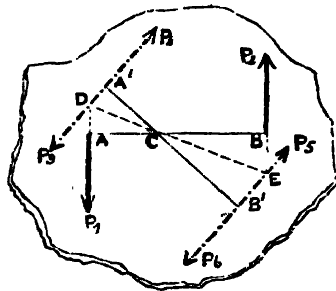
Thus, if the arm of a couple be 4 inches in length, and the forces acting at its extremities be both 5 pounds, and the arm of another couple be 6 inches, and the forces at its extremities be both 8 pounds, the *moments* of these couples will be represented by the numbers 20 and 48, and a line 20 inches in length perpendicular to the plane of the first, and another of 48 inches perpendicular to that of the second, will represent their respective axes.

PROPOSITION XII.

The arm of a couple may be turned round any point in that arm, in the plane of the couple, without altering the conditions of equilibrium.

Let A B represent the arm of the couple, P_1 and P_2 the forces acting at A and B.

In A B take any point C, turn A B round C into the new position A′ C B′.



Now at the points A′ and B′ we may apply equal and opposite forces, P_3, P_4, P_5, and P_6, perpendicular to the line A′ B′, and each equal to the force P_1 or P_2, without disturbing the conditions of equilibrium of the body on which the couple P · A B is supposed to act.

Produce the line A P_1 to meet A′ P_3 in the point D, and P_2 B to meet B′ P_5 in E. Join C D and C E.

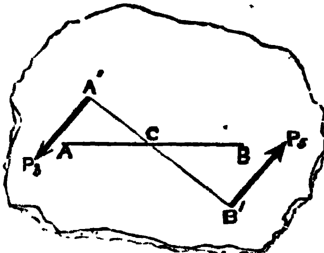
Because in the triangles A′ D C and A D C, the angles at A and A′ are right angles, the side D C common, and A′ C = A C.

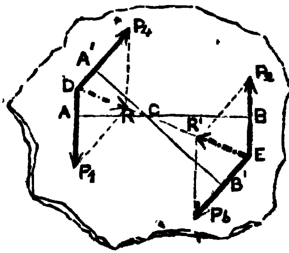
Therefore A′ D = A D, and the angles at D and C are bisected by C D.

Similarly, it may be shown that the angles at C and E are bisected by C E.

Hence C D and C E are both in the same straight line.

Since any force may be transferred from its point of application to any other point in the line of its action, we may remove the forces P_1 and P_4 from the points A and A′ to the point D, and the forces P_2 and P_6 from B and B′ to E. For the sake of clearness, the new position of the forces will then be represented by the two following figures—





one representing the positions of the two forces P_3 and P_5, which have not had their points of application altered, forming a couple, and acting at the extremities of the arm A′ B′; and the other, the positions of the forces whose points of application have been changed.

Now, completing the parallelograms P_1 D P_4 R and E P_2 R′ P_6; D R and E R′ their

diagonals will represent the resultants of the forces P_1 and P_4 acting at D, and P_3 and P_6 acting at E. And since the four forces P_1, P_4, P_3 and P_6 are equal, as are also the angles at D and E; the forces represented by D E and E E' will be equal; and since they attain opposite directions in the same straight line D C E, they will counteract each other.

Hence on the whole we have replaced the couple P : A B by the couple P · A' B', without altering the conditions of equilibrium.

PROPOSITION XIII.

The effect of a couple will not be altered if its arm be removed to any position parallel to itself, either in its own plane or else in one parallel to it.

Firstly let the arm A B of the couple P · A B be removed to the new position A' B' (Fig. 1), A' B' being parallel and equal to A B and in the same plane.

The conditions of equilibrium will not be altered if we apply to A' two forces P_3 and P_4, acting in opposite directions perpendicularly to A' B', and both equal to P_1 or P_2.

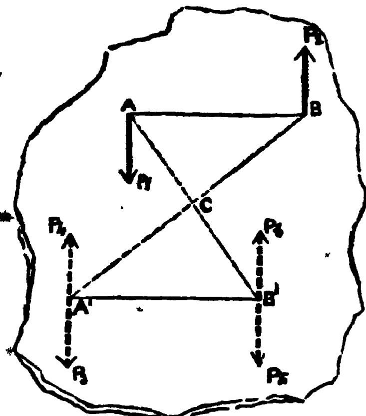


Fig. 1.

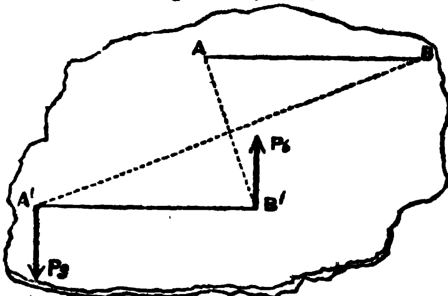


Fig. 2.

Similarly we may apply at B' the forces P_5 and P_6, each equal to the former and perpendicular to A' B'.

Join A' B, A B' meeting in C. Then since A B is parallel and equal to A' B', we have in the triangles A B C, A' B' C, the angles at A and B equal to the angles at B' and A', and also A B = A' B'. Hence A C = B' C and B C = A' C.

Now for the sake of clearness we may suppose the six forces represented above divided into two groups, as in Figs. 2 and 3, one (Fig. 2) consisting of the arm A' B' acted on by the equal and opposite forces P_3 and P_6, forming a couple P · A' B'; and the other of the four equal and parallel forces; P_1 and P_5 acting at A and B' in one direction, and P_4 and P_2 at A' and B in the opposite direction.

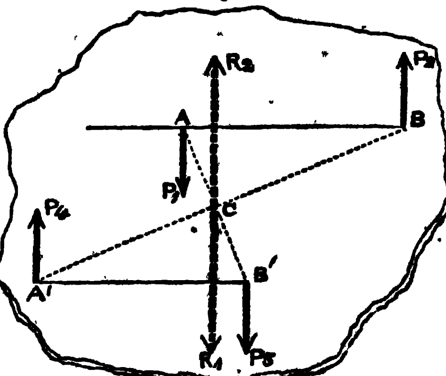


Fig. 3.

Now by Prop. XI., since A C = B′ C and $P_1 = P_5$, P_1 and P_5 acting at A and B′ will be equivalent to a single force C R_1 = 2 P acting at C in the direction C R_1 parallel to A P_1 or B′ P_5. Similarly P_4 and P_2 acting at A′ and B will be equivalent to C R_2 = 2 P acting at C in the direction C R_2 parallel to A′ P_4 or B P_2. But the two equal forces C R_1 and C R_2 acting on C in opposite directions in the same straight line will counteract each other.

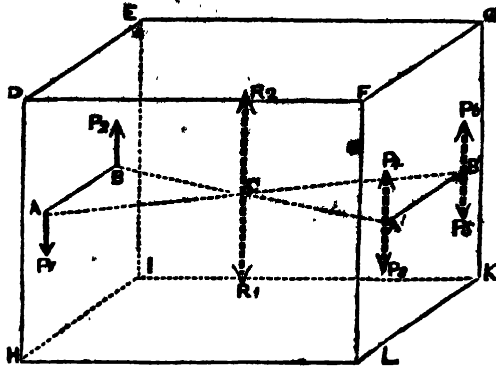


Fig. 4.

Hence the group of forces P_1, P_2, P_4, and P_5 will neutralise each other, and we shall only have the effect of the couple P · A′ B′ left, as represented in the other group (Fig. 4).

On the whole, therefore, we have replaced P · A B by the couple P · A′ B′, the arm A′ B′ being parallel and equal to the arm A B and in the same plane with it.

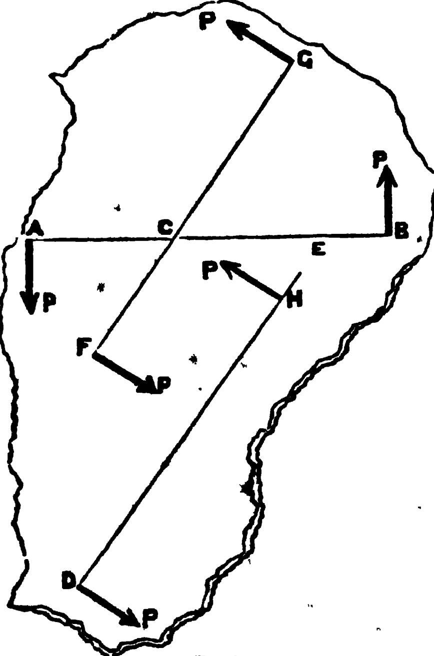


Fig. 5.

Secondly, let the arm A B be removed to the position A′ B′ parallel to A B, in a plane parallel to the plane of the couple P · A B (Fig. 4).

Let D E H I represent the plane of the couple P · A B, F G K L a plane parallel to D E H I, the plane to which the arm A B is supposed to be removed.

For the sake of clearness we may consider these planes as opposite faces of the parallelopiped D E F G H I K L.

Then A′ B′, the new position of the arm, will be drawn in the plane F G K L, parallel and equal to A B in the plane D E I H.

Join A B′, B A′ meeting in C, and apply, as in the preceding case, equal and opposite forces P_3, P_4, P_5, P_6 at A′ and B′, each equal and parallel to P_1 or P_2. Then Euc., B. xi., Prop. 2, the triangles A B C, A′ B′ C are in the same plane, and as in the preceding case A C = B′ C and A′ C = B C.

The parallel forces P_1 and P_5 acting at A and B′ are equivalent to C R_1 = 2 P acting

at C parallel to P_1 or P_2; and the parallel forces P_3 and P_4 acting at B and A' ar equivalent to C R_2 = 2 P acting at C parallel to P_3 or P_4. The forces C R_1 and C R_2 neutralise each other, leaving, as in the previous case, the forces P_3 and P_2 acting at A' and B' as a couple P · A' B'.

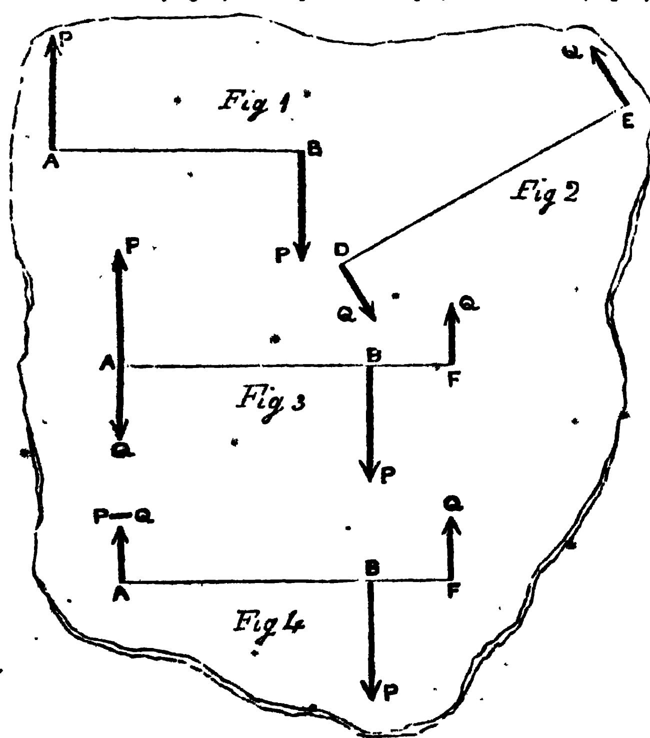
Combining the two last proportions, we see that the arm of a couple may be transferred from any position in its own plane, to any other position in that plane, or to any plane parallel to it.

Thus, if we wish to transfer the couple P · A B (Fig. 5) to the position, in which one extremity of its arm shall correspond to the point D, and the arm itself be in the direction D E, all we have to do is to take any point C in A B, and through C draw F G parallel to D E. Then by Prop. XII. P · A B may be transferred to the position P · F G, and from that by Prop. XIII. to the position P · D H. F G and D H being both equal to A B.

PROPOSITION XIV.

Couples acting in the same plane or in planes parallel to each other, will be equal if their moments be equal.

Let P · A B (Fig. 1) be a positive couple, and Q · D E (Fig. 2) a negative one, either acting in the same plane with P · A B or else in one parallel to it. P being greater than Q, and D E greater than A B.



Then the couple Q · D E can be transferred so as to have its arm in such a position that one extremity D shall coincide with A (Fig. 1), and its arm be the same line as A B. This is represented in Fig. 3, A F being equal to D E. Now, the opposite forces P and Q acting at A, are equivalent to a single force P—Q acting at A in the direction A P, as shown in Fig. 4.

Hence the effect of the two opposite couples P · A B and Q · D E on the rigid body,

on which they are supposed to act, will be equivalent to the three parallel forces P—Q, P and Q acting at the points A, B and F in the same straight line A B F.

Now, if these couples P · A B and Q · D E, are such as to neutralize each other or produce equilibrium, the equilibrium will not be disturbed by the alteration we have made in the position of their arms. Hence, the parallel forces P—Q, P and Q acting in the points A, B and F in the line A B F, must in this case produce equilibrium, and by Prop XI. we shall have

$$(P - Q)\, A B = Q \cdot B F$$

or $P \cdot A B = Q \cdot A B + Q \cdot B F = Q\,(A B + B F) = Q \cdot A F = Q \cdot D E$;

which shows that the moments of the two couples are equal. But a negative couple P · A B would counteract the positive couple P · A B if their arms were in the same position.

Hence, two negative couples will be equivalent to each other if their moments be equal, provided only that they act in the same plane or in planes which are parallel to each other.

The same reasoning will apply to positive couples.

PROPOSITION XV.

To find the resultant couple of two couples, which do not act in the same plane on a rigid body.

For the sake of clearness, we shall suppose the planes in which the two couples act,

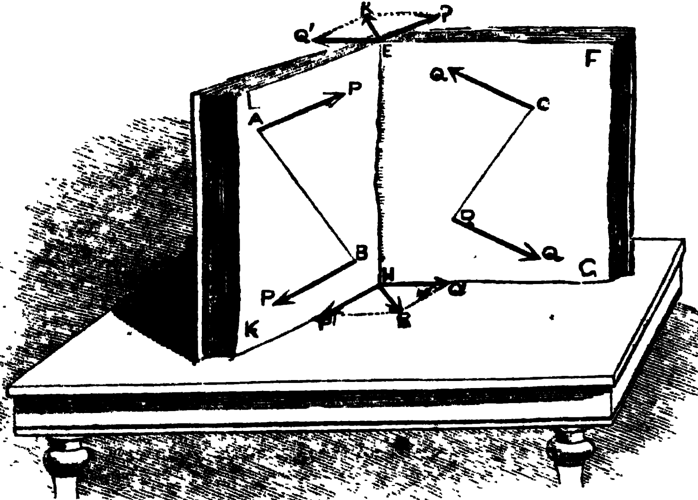


Fig. 1.

represented by the open pages of a book E F G H K L, standing upon a table (Fig. 1), and inclined to each other at an angle θ. E H will be the intersection of the two pages or planes, which are both supposed to be perpendicular to the surface plane of the table.

Let P · A B be the couple acting in the plane L E H K, Q · C D that acting in the plane E F G H.

By the previous proposition the couple $P \cdot A B$ may be replaced by the couple $P' \cdot E H$ acting in the same plane $L E H K$ provided $P' \cdot E H = P \cdot A B$,

$$\text{or } P' = P \cdot \frac{A B}{E H}$$

In $H K$ take $H P' = P \cdot \frac{A B.}{E H}$

Produce $L E$ to [illegible] and make $E P' = H P'$.

Then $P' E H P'$ will represent in magnitude and direction the couple $P' \cdot E H$ which replaces $P \cdot A B$.

In a similar manner $Q' E H Q'$ will represent in magnitude and direction the couple $Q' \cdot E H$, which will replace $Q \cdot D C$ provided $Q' H$ or $Q' = Q \cdot \frac{C D}{E H}$.

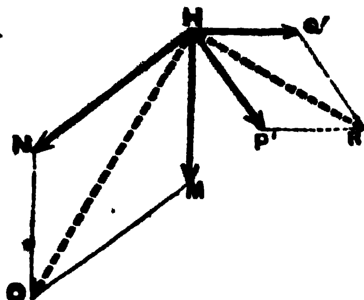


Fig 2.

Completing the parallelogram $P' H Q' R$ on the plane of the table, and joining $H R$, $H R$ its diagonal will represent a force R in magnitude and direction, which will replace the forces P' and Q'.

Since the plane of the table is at right angles to the planes $E F G H$, and $L E H K$, and consequently Euc. B. XI. Prop. 19 to their intersection $E H$. Therefore $H R$ in the plane of the table will be at right angles to $E H$.

Similarly, P' and Q' acting at E will be replaced by a force R, represented by $E R$ acting at right angles to $E H$.

Hence, on the whole we have replaced the couples $P \cdot A B$ and $Q \cdot C D$, by a single couple $R \cdot E H$, whose arm lies in the intersection of the planes in which the couples $P \cdot A B$ and $Q \cdot C D$ act.

Let us now suppose the parallelogram $H Q' R P'$, which we have previously drawn on the plane of the table, to be drawn as in Fig. 2, on the plane of the paper.

Draw $H M$ perpendicular to $H Q'$, $H O$ perpendicular to $H R$, and $H N$ perpendicular to $H P'$.

Then if θ = angle $Q H P'$, $M H N$ will $= \theta$, the angle $M H O$ will $= Q' H R$, and $N H O = P' H R$.

Take $H M = Q' \cdot H E$, $H O = R \cdot H E$, and $H N = P' \cdot H E$. Join $M O$ and $N O$

Then $H M$, $H O$, and $H N$ will be the axes of the couples $Q' \cdot H E$, $R \cdot H E$, and $P' \cdot H E$.

Because $H P' R Q'$ is a parallelogram; therefore, the angle $H P' R = 180° - \theta$ and Trigonometry, page 322.

$$H R^2 = H P'^2 + P' R^2 - 2 H P' \cdot P' R \cos H P' R = H P'^2 + P' R^2 + 2 H P' \cdot P' R \cdot \cos \theta$$

$$\text{or } R^2 = P'^2 + Q'^2 + 2 P' \cdot Q' \cdot \cos \theta$$

Multiplying both sides of the above equation by $H E^2$, we have

$$R^2 \cdot H E^2 = P'^2 \cdot H E^2 + Q'^2 \cdot H E^2 + 2 \cdot P' \cdot H E \cdot Q' \cdot H E \cos \theta$$

but $H M = Q' \cdot H E$, $H O = R \cdot H E$, and $H N = P' \cdot H E$ by construction.

Hence, substituting these values in the equation, we have

$$H O^2 = H N^2 + H M^2 + 2 H N \cdot H M \cdot \cos \theta = H N^2 + H M^2 - 2 H N \cdot H M \cos N H M.$$

An equation which is identically the same as that we should arrive at if we suppose $H M O N$ a parallelogram whose diagonal is $H O$.

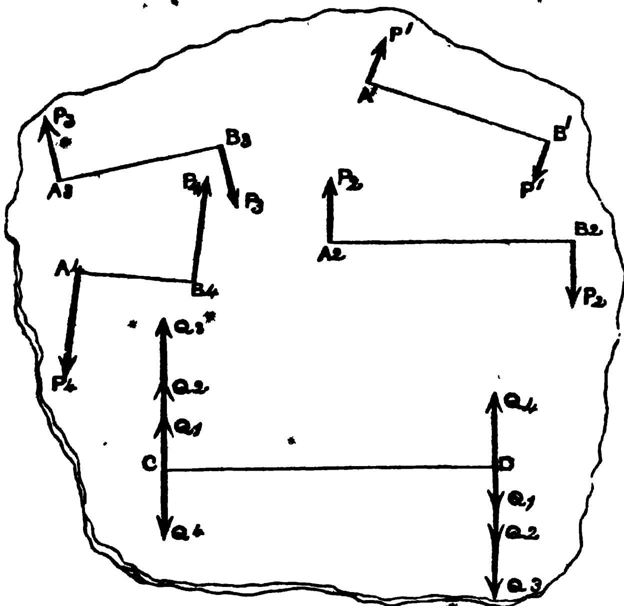
Hence, if two sides of a parallelogram represent the axes of two component couples, its diagonal represents the axis of the resultant couple.

Similarly it may be shown that if the three edges of a parallelopiped represent the axes of three component couples, the diagonal of the parallelopiped will give the magnitude and direction of the axis of the resultant couple.

PROPOSITION XVI.

When any number of couples act in the same, or in parallel planes; the moment of the resultant couple is the algebraical sum of the moments of the component couples.

Let $P_1 \cdot A_1 B_1$, $P_2 \cdot A_2 B_2$, $P_3 \cdot A_3 B_3$ be three positive, and $P_4 \cdot A_4 B_4$ a negative couple acting in the same plane.



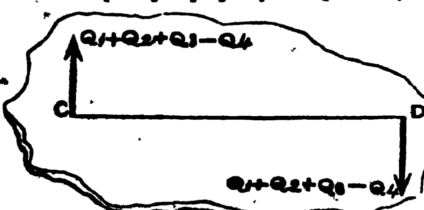
Draw any arbitrary line C D, and apply at opposite extremities of it, two equal and opposite forces Q_1, such that $Q_1 \cdot C\,D = P_1 \cdot A_1\, B_1$ or $Q_1 = P_1 \cdot \frac{A_1 B_1}{C\,D}$.

Then Prop. XIV. the couple $P_1 \cdot A_1 B_1$ may be replaced by the couple $Q_1 \cdot C\,D$.

Similarly apply at C and D two additional equal and opposite forces Q_2 such that

$$Q_2 = P_2 \cdot \frac{A_2 B_2}{C\,D}$$

The couple $P_2 \cdot A_2 B_2$ may be replaced by the couple $Q_2 \cdot C\,D$.



Also the couple $P_3 \cdot A_3 B_3$ may be replaced by the couple $Q_3 \cdot C\;D$, provided $Q_3 = P_3 \cdot \frac{A_3 B_3}{C\,D}$

If one of the couples as $P_4 \cdot A_4 B_4$ acts in the opposite direction to the others, it will be replaced by the couple $Q_4 \cdot C\,D$, where $Q_4 = P_4 \cdot \frac{A_4 B_4}{C\,D}$

The force Q_4 acting in the opposite direction to the forces Q_1, Q_2 and Q_3.

Hence, on the whole, we have replaced the four couples $P_1 \cdot A_1 B_1$, $P_2 \cdot A_2 B_2$, $P_3 \cdot A_3 B_3$, $P_4 \cdot A_4 B_4$, acting in any direction on a rigid body in the same plane by four couples, acting at the extremities of the same arm.

This is evidently equal to a single couple whose arm is C D, with equal and opposite forces $Q_1 + Q_2 + Q_3 - Q_4$ acting at each extremity.

Hence we have

$$(Q_1 + Q_2 + Q_3 - Q_4) \cdot C D = Q_1 \cdot C D + Q_2 \cdot C D + Q_3 \cdot C D - Q_4 \cdot C D$$
$$= P_1 \cdot A_1 B_1 + P_2 \cdot A_2 B_2 + P_3 \cdot A_3 B_3 - P_4 \cdot A_4 B_4$$

or the resultant couple is the algebraical sum of the moments of the component couples.

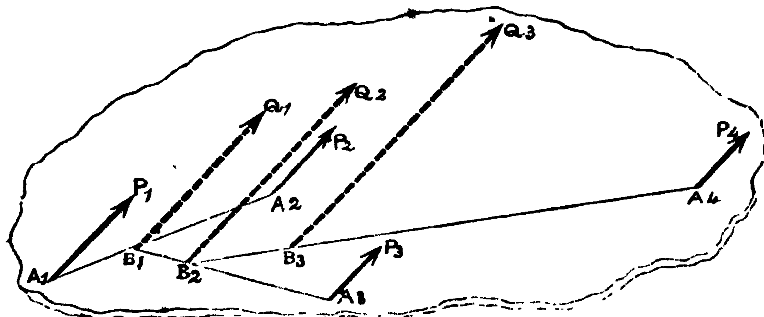
The same reasoning may be extended from four to any number of couples, and by Prop. XIII. to couples acting in parallel planes.

Theory of Couples.—The propositions from 12 to 16 contain the fundamental principles of what has been called the *theory of couples*; for this beautiful theory, which was introduced into the science of Mechanics about forty years since, we are indebted to the distinguished mathematician, M. Poinsot. After we have extended our 11th proposition from two to any number of parallel forces, we shall again return to the theory of couples, and determine by its aid the conditions of equilibrium for any number of forces acting on a rigid body.

PROPOSITION XVII.

To find the magnitude and direction of the resultant of any number of parallel forces acting on a rigid body in the same plane.

Let four parallel forces, P_1, P_2, P_3, and P_4, acting in the same plane on the points



A_1, A_2, A_3, and A_4, be represented in magnitude and direction by $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$.

Join $A_1 A_2$, then by Prop. XI. $B_1 Q_1 = P_1 + P_2$ applied at a point B_1 in $A_1 A_2$ such that

$$A_1 B_1 = \frac{P_2}{P_1 + P_2} A_1 A_2$$

and drawn parallel to $A_1 P_1$ or $A_2 P_2$ will represent Q_1 the resultant of the parallel forces P_1 and P_2 in magnitude and direction.

Then join $B_1 A_3$, in $B_1 A_3$ take a point B_2 such that

$$B_1 B_2 = \frac{P_3}{Q_1 + P_3} \cdot B_1 A_3 = \frac{P_3}{P_1 + P_2 + P_3} \cdot B_1 A_3$$

through B_2 draw $B_2 Q_2$ parallel to $A_3 P_3$ and $= P_1 + P_2 + P_3$.

$B_2 Q_2$ will represent Q_2 the resultant of P_3 and Q_1, or of P_3, P_2, and P_1 in magnitude and direction.

Again, join $B_2 A_4$ in $B_2 A_4$, take a point B_3 such that

$$B_2 B_3 = \frac{P_4}{Q_2 + P_4} B_2 A_4 = \frac{P_4}{P_1 + P_2 + P_3 + P_4} B_2 A_4$$

Through B_3 draw $P_3 Q_3 = Q_2 + P_4 = P_1 + P_2 + P_3 + P_4$ parallel to $A_4 P_4$.

Then $B_3 Q_3$ will represent Q_3 the resultant of the forces Q_2 and P_4, or of the four parallel forces P_1, P_2, P_3, and P_4 in magnitude and direction.

The same reasoning may be extended to any number of parallel forces.

It is sometimes far more convenient to refer these various points, A_1, A_2, A_3, A_4, B_1, B_2, B_3, &c., to two fixed arbitrary lines or axes drawn at right angles to each other, as in Props. VIII. and IX.

Let O X and O Y, drawn through the point O at right angles to each other, be chosen as arbitrary *rectangular axes*, to which the points A_1, A_2, &c., B_1, B_2, &c., are to be referred.

Through A_1, B_1, and A_2 draw $A_1 M_1$, $B_1 N_1$, and $A_2 M_2$ perpendicular to O X.

Then the lines O M_1 and $M_1 A_1$, which determine the position of the point A_1 with respect to the axes O X and O Y, are called the *rectangular co-ordinates of the point* A_1.

Similarly, O N_1 and $N_1 B_1$ are the *rectangular co-ordinates* of B_1; and O M_2 and $M_2 A_2$ are those of the point A_2.

Let O M_1 be represented by the symbol x_1, O M_2 by x_2, $A_1 M_1$ by y_1, and $A_2 M_2$ by y_2.

Through A_1 draw A_1 R parallel to O X, cutting $B_1 N_1$ in S and $A_2 M_2$ in R.

Then from the construction of the figure, it is evident that the angles at S, R, M_1, N, and M_2 are right angles; consequently $A_1 S = M_1 N_1$, $A_1 R = M_1 M_2$, and also $A_1 M_1$, $S N_1$, and $R M_2$, are equal to each other.

Now, by the previous part of the proposition, the point B_1 was so taken in $A_1 A_2$ that

$$A_1 B_1 = \frac{P_2}{P_1 + P_2} A_1 A_2, \text{ or } \frac{A_1 B_1}{A_1 A_2} = \frac{P_2}{P_1 + P_2}.$$

Again, because A_2 R is parallel to B_1 S, therefore Euc. B. vi. p. 2.

$$\frac{A_1 B_1}{A_1 A_2} = \frac{A_1 S}{A_1 R} = \frac{B_1 S}{A_2 R}.$$

But $\frac{A_1 S}{A_1 R} = \frac{M_1 N_1}{M_1 M_2} = \frac{O N_1 - O M_1}{O M_2 - O M_1} = \frac{O N_1 - x_1}{x_2 - x_1}$.

Hence, since $\frac{A_1 S}{A_1 R} = \frac{A_1 B_1}{A_1 A_2} = \frac{P_2}{P_1 + P_2}$,

we have $\frac{O N_1 - x_1}{x_2 - x_1} = \frac{P_2}{P_1 + P_2}$.

Or $(O N_1 - x_1)(P_1 + P_2) = P_2 x_2 - P_2 x_1$.

And $O N_1 (P_1 + P_2) - P_1 x_1 - P_2 x_1 = P_2 x_2 - P_2 x_1$.

Hence $O N_1 (P_1 + P_2) = P_1 x_1 + P_2 x_2$.

And $O N_1 = \frac{P_1 x_1 + P_2 x_2}{P_1 + P_2}$.

Again, $\frac{B_1 S}{A_2 R} = \frac{B_1 N_1 - S N_1}{A_2 M_2 - R M_2} = \frac{B_1 N_1 - A_1 M_1}{A_2 M_2 - A_1 M_1} = \frac{B_1 N_1 - y_1}{y_2 - y_1}$.

But $\frac{B_1 S}{A_2 R} = \frac{A_1 B_1}{A_1 A_2} = \frac{P_2}{P_1 + P_2}$.

Hence $\frac{B_1 N_1 - y_1}{y_2 - y_1} = \frac{P_2}{P_1 + P_2}$.

And $(B_1 N_1 - y_1)(P_1 + P_2) = P_2 y_2 - P_2 y_1$.

Or $B_1 N_1 (P_1 + P_2) - P_1 y_1 - P_2 y_1 = P_2 y_2 - P_2 y_1$.

And $B_1 N_1 = \frac{P_1 y_1 + P_2 y_2}{P_1 + P_2}$.

If, now, from B_2 and A_3 we draw $B_2 N_2$ and $A_3 M_3$ perpendicular to $O X$, and represent $O M_3$ by x_3 and $A_3 M_3$ by y_3.

Then, by a similar construction and demonstration to that used for the points A_1, B_1 and B_2, we can show that

$$\frac{B_1 B_2}{B_1 A_3} = \frac{B_1 N_1 - B_2 N_2}{B_1 N_1 - A_3 M_3} = \frac{B_1 N_1 - B_2 N_2}{B_1 N_1 - y_3}.$$

But it has been shown that $\frac{B_1 B_2}{B_1 A_3} = \frac{P_3}{P_1 + P_2 + P_3}$.

Hence $\frac{P_3}{P_1 + P_2 + P_3} = \frac{B_1 N_1 - B_2 N_2}{B_1 N_1 - y_3}$.

Or $P_3 \cdot B_1 N_1 - P_3 y_3 = B_1 N_1 (P_1 + P_2) + P_3 \cdot B_1 N_1 - B_2 N_2 (P_1 + P_2 + P_3)$.

But $B_1 N_1 = \frac{P_1 y_1 + P_2 y_2}{P_1 + P_2}$, or $B_1 N_1 (P_1 + P_2) = P_1 y_1 + P_2 y_2$, and substituting this value in the above equation, and transposing, we have $B_2 N_2 (P_1 + P_2 + P_3) = P_1 y_1 + P_2 y_2 + P_3 y_3$.

Or $B_2 N_2 = \frac{P_1 y_1 + P_2 y_2 + P_3 y_3}{P_1 + P_2 + P_3}$.

In a similar manner it may be shown that

$$O N_2 = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3}{P_1 + P_2 + P_3}.$$

If, now, perpendiculars $B_3 N_3$ and $A_4 M_4$ be drawn from B_3 and A_4 to $O X$,

And $O M_4 = x_4$, and $M_4 A_4 = y_4$, we shall have by the carrying out the same method of demonstration

$$B_3 N_3 = \frac{P_1 y_1 + P_2 y_2 + P_3 y_3 + P_4 y_4}{P_1 + P_2 + P_3 + P_4}.$$

$$\text{And } O N_2 = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + P_4 x_4}{P_1 + P_2 + P_3 + P_4}.$$

We might proceed from the case of four forces to five, from five to six, and so on; so that, assuming the symbols $\bar{x}$ and $\bar{y}$ to represent the rectangular co-ordinates of the point of application of the resultant of n parallel forces P_1, P_2, P_3, &c., P_n referred to the arbitrary axes O X and O Y; we shall have

$$\bar{x} = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + \&c., + P_n x_n}{P_1 + P_2 + P_3 + \&c., + P_n}.$$

$$\text{And } \bar{y} = \frac{P_1 y_1 + P_2 y_2 + P_3 y_3 + \&c., + P_n y_n}{P_1 + P_2 + P_3 + \&c., + P_n}.$$

If any of the forces act in an opposite direction to the others, they must be taken with the negative sign; the co-ordinates of the various points of application of the forces must also be taken with the proper signs, determined by their position with regard to the axes.

PROPOSITION XVIII.

To find the magnitude and direction of the resultant force and resultant couple of any number of forces acting on a rigid body in the same plane, and the conditions under which there will be equilibrium.

Let P_1, P_2, P_3, and P_4, be four forces represented in magnitude and direction by $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$; acting on a rigid body in the same plane. A_1, A_2, A_3, and A_4, being the points of application of these forces to the rigid body. In the plane in which the forces act, take any two straight lines O X, O Y, at right angles to each other as rectangular axe.

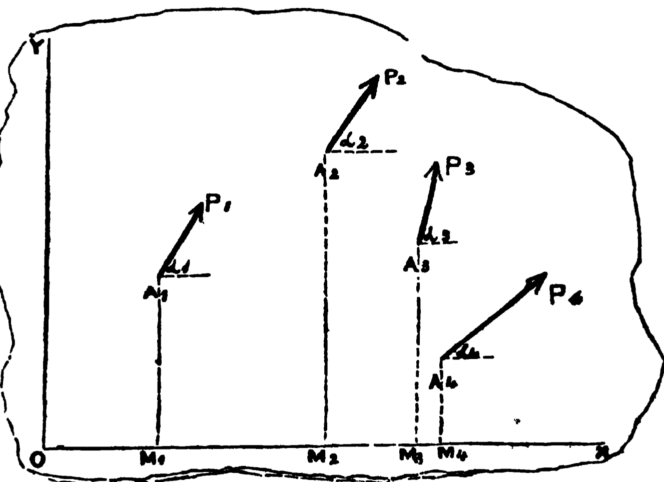


Fig. 1.

Let $O M_1 = x_1$, $A_1 M_1 = y_1$, be the rectangular co-ordinates of A_1, referred to the axes O X and O Y.

$O M_2 = x_2$, $A_2 M_2 = y_2$, those of A_2; $O M_3 = x_3$, $A_3 M_3 = y_3$ those of A_3; and $O M_4 = x_4$, $A_4 M_4 = y_4$ those of A_4

Also let a_1, a_2, a_3, and a_4 be the angles that $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$ make with lines drawn through A_1, A_2, A_3, and A_4 parallel to O X.

For the sake of clearness we will first confine our attention to the force P_1, acting at the point A_1.

Through O (Fig. 2) draw O a_1 at right angles to $P_1 A_1$ produced.

Also through O draw $P_1'' P_1'''$ parallel to $A_1 P_1$.

Make O P_1'' and O P_1''' both equal to $A_1 P_1$.

Then without disturbing the equilibrium of the body we may introduce two equal and opposite forces P_1, represented in magnitude and direction by O P_1'' and O P_1''', acting at O, and also transfer the point of application of P_1 from A_1 to a_1.

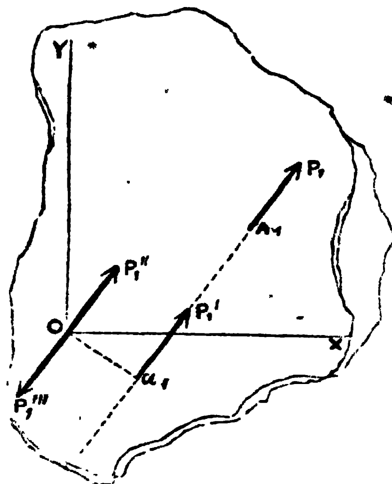


Fig. 2.

Hence, on the whole the effect of the force P_1 acting at A_1 is equivalent to a force P_1 acting at O in the direction O P_1'' parallel to $A_1 P_1$, and two equal and opposite forces represented in magnitude and direction by O P_1''', $a_1 P_1'$ acting at right angles to O a_1 and forming a couple $P_1 \cdot Oa_1$, or a couple whose moment is P_1 multiplied by the perpendicular from O on the line $A_1 P_1$ produced.

In a similar manner the force P_2 (Fig 1) acting at A_2 may be replaced by a force P_2 acting at O, parallel in direction to $A_2 P_2$ and a couple whose moment is P_2 multiplied by the perpendicular from O on $A_2 P_2$ produced.

The force P_3 acting at A_3 may be replaced by a force P_3 acting at O parallel to $A_3 P_3$, and a couple whose moment is P_3 multiplied by the perpendicular from O on $A_3 P_3$ produced.

And, lastly, P_4 acting at A_4 may be replaced by a force P_4 acting at O parallel to $A_4 P_4$, and a couple whose moment is P_4 multiplied by the perpendicular from O on $A_4 P_4$ produced.

On the whole, therefore, the forces P_1, P_2, P_3, and P_4 acting at the points A_1, A_2, A_3, and A_4, may be replaced by forces P_1, P_2, P_3, and P_4, all acting at O in directions parallel to $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, whose resultant will be a single force whose magnitude and direction may be found by Prop. II.; and four couples whose arms will have a common extremity O, and whose moments will be equal to P_1, P_2, P_3, and P_4, each respectively multiplied by the perpendicular from O on the original direction of the force; or on $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$ produced.

By Prop. XVI., these four couples will be equivalent to a single couple whose moment is equal to the algebraical sum of their moments.

The single resultant force acting at O shows the tendency of the four forces P_1, P_2, P_3, and P_4, acting at O to move O in some rectilinear direction, while the resultant couple gives their tendency to twist the body in some direction round the point O.

In order, therefore, that the rigid body should be in equilibrium when acted on by these forces, these resultants must each be equal to nothing, since if the resultant force

lone were equal to nothing, the resultant couple would twist the body round O, as a fixed point; or if the moment of the resultant couple alone were equal to nothing, the body would move so as to keep O in a straight line.

Hence the *conditions of equilibrium* of four forces P_1, P_2, P_3, and P_4, acting on a rigid body in the same plane at the points A_1, A_2, A_3, and A_4 in the directions $P_1 A_1$, $P_2 A_2$, $P_3 A_3$, and $P_4 A_4$ are two.

First.—The resultant of four forces, respectively equal to P_1, P_2, P_3, and P_4, acting on any point O of the body, in directions respectively parallel to $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, must be equal to zero.

Second.—The algebraical sum of the moments of the four couples, whose arms are the perpendiculars drawn from O on $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$ produced, and whose forces are respectively equal to P_1, P_2, P_3, and P_4, must likewise be equal to zero.

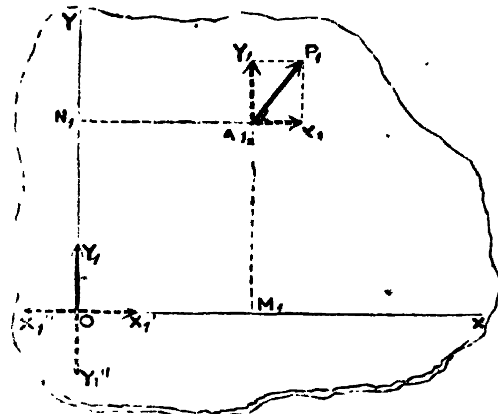


Fig. 3.

This latter condition is technically called taking the moments about the point O.

The reasoning above used for four forces may readily be extended to any number; it must also be observed that the position of the point O is perfectly arbitrary in the solution of problems. It is generally so chosen as to facilitate the solution, and to avoid unnecessary labour.

Instead of pursuing the preceding method, it is frequently advisable to resolve each of the forces P_1, P_2, P_3, and P_4 into two, acting in directions parallel to the arbitrary axes O X and O Y (Fig. 1), as in Props. VIII and XI.

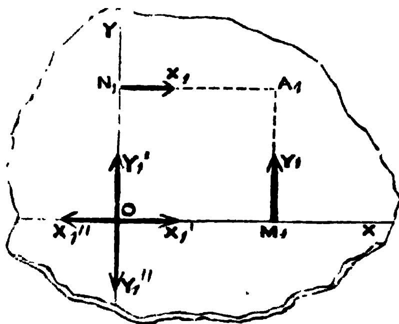


Fig. 4.

Confining, as before, our attention first to the force P_1, acting at A_1 in the direction $A_1 P_1$.

Through A_1 (Fig. 3) draw $A_1 X_1$ and $A_1 Y_1$ parallel to O X and O Y respectively; and through P_1, $P_1 X_1$, and $P_1 Y_1$ perpendicular to $A_1 X_1$, and $A_1 Y_1$. Again through A_1 draw $A_1 N_1$ perpendicular to O Y, and $A_1 M_1$ perpendicular to O X.

The force P_1 acting at A_1 may be replaced by two forces X_1 and Y_1 at right angles to each other, represented in magnitude and direction by $A_1 X_1$ and $A_1 Y_1$ (Prop. III).

Without disturbing the conditions of equilibrium, two equal and opposite forces

represented in magnitude and direction by $O X_1'$ and $O X_1''$, each equal to $A_1 X_1$, may be applied to O in the direction O X.

And two equal and opposite forces $O Y_1'$, $O Y_1''$, each equal to $A_1 Y_1$ may be applied to O in the direction O Y.

Also (Fig. 4) the force X_1 may be transferred from A_1 to N_1, and the force Y_1 from A_1 to M_1.

The forces now acting on the body as in Fig. 4, may be divided into two groups, one as in Fig. 5, consisting of the forces X_1 and Y_1 acting at O, and represented in magnitude and direction by $O X_1'$ and $O Y_1'$; and the other as in Fig. 6, of the couples whose moments are X_1 multiplied by $O N_1$, and Y_1 multiplied by $O M_1$.

The tendency of the couple $X_1 \cdot O N_1$ is evidently to twist the body in the opposite

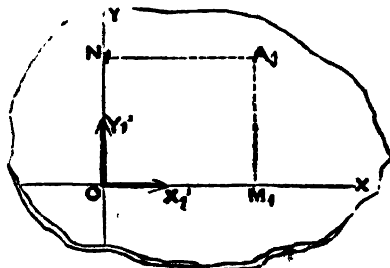


Fig. 5.

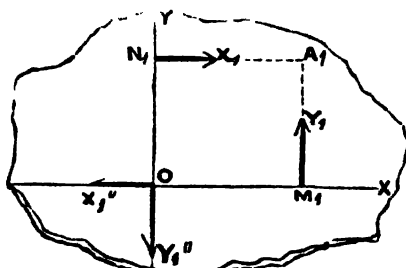


Fig. 6.

direction to that in which the couple $Y_1 \cdot O N_1$ has a tendency to twist it. Hence, if one be considered positive, the other must be negative. By construction $A_1 M_1 = y_1$, and $A_1 N_1 = M_1 O = x_1$.

Hence the resultant moment of these two couples, Prop. XVI. will be

$$Y_1 \cdot x_1 - X_1 \cdot y.$$

On the whole, therefore, we have replaced the force P_1 acting at A_1 by the forces X_1 and Y_1 acting on O in the direction of the axes O X and O Y, and a couple whose moment is equal to $Y_1 \cdot x_1 - X_1 y_1$.

Referring to Fig. 3, we see that $A_1 X_1 = A_1 P_1 \cos a_1$, and $A_1 Y_1 = A_1 P_1 \sin a_1$.

Hence $X_1 = P_1 \cos a_1$ and $Y_1 = P_1 \sin a_1$.

In a similar manner it may be shown the force P_2 acting at A_2 may be replaced by the forces X_2 and Y_2 acting at O in the directions O X and O Y, and the couple whose moment is $Y_2 x_2 - X_2 y_2$, where $X_2 = P_2 \cos a_2$ and $Y_2 = P_2 \sin a_2$.

Likewise P_3 acting at A_3 may be replaced by X_3 and Y_3 acting at O in the directions O X and O Y and the couple whose moment is $Y_3 x_3 - X_3 y_3$, where $X_3 = P_3 \cos a_3$ and $Y_3 = P_3 \sin a_3$.

Lastly, P_4 acting at A_4 may be replaced by X_4 and Y_4 acting at O in the directions O X and O Y and the couple whose moment is $Y_4 x_4 - X_4 y_4$, where $X_4 = P_4 \cos a_4$ and $Y_4 = P_4 \sin a_4$.

On the whole, therefore, the four forces P_1, P_2, P_3, and P_4, acting at A_1, A_2, A_3 and A_4, in the directions $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, have been replaced by four forces X_1, X_2, X_3, and X_4, acting at O in the direction O X, whose resultant is a single force equal to $X_1 + X_2 + X_3 + X_4$, acting at O in the direction O X; by four forces Y_1, Y_2, Y_3, and Y_4, acting at O in the direction O Y, whose resultant is a single force equal to

$Y_1 + Y_2 + Y_3 + Y_4$ acting at O in the direction O Y; together with four couples, whose resultant couple, Prop. XVI., will be one whose moment is equal to

$$(Y_1 x_1 - X_1 y_1) + (Y_2 x_2 - X_2 y_2) + (Y_3 x_3 - X_3 y_3) + (Y_4 x_4 - X_4 y_4).$$

The two forces $(X_1 + X_2 + X_3 + X_4)$ acting at O in the direction O X, and $(Y_1 + Y_2 + Y_3 + Y_4)$ acting at O in the direction O Y, will have a single resultant which may be found by Prop. I.

In order that the body may be in a position of equilibrium when acted on by the forces P_1, P_2, P_3, and P_4, applied at A_1, A_2, A_3, and A_4, in the directions $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, this resultant must be zero, and so must the moment of the resultant couple.

In this case, excluding those conditions which belong to the couples, referring to Prop. IX., we shall have two conditions for the forces acting at O in the directions O X and O Y.

$$X_1 + X_2 + X_3 + X_4 = 0$$

$$\text{And } Y_1 + Y_2 + Y_3 + Y_4 = 0$$

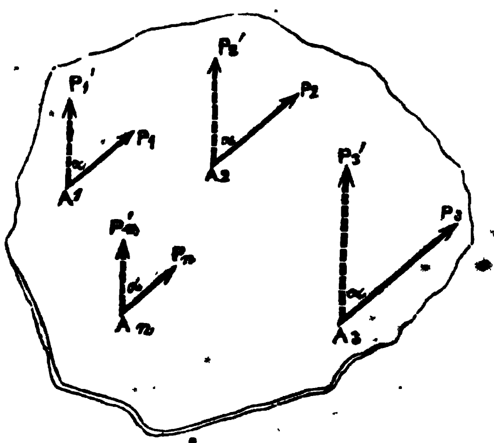
The moment of the resulting couple being also zero, gives us a third condition.

$$(Y_1 x_1 + X_1 y_1) + (Y_2 x_2 - X_2 y_2) + (Y_3 x_3 - X_3 x_3) + (Y_4 x_4 - X_4 y_4) = 0.$$

Where $X_1 = P_1 \cos a_1$, $X_2 = P_2 \cos a_2$, $X_3 = P_3 \cos a_3$, $X_4 = P_4 \cos a_4 \cdot Y_1 = P_1 \sin a_1$, $Y_2 = P_2 \sin a_2$, $Y_3 = P_3 \sin a_3$, and $Y_4 = P_4 \sin a_4$.

The above reasoning may readily be extended from four to any number of forces.

We have for the sake of simplicity drawn the directions of the forces P_1, P_2, P_3, and P_4, in such a manner that their co-ordinates and resolved portions should be positive. In other cases we must remember that if the resolved part of any force act in an opposite direction to that we have drawn, it must be considered negative; and if one or both of the co-ordinates of the points of application of the force be negative, we have only to use the negative sign. Substituting these signs carefully in the above formulæ, we many extend them so as to include every possible case of any number of forces acting on a rigid body in the same place



Centre of Gravity.—By Prop. XVII. we found that if any number of parallel forces represented in magnitude and direction by P_1, P_2, P_3, &c., P_n, acting on a rigid body in the same plane, at points A_1, A_2, A_3, &c., A_n, be referred to rectangular axes, and x_1, y_1; x_2, y_2; x_3, y_3; &c., x_n, y_n, be the rectangular co-ordinates of A_1, A_2, A_3, &c., A_n referred to these axes.

The resultant force will be equal to $P_1 + P_2 + P_3 + P_n$, acting in a direction parallel to P_1, P_2, &c., and applied at a point whose co-ordinates $\bar{x}$ and $\bar{y}$ may be found by the formulæ.

$$\bar{x} = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + \text{\&c.}, + P_n x_n}{P_1 + P_2 + P_3 + \text{\&c.}, + P_n}$$

$$\bar{y} = \frac{P_1 y_1 + P_2 y_2 + P_3 y_3 + \&c., + P_n y_n}{P_1 + P_2 + P_3 + \&c., + P_n}.$$

From these formulæ it is obvious that the position of the point of application of the resultant of the parallel forces is independent of their direction. Hence, if each of the forces represented in magnitude and direction by $A_1\ P_1$, $A_2\ P_2$, $A_3\ P_3$, and $A_n\ P_n$, be turned through the same angle a, about the points A_1, A_2, A_3, &c., A_n; into the position shown by the dotted lines, the point of application of the resultant will not move.

For this reason this point is called the *centre of the parallel forces.*

We have seen (Properties of Matter, page 12), that the weight of a heavy body is produced by the earth's attraction on each of the material particles of which it is composed, and that this attraction is the same for all kinds of matter. Hence, a cubic inch of iron weighs more than a cubic inch of wood; not because the earth's attraction is greater for particles of iron than for those of wood, but because a cubic inch of the former contains a greater number of gravitating particles than the latter.

The attraction which the earth exerts on all masses near its surface, on account of its larger relative mass, is so great, that for all masses which are not very large, we may neglect the attraction which the particles of these masses or those in their neighbourhood exert on one another. We can, on this account, rapidly determine with considerable accuracy the direction of the earth's attraction, or *gravity.*

Let a small weight of lead or brass P, be fixed to one extremity of a thin flexible string A P, and suspended from a fixed point A. The weight P, after oscillating for some little time, will, if undisturbed, rest in such a position that the string A P shall point to the earth's centre. Such an instrument is called a *plummet* or a *plumb-line,* and the line in which it rests the *vertical.* If two plumb-lines be suspended from points situated at different parts of the earth's surface, they cannot be exactly parallel, but must make a certain angle with each other, unless the one point lie in the antipodes of the other, in which case the directions will be in the same straight line.

For all practical purposes, on account of the comparative greatness of the earth's radius, we may say that two plumb-lines of any ordinary length, suspended within any apartment, however large, will be parallel to each other, since if in still water, at a distance of 200 yards from each other, we were to sink two plummets a mile and a half in length, they would not deviate further than 3 inches from perfect parallelism.

It is true that a large mass of matter, such as a mountain, will slightly deflect, as according to the theory of gravitation it ought, the plumb-line from the true vertical, but in ordinary cases this source of error may be neglected. Again, the attraction of the earth will vary according to the distance from its centre of the particle or body attracted; but this variation may also be neglected for bodies which are small when compared with the magnitude of the earth.

Hence, in most cases, even such, for instance, as the largest line of battle-ship, without introducing any sensible error, we may regard the attraction of the earth, on any mass of matter, as acting on every material particle composing that mass, in directions parallel to the plumb-line suspended near it.

The *centre of the parallel forces,* produced by the weights of the material particles of which any heavy body is composed, is called the *centre of gravity of that body.*

Hence, Prop. XVII. enables us to find the position of the centre of gravity of any number of heavy particles whose weights and positions are known.

From the properties of the *centre of parallel forces* we have already demonstrated, it follows that the *centre of gravity* of a heavy body *is that point within or without the body at which the whole of its weight may be conceived to act; and the body will produce the same mechanical effect, as if we were to suppose the whole of its weight concentrated in that point.* This enables us to extend Prop. XVII., to find the common centre of gravity of several bodies, whose weights and the position of whose respective centres of gravity are known.

Again, because the parallel forces exerted by the weights of the material particles of a body will have the same direction in whatever position it is placed, its centre of gravity will not change its position with respect to the body for any change in the position of the body. Hence, if the centre of gravity be fixed, the body will balance about it in every position, because the resultant of the weights of every one of its elementary particles will pass through the fixed point (the centre of gravity) in every position in which the body can be placed.

PROPOSITION XIX.

If a heavy body be in equilibrium when suspended from a point, or when resting on a point in contact with another body, its centre of gravity will be in the vertical line passing through the point of suspension or contact.

Let Figs. 1, 2, and 3 represent a section of the heavy body, taken through the plane passing through its centre of gravity G, and its point of suspension or contact A.

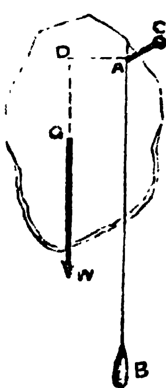


Fig. 1.

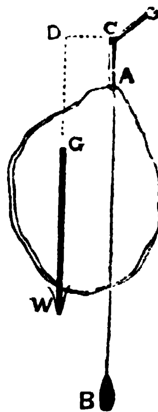


Fig. 2.

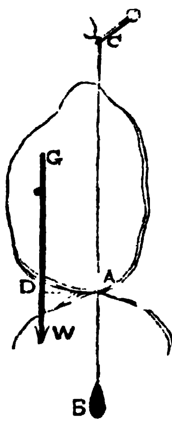


Fig. 3.

In Fig. 1 we suppose the body supported by a pin A C, passing through a hole at A, about which it can move freely; in Fig. 2 it is suspended from a point at A, by a string fixed at C; and in Fig. 3 it is supported by another body with which it is in contact at A.

In all three cases let the vertical be represented by the plumb-line A B, the centre of gravity must lie in that line. If it do not, let it have some other position, as G.

From our definition of the centre of gravity, the weight of the body will produce the same effect upon the body as if, being destitute of weight, a force equal to its weight, acting vertically downwards, was applied to its centre of gravity.

Let G W = W represent this force, the weight of the body, in magnitude and direction, and we then have the case of a rigid body without weight in equilibrium when acted on by the force W acting at G, and the reaction or tension at the point A.

Through A draw A D perpendicular to G W or G W produced. Then by the principle of the transmission of force, W may be transferred from G to D, and W · A D will be the moment of a force tending to twist the body round the point A, which is

counteracted by no other force. Consequently, equilibrium can only exist if W or A D be equal to nothing. But by the terms of the proposition, W cannot be equal to nothing. Hence, there can only be equilibrium where A D is nothing, in which case G must lie in the line A B.

In the same manner it can be shown that if a heavy body balance on a given straight line—as, for instance, the sharp edge of another body which is a straight line—its centre of gravity will lie in that straight line.

If any homogeneous heavy body be of a form which is symmetrical with respect to a certain point or line, the centre of gravity will be in that point or line; for the very idea of symmetry requires, that if any point be taken in the body, there must be another point in that body equidistant from the point or line about which it is symmetrical. Hence, since the centre of gravity is the centre of the parallel forces produced by the weights of the material particles composing the body; if it be homogeneous, that is, composed of particles of the same weight, and distributed uniformly throughout its

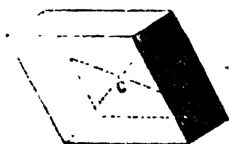


Fig. 1.



Fig. 2.

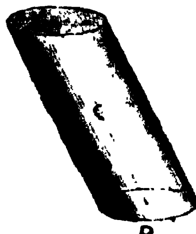


Fig. 3.

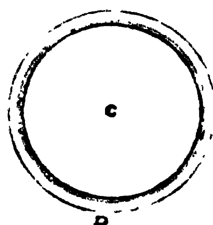


Fig. 4.

substance, any one particle in the body will be balanced about the point or line around which its form is symmetrical, by another particle equal to it in weight, and equidistant from that point or line.

The centre of gravity, therefore, of a sphere, will be the centre of the sphere; that of a cube or oblique parallelopiped (Fig. 1), the point where two diagonals intersect each other. The centre of gravity of a right (Fig. 2) or oblique (Fig. 3) cylinder will be in the middle of its axis; and that of a ring (Fig. 4) will be the centre of the ring.

The centre of gravity of a hollow sphere or cylinder will be the same as if it were solid, provided it be of the same thickness throughout. From this it follows that the centre of gravity of a body need not necessarily be a point within it.

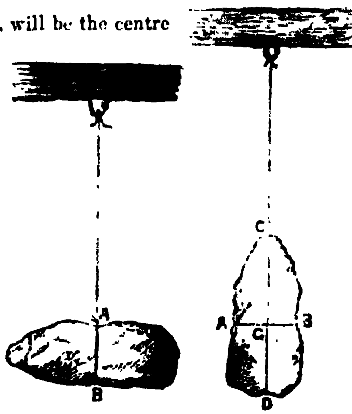


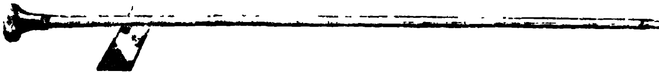
Fig. 1. Fig. 2.

Experimental Determination of the Centre of Gravity.—The preceding proposition affords a means in many instances of determining the position of the centre gravity of a body experimentally.

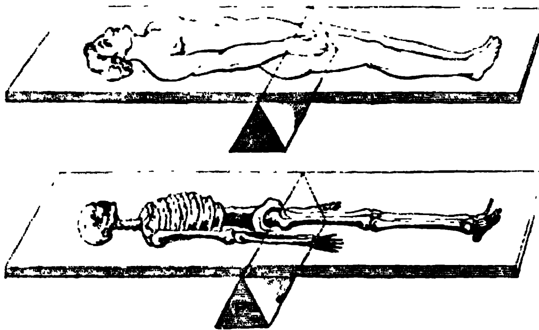
Thus, if a body be suspended by a string, attached to the point A (Fig. 1), the centre of gravity will lie in the vertical line A B passing through the body when it has

attained a position in which it is perfectly at rest. We now suspend it (Fig. 2) from another point C; let C D be the position of the vertical when it is at rest, this line will intersect the previous line A B in the point G, which will be the centre of gravity of the body. In many cases we shall thus be enabled to estimate the position of the centre of gravity. If the body A C B D (Fig. 2) be bounded by plane surfaces, and be of uniform thickness throughout, we know that if the lines A B and C D be traced on both parallel surfaces, the centre of gravity will lie in the middle of the straight line joining the two points of intersection of these lines.

To find the centre gravity of a walking-stick, which is supposed to be symmetrical



with respect to an axis passing through its centre, and has a heavy head or handle, we have only to balance it on the sharp edge of a body, as indicated in the figure, and we know the centre of gravity will lie in the point where the vertical plane passing through the edge intersects the axis of the stick.



In these experimental determinations of the centre of gravity it does not signify whether the body be homogeneous or not. thus, the adjacent figures show the method used by Desaguliers for estimating the centre of gravity of a human body or skeleton, in the positions there indicated.

Stable and Unstable Equilibrium.—Theoretically we say that a heavy body under the action of gravity, may be in equilibrium in a certain position, though we may not be able practically or experimentally to demonstrate it, because the slightest disturbance of the position of the body may destroy the conditions of its equilibrium. Thus, theoretically, a cone will be in a position of equilibrium whether it rest on a table (Fig. 1) on its apex, or (Fig. 2) on its base. In the former case, however, the slightest possible movement to the one side or the other will destroy the equilibrium. The equilibrium which is theoretically but not practically possible, is called *unstable*, while that which is practicable is called *stable*. An egg will rest on its side in a position of *stable equilibrium*, while the attempt to balance it on one of its extremities will only be successful

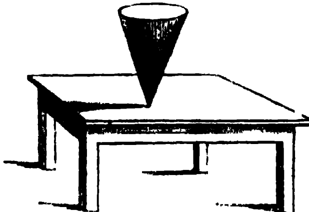


Fig. 1.

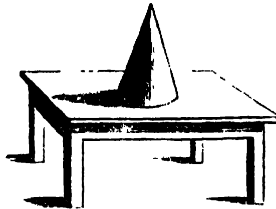


Fig. 2.

by imitating the well-known method Columbus used to destroy its *unstable equilibrium.* A body may be said to be in *stable equilibrium* if after a slight disturbance it recovers its position of equilibrium; and in *unstable equilibrium* if after a slight disturbance it does not recover it. When a body is supported by its centre of gravity it will rest in every position about that point; this has been called *indifferent equilibrium.* Thus, a circle or any plane figure, of uniform thickness, will rest in every position on an axis passing through its centre of gravity.

PROPOSITION XX.

The equilibrium of a body will be stable or unstable according as its centre of gravity is in the lowest or highest position possible.

If a body be in a state of equilibrium, suspended by an axis at A (Fig. 1), about which it is capable of moving freely, or resting on a point A (Fig. 2), any slight disturbance will cause the body to move to the right or left, as shown by the dotted lines, and the centre of gravity to describe a small arc G G′; but the centre of gravity having moved, however slightly, below the point G, the tendency of the weight of the body will evidently be to remove the centre of gravity further from the original position G. But if the centre of gravity be in its lowest position G (Fig. 3) after any disturbance which should remove the point G to G′, the tendency of the weight will be to restore the body to its original position, and the point G′ to G.

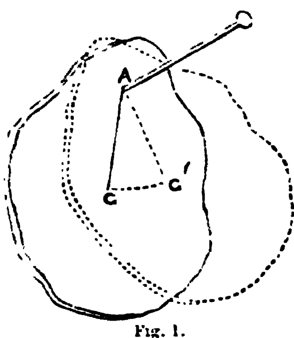


Fig. 1.

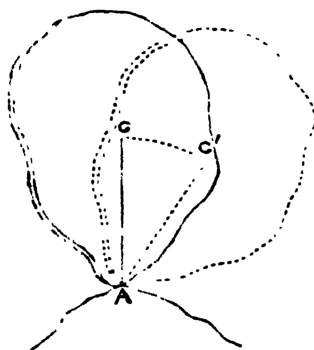


Fig. 2.

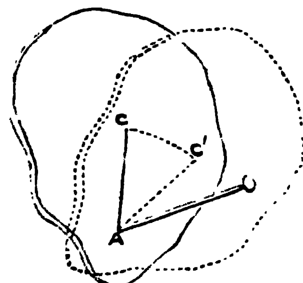
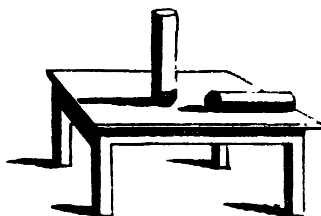
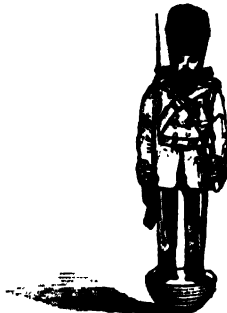


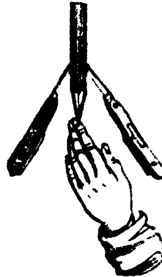
Fig. 3.

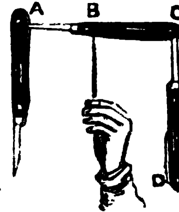

Fig. 4.

The tendency of the centre of gravity to recover its place if removed from its lowest to a higher position is well illustrated by fixing the half of a bullet (Fig. 4), or any hemispherical heavy metallic body, to a cylinder of elder-pith, cork, or any other light substance. The centre of gravity of the whole will lie somewhere in the heavy hemisphere; and if the cylinder be removed to any position, except it be laid almost absolutely on its side, it will

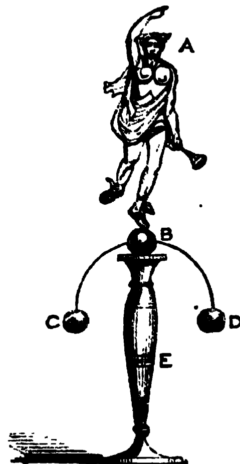
speedily recover its upright position, as shown in the figure. Toys are constructed on this principle, the cylinder being cut into the form of a soldier; a regiment of such mimic troops being pressed nearly to the ground by passing a stick over them, seem immediately to spring up and recover their position as if by magic. The toys called *tumblers*, made of plaster of Paris with a hemispherical bottom loaded with iron or lead to bring the centre of gravity to the lowest position possible, also owe their amusing properties to the same principle. Screens have been invented which, by this contrivance, right themselves after being pressed down. The annexed illustrations show how a pointed stick may be easily balanced on the tip of the finger by fixing two pen-knives in its side, thus converting an unstable into stable equilibrium; and how three pen-knives, placed in the position A, B, C, and D, may be kept in equilibrium on the point of a needle held in the hand in both cases the centre of gravity must fall below the point on which the bodies are balanced.







A mall figure A with its foot fixed to a sphere B, through which passes a bent wire, having two leaden balls C and D attached to its extremities, if placed loosely on a stand E will speedily recover its upright position after being moved from it: the figure being so constructed that the centre of gravity of the three bodies A, C, and D, falls below the point of support, where the sphere B rests on the stand E.



The apparent paradox of a double cone ascending an inclined plane by its own weight, is produced by the construction of the cone and plane being such that the centre of gravity of the cone really descends, and by its descent causes the cone to ascend the plane.

Construct an inclined plane of two equal pieces of straight wire A C and B C (Fig. 1), fixed at their extremities to three upright pieces A E, B F, and C D, standing perpendicular to the horizontal stand D E F: A E and B F being equal to each other, and C D less than A E or B F.

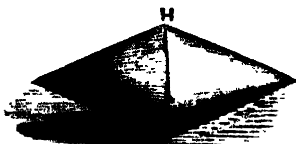
Let H be a double cone consisting of two right cones united together by their circular bases.

If the distance A B or E F be equal to or less than that between the two vertices of the cone, and the difference between A E and C D less than the radius of the circular base of the double cone, upon placing the cone near C with its circular base between the wires A C and B C it will roll up the inclined plane till its extremities are stopped by the upright supports at A and B.

That the centre of gravity really descends in this case will be readily seen by the

diagram (Fig. 2), which represents an imaginary section of the inclined plane through C D, perpendicular to E F, represented by the lines G' O, O D, C D, G' C (Fig. 1).

Let the circles K L M, K' L' M' (Fig. 2) represent the two positions of the circular base of the cone at the commencement and end of its ascent up the inclined plane; G and G', the centres of these circles, will represent the position of its centre of gravity at these periods, and the line G G' the path of the centre of gravity during the ascent; since the centre of gravity of the double cone is the centre of its circular base. By construction, the angle G' O D is a right angle. Through C' and G' draw C' N and G' R parallel to O D', and the radius G M of the circle K L M perpendicular to C' N. Then if M' be the point where the circle K' L' M' cuts G' O, G R or N M' will evidently represent the vertical descent of the centre of gravity of the cone, while the cone itself is ascending the plane. And since G N (Fig. 2) is equal to the difference between the two supports A E and C D (Fig. 1) of the inclined plane, in order that the cone may ascend by the descent of its centre of gravity, this difference must be less than the radius G' M' (Fig. 2) of the base of the cone.



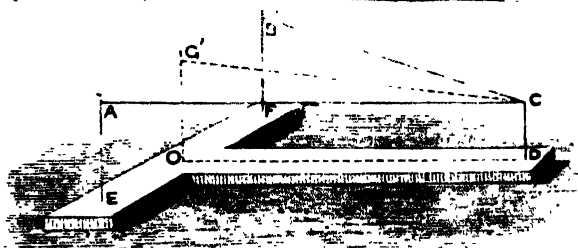


Fig. 1.

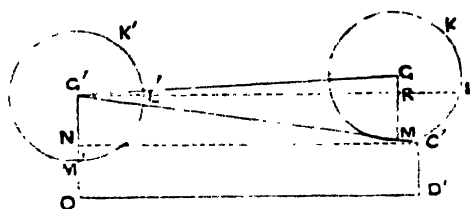


Fig. 2.

PROPOSITION XXI.

A body placed on a plane horizontal surface will stand or fall according as the vertical line drawn through its centre of gravity passes within or without its base.

Let A B C (Figs. 1 and 2) represent sections of two bodies, by vertical planes passing through their centre of gravity G, having their bases A B placed in contact with a horizontal plane.

In Fig. 1 the vertical G W passing through G falls within A B, and in Fig. 2 without A B.

The whole effect of the weight of the body is equivalent in both cases to a single force equal to that weight applied at G in the direction G W.

In Fig. 1 this force is evidently destroyed by the reaction of the plane, and the body will stand. In Fig. 2 this force has a moment about the point B, which is not counteracted by the resistance of the plane; and consequently the body will turn round B

till it falls in such a position that G W will lie within the base on which it rests. This property is readily shown, experimentally, by taking two oblique cylinders (Figs. 3 & 4) of such lengths, that the vertical passing through the centre of gravity of the one, when placed on a table, shall fall within its circular base and the other without. The former will stand and the latter will fall when placed with their circular bases in contact with the table.

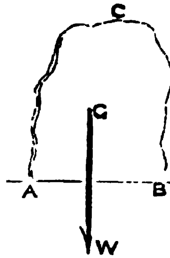


Fig. 1.

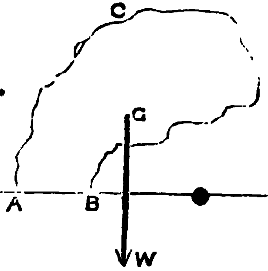


Fig. 2.

If the surface on which the body rests be inclined instead of horizontal, the same

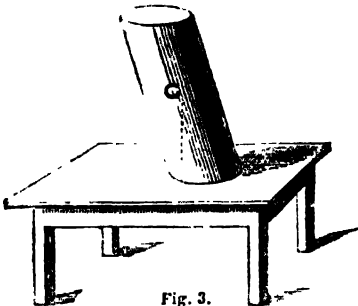


Fig. 3.

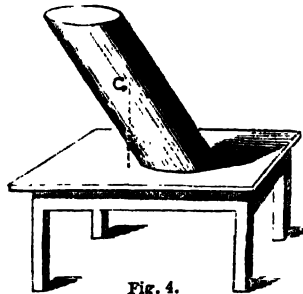


Fig. 4.

effect will be produced, provided the body be prevented from sliding down the inclined plane by friction or some other force.



Thus, a loaded waggon passing along an inclined road will not be overturned so long as the vertical G A passing through the centre of gravity G of the load and waggon falls within the points where the wheels touch the road. As soon as the inclination of the road becomes so great, or the position of the centre of gravity of the load be such that the vertical falls without these points, the waggon will be overturned.

The broader the base and the lower the centre of gravity of the body, the firmer will be its stability; a waggon carrying a load of straw is more likely to be overturned than if it were laden with an equal weight of iron or some other heavy material; because in the first instance the distance of the centre of gravity of the load and waggon from the ground is much higher than in the latter.

The round tower which is the belfry of the cathedral at Pisa, is 190 feet high, and deviates from the perpendicular about 14 feet. At Bologna there is a square tower called Garisenda, 134 feet high, and deviating 9 feet from the perpendicular. Both these buildings are supposed to owe their inclination from the depression of the ground under their foundations; and they have not been overthrown because the vertical, passing through their centre of gravity, still falls within their base.

Desaguliers, in his "Course of Experimental Philosophy," has the following interesting observations on the position of the centre of gravity in animal bodies when at rest, or in motion, in various positions. "When we stand upright, with our feet as represented in Fig. 1, the line of direction (*i. e. the vertical passing through the centre of gravity*) goes through the point C and passes between our feet to D, and we may move our heads from E to F or G, and our bodies forwards, backwards, or sideways as far as I or II, without danger of falling, or stirring our feet, as long as the line of direction traverses no farther than I A or II B, and falls anywhere within the space A B, which in this situation of our feet makes a pretty large base. But if we set one foot before the other, as in Fig. 2, a little push sideways will make the line of direction (which went through C) fall out of the base to the right or left towards E or B; in which case a man must fall if he does not quickly remove his feet to the position of Fig. 1.

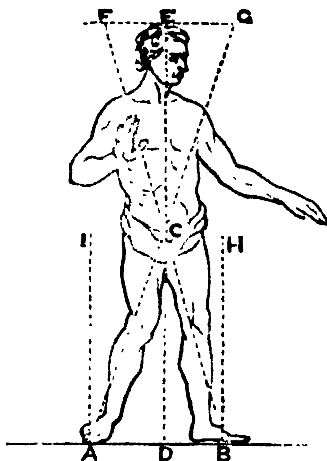


Fig. 1.

"When we stand upon either leg we must bring our body so much over the foot, that the centre of gravity being directly over it, the line of direction may go through the sole of it; and in walking, the line of direction must travel through every place where each foot is set down, going successively through the points E, A, D, B (Fig. 3), while the centre of gravity goes through the points G, C, F, &c., so that unless a man in walking straight forward sets one foot directly before the other, the line of direction will not describe a straight line upon the plane where the man walks, but an indented line; that is, angles to the right and left, whilst the body of the man goes on in a waddling motion. This we see in the walking of fat people, and all others that straddle in their gait. The line of direction going through the points A, B, C, D, E, describes a straight line in Fig. 4, where the feet are set before one another; but when the motion of one foot is in a parallel line with the motion of the other, an indented line is described by the motion of the centre of gravity above, and the line of direction as it cuts the ground at A, B, C, D, E (Fig. 5).



Fig. 2.

"It is not strictly true that any man in his common walk sets one foot so exactly before the other as to carry on the bottom of his line of direction in a straight line as represented in Fig. 4, because if a straight line be drawn with chalk, it is difficult to walk straight along it; but the plainest proof is the observation of two upright sticks, of about the height of a man, the one painted white and the other black, and set up

about ten yards beyond one another, in the same line that a man walks towards them; for in such a case, though he keep one eye shut, the last stick will appear sometimes on the right and sometimes on the left of the first; and the more so the nearer the man comes to the sticks. Rope-dancers, indeed, go in a straight line; but it is what they have learned by art, and inured themselves to by long practice; yet they must, even after all, have helps to keep their centre of gravity over the rope. They generally keep their eyes on some distant point in the same plane as the rope. They have commonly a long pole loaded at the ends with the balls of lead B *b*, by the motion of which they can alter the position of the common centre of gravity of their body and the pole; as, for example, the centre of gravity of the rope-dancer C A (Fig. 6) being at A, his line of direction should go through *a*, off from the rope; but by moving the pole towards B, the common centre of gravity of the man and pole is brought to C, and the line of direction C D goes through the rope. Those that are well skilled in this art will sometimes use their arms only instead of a pole; and it is very common for several of them to dance with a flag, with which they strike the air the same way that the centre of gravity goes when the line of direction does not go through the rope; and by the reaction of the air the centre of gravity is brought back to its proper place.

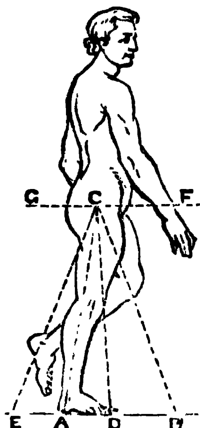


Fig. 3.

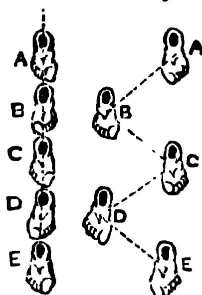


Fig. 4. Fig. 5.

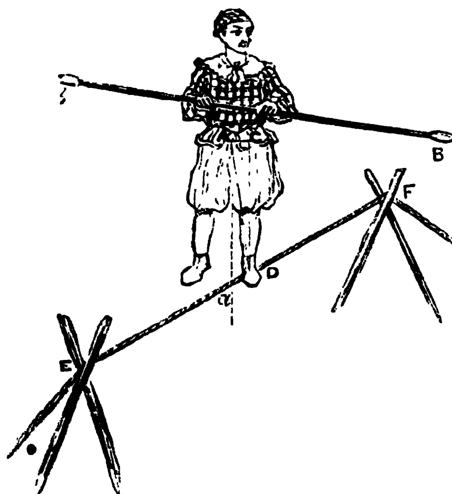


Fig. 6.

"The ancients observing that horses and other quadrupeds, in galloping, lift up their two fore feet, and then their hind feet as soon as the fore feet are set down, did imagine that in walking, as well as pacing and trotting, a horse has two feet off of the ground at one time; and accordingly in their brass or marble statues, they have represented their horses with two legs off of the ground diagonally opposite—as the right before and the left behind, or left before and right behind. The modern statuaries have also fallen into the same error, because in the quick walking of

a horse, the eye cannot well distinguish; and therefore Borelli (*De motu animalium*) has shown from mechanical principles, that the motion of raising two feet at once in walking cannot be consistent with the wisdom and simplicity of nature."

"Let us consider a horse as an oblong machine sustained by the four legs, as four props or columns, resting on the points A, B, C, D (Fig. 7), which make a rectangular quadrilateral figure; then the line of direction will fall perpendicularly on E, a point in or near the centre of the quadrilateral figure, which will make the station or standing of the horse the most firm. The progressive motion begins by one of the hind feet—as, for example, the left hind foot C, which, by strongly pressing back the ground, moves forward the centre of gravity, and consequently carries on the line of direction from E to G as itself moves from C to F. This done, the foot B is raised and carried forward as far as H, which motion of the foot is easy, because the line of direction first falls within the triangle A B D; secondly, within the trapezium A B F D—that is, the body of the horse is sustained by three or by four columns. Lastly, the three feet A, D, F, remaining firm, and taking the line of direction at G, immediately the left fore foot B is carried forward to H; and by the impulse already made, the centre of gravity is also carried over I—namely, the central point of the rhomb A H F D. The motion of the two left feet being completed, the impulse and motion of the right hind foot D begins, and then that of the right fore foot, and so on in the manner above described, as the animal moves forward.

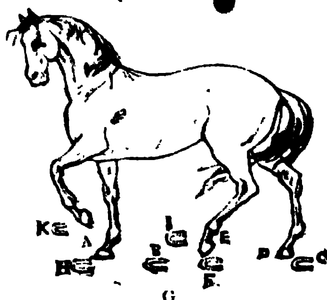


Fig. 7.

"Ducks, geese, and the greatest part of the water-fowl, whose legs are set wide asunder for the convenience of their swimming, and turning quick in the water, have always a waddling motion upon land; but a cock, a stork, an ostrich, and most other birds that are not web-footed, walk almost directly forward without waddling (especially when they walk slowly), having their legs so placed as to put one foot before the other with greater ease. Thus quadrupeds seldom or never waddle, because they have commonly three feet upon the ground at a time; so that however the base receiving the line of direction alters from a quadrangular to a triangular figure, that part of it in which the line of direction falls is always in or near the same line."

"When a man stands in a firm posture (Fig. 1) A B, the distance of his feet is the length of a quadrilateral figure, whose breadth is nearly the length of the feet, and D is the point under the centre of gravity C, where the line of direction falls. Let the lines A C and B C be drawn, then let these two lines and D C be continued to the points E F G so as to make the triangles E C G and A C B equal and similar, as long as the line F D (or a plane going through it) cuts the whole body of the man into two equal parts, the centre of gravity will be at C, and C D will be the line of direction. But if the body be inclined towards the left hand H, the centre of gravity will move from C to H, the line of direction will become H B, and the right foot being easily removed from A, may be carried on beyond B, by which means the man will go on towards the left. In like manner, by inclining towards I, the line of direction will be removed to I A, and the man go to the right. When a man stands upon one foot it is with some difficulty.

For example, let the line of direction be C D (Fig. 3); by the motion of the blood and lungs, and other animal motions, the centre of gravity will be apt to vacillate or totter towards F or G on either side about the centre of motion D, where now the base is but small. If the line of direction comes to B, the man must fall forwards, backwards if to E; and though A be under the heel of the foot, yet in the motion of the said line of direction from D to A, the body will be apt to go towards E, and so bring the line of direction beyond the base. This will more probably happen in the side motion of the body; so that the body will be in danger of falling, unless the right foot be put down towards that side where the body inclines. Birds stand upon one foot much more easily than men, because their line of direction being much shorter, and the base of one foot a large rhomboidal figure made by the four claws, the line of direction cannot go out of that base, unless the centre of gravity rises, which is impossible without a violent motion."

When a porter carries a burden upon his shoulders, he must stoop, because, if he should stand upright, the common centre of gravity of the man and burden would be so far brought back, that the line of direction would fall behind the feet.

PROPOSITION XXII.

A heavy body rests upon a horizontal plane, to find the pressures produced by its weight upon the points of contact by which it is supported.

When a heavy body rests upon a horizontal plane, the pressures produced by its weight on the points on which it is in contact with the plane can be determined, if these points do not exceed three in number. When the points of contact are more than three, the pressure upon each is indeterminate.

If the body rest upon a single point, it is evident that the pressure on this point is equal to the weight of the body.

When the body rests upon two points A and B, as in Fig. 1, where the body is supposed to consist of a circular disc and a cylindrical stick, fixed perpendicularly to its centre: the pressures on these points may readily be determined.

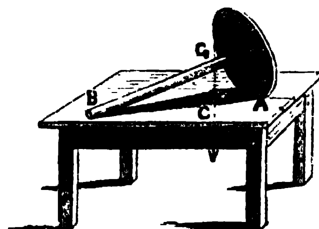


Fig. 1.

Let G be the centre of gravity of the body resting on the plane.

This body is evidently kept in equilibrium by its weight, and the equal and opposite reactions produced by the two pressures on the points A and B.

The vertical passing through G must therefore pass through the line joining A and B.

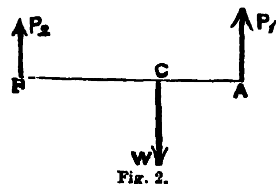


Fig. 2.

Let C be the point where the vertical cuts A B; the weight of the body acting at G may be transferred from G to C.

Then (Fig. 2) we have two reactions P_1 and P_2 acting upwards, perpendicular to the plane at A and B; and a weight, W equal to the weight of the body acting downwards, and also perpendicular to the plane.

Hence Prop. XI. $P_1 + P_2 = W$, and $P_2 \cdot B\,C = P_1 \cdot A\,C$.

Two equations from which P_1 and P_2 may readily be determined, when the position of the centre of gravity G of the body is known.

The reactions P_1 and P_2 will be equal and opposite to the pressures produced by the body at A and B.

When the body rests upon three points A, B and C (Fig. 3), we may still find the pressures produced by its weight on these points.

Join the points A, B and C. In order that there may be equilibrium, the vertical passing through G the centre of gravity of the body must fall within the triangle A B C. (Prop. XXI.)

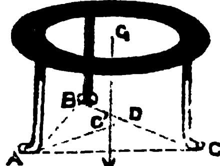


Fig. 3.

Let C' be the point where the vertical passing through G cuts the triangle A B C.

Join A C', and produce A C' to meet B C in D.

If W be the weight of the heavy body, we may transfer the force W from G to C'. This force, acting perpendicular downwards at C', may be resolved into two forces, W_1 acting at A, and W_2 acting at D, both perpendicular to the plane of the triangle A B C.

$$\text{Where } W_1 + W_2 = W, \text{ and } W_1 \cdot AC = W_2 \cdot CD.$$

The force W_2 acting at D may again be resolved into two others, W_3 acting at B and W_4 at C, both perpendicular to the triangle A B C.

$$\text{Where } W_3 + W_4 = W_2, \text{ and } W_3 \cdot BD = W_4 \cdot CD.$$

From these equations the forces W_1, W_3, and W_4, which will be the pressures exerted by the heavy body on the points A, B and C, may be readily determined.

If the heavy body rest on the plane by four points of support, as a table on its four legs A, B, C, D, Fig. 4, the pressures upon these points will be indeterminate if we consider the body perfectly rigid.

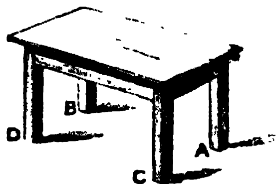


Fig. 4.

For since it may be supported by any three of the points of contact, the pressure on the fourth may be either nothing or some finite proportion of the weight of the body. The method used in the two preceding cases to determine the pressures will fail in this. The only condition we have is that the sum of the pressures on the points A, B, C and D must be equal to the weight of the body; and if we consider them as weights acting perpendicularly to the plane on which the table rests, the centre of gravity of these weights will lie in the vertical, passing through the centre of gravity of the table.

The same reasoning may be extended to the cases where the points of contact are more than four.

The Magic Clock.—The influence of the centre of gravity on the position of equilibrium of a heavy body, which can only turn round a fixed horizontal axis, is well illustrated by an ingenious contrivance called the *magic clock.*

A transparent glass dial-face has a hole pierced through its centre at C (Fig. 1); in this is fixed a horizontal axis; on this axis is placed the hour-hand of the clock, which can move freely round it to the right or left. The extremity of the hand opposite the pointer B is terminated by a hollow ring; in this ring there is a heavy spherical ball A, capable of moving freely round it. This ball is made to move uniformly round the interior of the ring, by a watch-movement concealed in the hand, once in twelve hours,

in the direction indicated by the arrow in Fig. 1. The weight of the ball A is so proportioned to that of the hand with its concealed watch-work, that as A moves round the interior of the ring G, the common centre of gravity of the ball and hand would describe a circle round the centre of the axis C, if the hand were fixed in one position. Since the hand can move freely round C, for every new position of A the hand will assume that position which shall make C G vertical (Prop. XIX.) As A, therefore, is carried round the ring uniformly once in twelve hours, the pointer of the hand B moves with the same uniformity round the dial in the opposite direction, and indicates the hour.

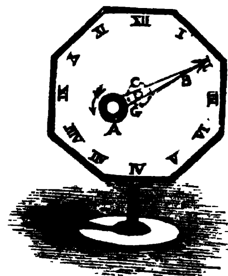


Fig. 1.

The ball A, and the watch-movement which causes it to turn round the ring, being both concealed, the hand seems to move by itself, as if by magic. The remarkable property that the hour-hand being made to move by your finger backwards or forwards, when left to itself, will, after a few oscillations, resume its position, and point to the correct hour, adds considerably to the illusion.

Let D (Fig. 2) represent the centre of the ring round which the spherical body

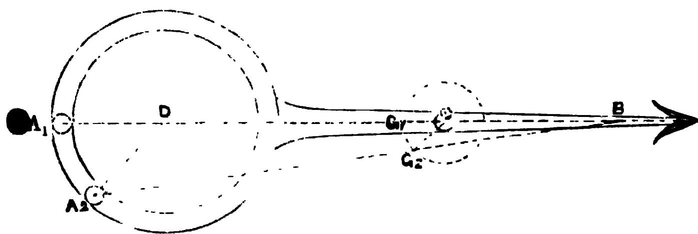


Fig. 2.

circulates; A_2, the centre of the spherical body; C, the centre of the axis round which the hand moves; B, the centre of gravity of the hand and its concealed watchwork, exclusive of the moveable spherical body; G_2, the common centre of gravity of the hand and spherical body. Also, let W represent the weight of the hand and works exclusive of the sphere, whose weight is represented by W'.

Then, if W and W' be so chosen that $W \cdot B\,C = W' \cdot C\,D$, the common centre of gravity G_2 of the hand and moveable sphere will describe a circle round C, if the hand be supposed to be fixed while A_2 describes a circle round D.

Let A_1 represent the position of the centre of the sphere, when it lies in B C D produced; G_1 the centre of gravity of the hand and sphere for this position.

Join D A_2, C G_2, and B G_2 A_2.

When the centre of the moveable sphere is in the position A_1, we have a weight W acting at B, and another W' at A_1, in directions parallel to each other; and since G_1 is the position of their resultant, we have (Prop. XI.)

$$W \cdot B\,G_1 = W' \cdot A_1\,G_1;$$

but by the construction of the instrument,

$$W \cdot B\,C = W' \cdot C\,D.$$

Hence $\frac{B G_1}{B C} = \frac{A_1 G_1}{C D}$, or $\frac{B C + C G_1}{B C} = \frac{A_1 C - C G_1}{C D}$.

Therefore $C D \cdot B C + C D \cdot C G_1 = A_1 C \cdot B C - B C \cdot C G_1$.

Or $C G_1 \cdot (C D + B C) = (A_1 C - C D) B C$.

And $C G_1 \cdot B D = A_1 D \cdot B C$.

Or $C G_1 = \frac{A_1 D \cdot B C}{B D}$.

When the centre of the sphere is in the position A_2, we have a weight W acting at B, and a weight W′ at A_2, in parallel directions. Hence (Prop. XI.) we have

$$W \cdot B G_2 = W' \cdot A_2 G_2.$$

$$\text{But } W \cdot B C = W' \cdot C D.$$

$$\text{Hence } \frac{B G_2}{B C} = \frac{A_2 G_2}{C D}.$$

Therefore Euc. B. vi. p. 2, the line $C G_2$ is parallel to $A_2 D$, and $B C G_2$ and $B D A_2$ are similar triangles; and consequently

$$\frac{C G_2}{A_2 D} = \frac{B C}{B D}, \text{ or } C G_2 = \frac{B C}{B D} \cdot A_2 D.$$

But $A_2 D = A_1 D$. Hence $C G_2 = \frac{A_1 D \cdot B C}{B D}$. The same value which we have before obtained for $C G_1$.

Hence for every position A_2 of the centre of the moveable sphere out of the line B C D, we shall have $C G_2$ parallel to $D A_2$, and equal to $C G_1$.

As A_1 therefore describes a circle round D, G_1 will describe

Fig. 3.

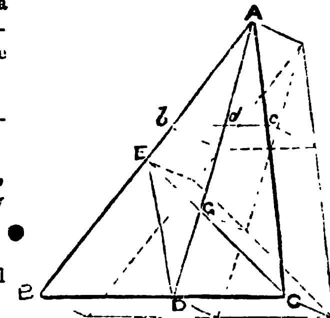
Fig. 4.

a circle round C in the same direction, and uniformly with it; the radius of this

circle being equal to the product of the radius of the circle described by the centre of the sphere, and the distance of B from O divided by B D.

When the centre of the sphere is in the position A_1, the hand will rest in such a position that C G_1 shall be vertical (Prop. XIX.), as shown in Fig. 3. When A_2 represents the position of the centre of the sphere, C G_2 will be vertical, and the hand will rest in the position shown by Fig. 4. Hence it is evident that, as the watch-movement concealed in the hand causes the centre of the sphere to move from A_1 to A_2, the extremity of the pointer of the hand will move through a similar arc in the opposite direction round C as its centre.

Mathematical Determination of the Centre of Gravity of a Heavy Body.—The application of Prop. XVII. to find the centre of gravity of a body of uniform or variable density, in most instances requires the aid of the differential and integral calculus before we can arrive at a solution. As this would lead us beyond the limits proposed for the mathematical department of our work, we give some instances of the determination of the centre of gravity of a few geometrical forms and solids of uniform density, depending upon the properties of the centre of gravity demonstrated in Prop. XIX.



EXAMPLE I.—*To find the centre of gravity of a triangular plate of uniform thickness and density.*

Let A B C be the surface of the triangular plate, bounded by two parallel triangular faces, and by parallelograms perpendicular to these faces.

Bisect B C in D. Join A D.

Through b any point in A B draw $b\ d\ c$ parallel to B C, and cutting A D in d.

Then the triangle A b d by construction is similar to the triangle A B D, and the triangle A d c to the triangle A D C.

Hence Euc. B. vi. p. 4, $\frac{A\,d}{A\,D} = \frac{b\,d}{B\,D}$, and $\frac{A\,d}{A\,D} = \frac{d\,c}{D\,C}$.

Therefore $\frac{b\,d}{B\,D} = \frac{d\,c}{D\,C}$, but B D = D C.

Hence $b\,d = d\,c$; and so it may be shown that every line drawn through any point in A B parallel to B C will be bisected by A D.

Now we may conceive the triangular plate, whose thickness is uniform, to be made up of a number of extremely thin slices, cut perpendicular to the surface and parallel to B C.

Each of these slices being symmetrical will balance about its centre, and therefore every one of them will balance about a line drawn parallel to A D, and passing through the centre of the thickness of the plate.

The whole triangular plate will therefore balance about this line.

Next bisect A B in E, join E C cutting C D in G.

It may be shown, as before, that the triangular plate A B C will balance about a line parallel to E C, passing through the centre of the thickness of the plate.

Since the centre of gravity of the plate lies in both the lines passing through the centre of the thickness parallel to C E and A D; the intersection of these two lines must be the centre of gravity.

Hence a point in the middle of the thickness of the plate below G will be the centre of gravity of the plate, provided its thickness and density be uniform throughout.

To find G, join D E.

Then because A B is bisected in E, and B C in D,

Therefore Euc. B. vi. Props. 2 and 4, E D is parallel to A C, and E D = ½ A C.

Hence the angle E D G = angle G C A, the angle at G is common, and therefore the triangle E G D is similar to the triangle A G C.

Therefore Euc. B. vi. P. 4, $\frac{D\,G}{D\,E} = \frac{A\,G}{A\,C}$.

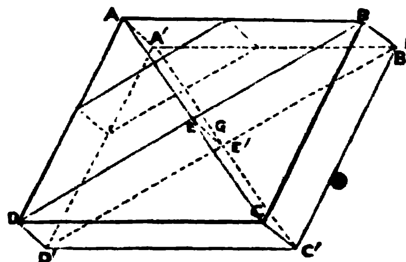
Or $D\,G = \frac{A\,G \cdot D\,E}{A\,C} = \frac{1}{2} \cdot \frac{A\,G \cdot A\,C}{A\,C} = \frac{1}{2}\,A\,G$.

And $A\,D = A\,G + D\,G = 2\,D\,G + D\,G = 3\,D\,G$.

Or $D\,G = \frac{1}{3}\,A\,D$.

EXAMPLE II.—*To find the centre of gravity of a plate of uniform thickness and density bounded by parallel planes, whose upper and lower surfaces are parallelograms.*

Let the parallelogram A B C D represent the upper surface of the plate, A′ B′ C′ D′ the lower; the other boundary planes, such as B C C′ B′, being perpendicular to A B C D.



Join B D and A C meeting in E, B′ D′ and A′ C′ meeting in E′. A C will bisect all lines drawn through the parallelogram parallel to B D, and B D will bisect all similar lines drawn parallel to A C.

Then we may conceive, as in the last example, the plate as made up of a number of parallel plates, each parallel to the plane B D D′ B′, each of which will balance about a line drawn through the centres of A A′ and C C′.

Again, we may conceive the plate as composed of a number of parallel plates parallel to A A′ C′ C, which will each balance about a line drawn through the centre of D D′ and B B′.

G, the intersection of these lines, or the centre of E E′, will therefore be the centre of gravity of the plate.

To find the centre of gravity of the surface of any rectilineal figure, we have only to divide it into triangles, find the centre of gravity of each triangle, suppose weights acting at each of these centres of gravity proportional to the surface of the triangle at whose centre of gravity it is supposed to act, and then find the centre of gravity of these weights, which will be the centre of gravity of the figure.

It will be found on calculation that the centre of gravity of any triangular surface is the same as that of three spheres, or any other body, whose weights are equal, placed with their centres of gravity in the angular points of the triangle. This also applies to a parallelogram, but not to all four-sided figures.

EXAMPLE III.—*To find the centre of gravity of a homogeneous solid of uniform density, whose form is that of a pyramid on a triangular base.*

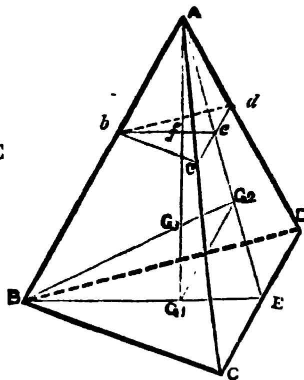
Let B C D be the base and A the vertex of the pyramid.

Bisect one of the sides of the base D C in E. Join A E, B E.

Take $E\,G_1$ one-third of B E and $E\,G_2$ one-third of A E.

Join $A\,G_1$ and $B\,G_2$ meeting in G_3, G_3 will be the centre of gravity of the pyramid.

In A C take any point c, through c draw $c\,d$ parallel to C D cutting A E in e, and $b\,c$ parallel to B C. Join $b\,d$ and $b\,e$ cutting $A\,G_1$ in f.



Then the plane $b\,c\,d$ is evidently parallel to the plane B C D, and the line $b\,e$ parallel to the line B E.

Hence by similar triangles $A\,E\,G_1$, $A\,e\,f$, $\dfrac{e\,f}{E\,G_1} = \dfrac{A\,e}{A\,E}$

and by similar triangles $A\,e\,b$, A E B, $\dfrac{b\,e}{B\,E} = \dfrac{A\,e}{A\,E}$

Therefore $\dfrac{e\,f}{E\,G_1} = \dfrac{b\,e}{B\,E}$ but $E\,G_1 = \frac{1}{3}\,B\,E$.

Hence $e\,f = \frac{1}{3}$ of $b\,e$.

Again, because $c\,d$ is parallel to C D, and C D is bisected in E, $c\,d$ is also bisected by A E.

f therefore is the centre of gravity of the triangle $b\,c\,d$.

In a similar manner it may be shown that the centre of gravity of any triangular slice of the pyramid parallel to B C D must lie in the line $A\,G_1$, and since we may conceive the pyramid made up of an infinite number of such parallel slices, the centre of gravity of the whole pyramid must lie in $A\,G_1$.

By similar reasoning it may be shown that the centre of gravity of the pyramid lies in the line $B\,G_2$.

Consequently, G_3, the intersection of the lines $A\,G_1$ and $B\,G_2$ will be the centre of gravity of the pyramid.

Join $G_1\,G_2$, then since $E\,G_2 = \frac{1}{3}\,A\,E$, and $E\,G_1 = \frac{1}{3}\,B\,E$, $G_1\,G_2$ is parallel to A B. Consequently the triangle $G_1\,G_3\,G_2$ is similar to the triangle G_3 A B, and the triangle $E\,G_1\,G_2$ to the triangle E B A.

Hence $\dfrac{G_1\,G_3}{A\,G_3} = \dfrac{G_1\,G_2}{A\,B}$ and $\dfrac{G_1\,G_2}{A\,B} = \dfrac{E\,G_1}{B\,E} = \frac{1}{3}$.

Therefore $\dfrac{G_1\,G_3}{A\,G_3} = \frac{1}{3}$ and $G_1\,G_3 = \frac{1}{3}\,A\,G_3$.

Or $G_1\,G_3 = \frac{1}{4}\,A\,G_1$.

Hence the centre of gravity of the pyramid is found to be in the line joining the vertex and the centre of gravity of the base, at a distance of a fourth of its length.

EXAMPLE IV.—*To find the Centre of Gravity of any Pyramid of uniform density, whose base is any polygon.*

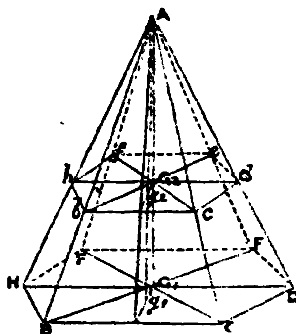
Let A be the vertex of the pyramid B C D E F H its polygonal base. Let G_1 be the centre of gravity of this base; join $A\,G_1$, and in $A\,G_1$ take G_2 so that $G_1\,G_2 = \frac{1}{4}\,A\,G_1$. G_2 will be the centre of gravity of the pyramid.

Join G_1 with each of the angular points of the base, through G_2 draw the plane $b\,c\,d\,e\,f\,h$ parallel to the base, and cutting A B in b, A C in c, &c., and join $G_2\,c$, $G_2\,d$, &c.

Let g_1 be the centre of gravity of the triangle $B\,G_1\,C$, join $A\,g_1$ cutting the triangle $b\,G_2\,c$ in g_2.

Then since the triangle $b\,G_2\,c$ is parallel to the triangle $B\,G_1\,C$ and $G_1\,G_2 = \frac{1}{4}\,A\,G_1$;

hence $g_1 g_2 = \frac{1}{3} A g_1$ and g_2, is the centre of gravity of the pyramid A B C G_1 on a triangular base G_1 B C, and it lies in the plane $b c d e f h$. In a similar manner it may be shown that the centres of gravity of every one of the triangular pyramids A G_1 C D, A G_1 D E, &c., into which the whole pyramid may be divided, all lie in the plane $b c d e f h$.



It follows, therefore, that the centre of gravity of the whole pyramid must lie in this plane.

Again the plane $b c d e f h$ is in every respect similar to the base, and G_2 its centre of gravity will therefore lie in the line A G_1. In like manner it may be shown that the centre of gravity of every section of the pyramid parallel to the base will lie in A G_1; hence the centre of gravity of the whole pyramid must lie in A G_1.

We have shown, therefore, that the centre of gravity of the whole pyramid must be in the plane $c d e f h$ and also in the line A G_1. Hence the centre of gravity must be their intersection G_2.

The centre of gravity of any pyramid of uniform density on a polygonal base, will lie in the line joining its vertex and the centre of gravity of the base, at a distance equal to $\frac{3}{4}$ of its length from the former, or $\frac{1}{4}$ of its length from the latter.

The above reasoning is altogether independent of the number of the sides of the polygon, and since we may conceive a curve to be made up of an infinite number of straight lines, or a polygon bounded by an infinite number of small sides, the above solution enables us to determine the centre of gravity of a cone with a curvilinear base.

Hence the centre of gravity of a right or oblique cone on any curvilinear base is found by joining the centre of gravity of the base with the apex of the cone, and taking a point in this line, equal to $\frac{1}{4}$ of its length, measuring this point from the centre of gravity of the base.

Machines.—Any instrument by means of which a force is communicated from one point to another, so as to keep at rest or set in motion a body acted on by another force, is called a machine. The simplest of these instruments are *cords*, *rods*, and *hard planes;* and these by their combinations form all complex machines, however various their forms and actions.

In statics we only consider machines when the forces acting upon them are in a state of equilibrium; motion will be produced by an addition to some of the forces which produce equilibrium. But the discussion of this branch of the subject belongs to dynamics.

For the sake of simplicity, and to enable us to apply the theoretical principles we have already proved, we consider *cords* as being destitute of weight and perfectly flexible, *rods* and *planes* as perfectly rigid, inflexible, and without weight. When necessary, we can take the weights of the elementary parts of our machines into consideration, by considering their weights as a force proportional to their weight applied to the centre of gravity in a vertical direction.

We also neglect the friction of all surfaces in contact with each other. It is evident, therefore, that our machines will be theoretical ones, which cannot exist in nature, and whose properties cannot be strictly proved by experiment. But if we determine the rigidity, flexibility, and friction of the substances composing our machine by experiment, and compare the force exerted by a real machine with the force it ought to

exert by theory, we may arrive at a knowledge of the retarding forces produced by friction or want of flexibility; and thus by our theoretical knowledge of the combination of machines, estimate the forces produced by any actual complex machine, the friction or flexibility of whose elements we have determined. (See *Pressure, Tension*, and *Flexible Cords*, page 41.)

Mechanical Powers.—The simplest combinations of these machines are called the *mechanical powers.* They are usually regarded as seven in number :—1, the lever; 2, the wheel and axle; 3, the toothed wheel; 4, the pulley; 5, the inclined plane; 6, the wedge; 7, the screw.

The wheel and axle, toothed wheel and pulley, may be regarded as modifications of the lever, and the wedge and screw as particular cases of the inclined plane.

The Lever.—The simplest form of a lever is a straight rod, supposed to be inflexible and without weight, resting on a fixed point somewhere in its length, about which it can turn freely, and having two forces applied at two other points of the rod.

The fixed point on which it rests, and about which it can turn, is called the *fulcrum*; one of the forces applied to it is called the *power* and the other the *weight.* The distances of the points of application of the power and weight from the fulcrum are called the *arms* of the lever.

There are three kinds of levers, distinguished by the relative position of the *power, weight*, and *fulcrum.*

Lever of the first kind.—In the lever of the first kind the power P (Fig. 1) and the weight W act in the same direction on opposite sides of the fulcrum F. A crow-bar (Fig 2), by means of which a man raises a heavy body W by placing one extremity B under W, and resting it on an obstacle C while he presses the extremity B, is an instance of a lever of the first kind.

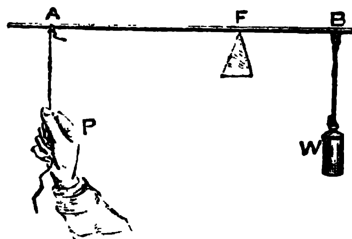


Fig. 1.

A poker is another. In this case the coals form the weight, the bar of the grate the fulcrum, and the hand the power.

The spade is a lever; the ground against which it is pressed when the handle is depressed, in order to turn up the earth in front of it, being the fulcrum.

Scissors and carpenters' pincers are examples of double levers of the first kind.

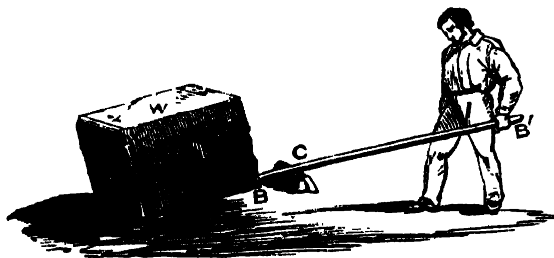


Fig. 2.

Lever of the second kind.—In the lever of the second kind the power P (Fig. 3) and the weight W act in opposite directions on the same side of the fulcrum F, the weight being nearer to the fulcrum than the power.

A cutting-knife (Fig. 4) and an oar are instances of levers of the second kind. In the case of the oar, the arm of the rower is the power, the pressure of the oar on the side of the

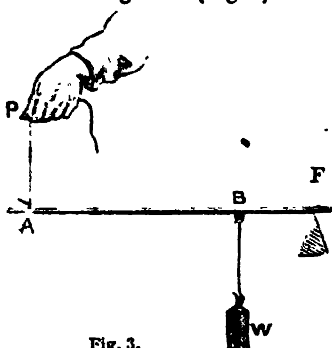


Fig. 3.

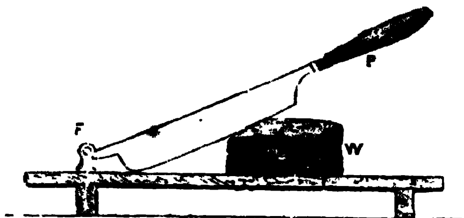


Fig. 4.

boat is the weight, and the point of the blade of the oar, which is for a moment stationary in the water, forms the fulcrum.

Nutcrackers give a good illustration of a double lever of the second kind.

Lever of the third kind.—In the lever of the third kind the power P (Fig. 5) and the weight W act as in the second kind, in opposite directions on the same side of the fulcrum; but in the third species of lever the power is nearer the fulcrum than the weight.

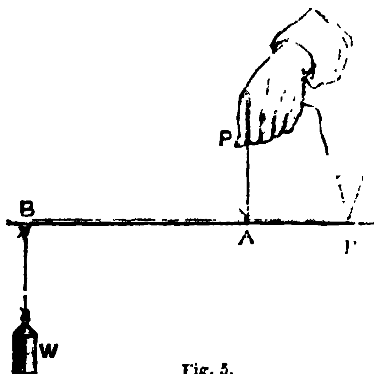


Fig. 5.

Most muscles in animals are generally inserted near the joint, as in Fig. 5, and act as levers of the third kind; the joint forms the fulcrum, the limb which the muscle moves, together with any resistance opposed to its motion, represents the weight, while the force exerted by the contraction of the muscle is the power. The treadle of a turning-lathe, or razor-grinder's machine, where the foot of the operator acts as the power, is a lever of the third kind.

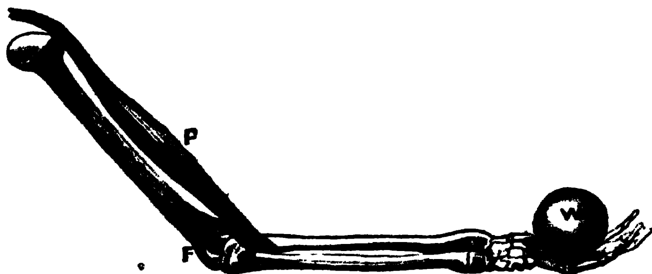


Fig. 6.

A pair of tongs, or a pair of sheep-shears, afford familiar instances of double levers of the third kind.

PROPOSITION XXIII.

To find the Conditions of Equilibrium and the Pressure on the Fulcrum when two Parallel Forces act in the same plane on a straight lever.

Let P and W be two parallel forces acting at A and B, on the straight lever A B whose fulcrum is F in each of the three kinds of levers.

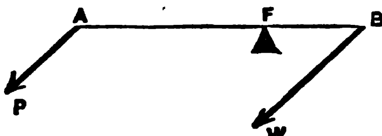


Fig. 1.

And let A P and W B (Figs. 1, 2, and 3) represent these forces in magnitude and direction.

In order that there may be equilibrium, the resultant of the two parallel forces must pass through F, and the pressure on that point will be equal to their resultant, and this will be destroyed by the resistance of the fulcrum.

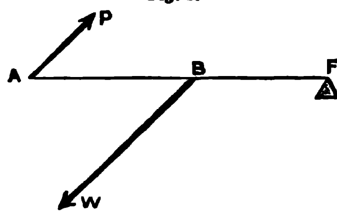


Fig. 2.

The proposition is therefore reduced to the case of Prop. XI., which we have already demonstrated, and from this we may learn that the pressure on F will be equal to P + W acting in a direction parallel to P or W.

And also that $P \cdot AF = W \cdot BF$ or $\frac{P}{W} = \frac{BF}{AF}$ in every case.

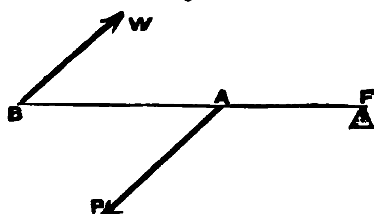


Fig. 3.

From this we find that in the lever of the first kind, the power may be less than, equal to, or greater than, the weight; in the second kind the power is always less; and in the third kind always greater than the weight.

In the preceding cases we have supposed the lever to be without weight; we shall show how to take the weight of lever into consideration in the cases of the balance and steelyard.

PROPOSITION XXIV.

To find the Conditions of Equilibrium of two Forces acting in the same plane on a Bent Lever, when the directions of the Forces are not Parallel.

Let A C B be a bent lever consisting of a portion of a plane rigid body, supposed to be destitute of weight. C the fulcrum about which the lever turns.

A and B the points of application of the two forces represented in magnitude and direction by A P and B W.

The effect of these forces will be to produce a pressure on C; and when there is equilibrium, this pressure must be destroyed by the reaction R of the fulcrum C, represented in magnitude and direction by C R.

Let α and β be the angles which A P and B W produced; make with a line C X

drawn through C at right angles to the vertical C Y passing through C. θ the angle C R makes with C X.

R and θ are two unknown quantities which we have to determine: through C draw C E and C D perpendicular to A P and B W produced.

We may regard our lever as a rigid body kept in equilibrium by the three forces P, W, and R. Resolving these forces into forces parallel to C X and C Y, X_1 and Y_1 being the resolved parts of P, X_2 and Y_2 those of W, and X_3 and Y_3 those of R.

Taking moments about C, we have by Prop. XVIII. the following conditions of equilibrium:—

(1) $X_3 + X_2 - X_1 = 0$, or $R \cos \theta + W \cos \beta - P \cos \alpha = 0$.

(2) $Y_2 - Y_3 - Y_1 = 0$, or $R \sin \theta - W \sin \beta - P \sin \alpha = 0$

And (3) $P \cdot CE - W \cdot CD = 0$.

From this last equation we obtain $P \cdot CE = W \cdot CD$, or

$$\frac{P}{W} = \frac{CD}{CE} = \frac{\text{Perpendicular on direction of W}}{\text{Perpendicular on direction of P}}.$$

From equations (1) and (2) we can determine R and θ, and consequently the magnitude and direction of the pressure on the fulcrum, which will be equal to R, and act in the direction opposite to C R.

Transposing equation (1) $R \cos \theta = P \cos \alpha - W \cos \beta$.

" " (2) $R \sin \theta = P \sin \alpha + W \sin \beta$.

Dividing one of these equations by the other, we have

$$\tan \theta = \frac{P \sin \alpha + W \sin \beta}{P \cos \alpha - W \cos \beta}, \text{ which determines } \theta.$$

Or, adding their squares, we have

$$R^2 (\cos^2\theta + \sin^2\theta) = P^2 (\cos^2\alpha + \sin^2\alpha) + W^2(\cos^2\beta + \sin^2\beta) - 2PW(\cos\alpha\cos\beta - \sin\alpha\sin\beta).$$

Hence, Trigonometry, pages 296 and 304,

$$R^2 = P^2 + W^2 - 2\,PW \cos(\alpha + \beta),$$

which determines the magnitude of R.

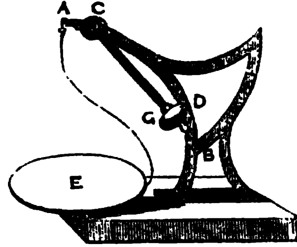
If we had supposed B instead of C the point of application of the fulcrum, and taken our moments about B, we should have had

$$\frac{P}{R} = \frac{\text{Perpendicular on direction of R}}{\text{Perpendicular on direction of P}} \cdot$$

Hence the condition of equilibrium in a lever of any kind, *is that the power must be to the weight inversely as the perpendiculars drawn from the fulcrum on their directions.*

The bent lever evidently includes the straight one, as in the latter case A, B, and C are in the same straight line.

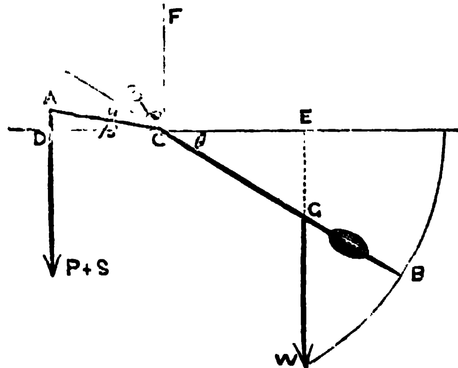
The Bent-lever Balance.—The bent-lever balance is a machine which, within certain limits, enables us to weigh substances without the use of weights; it consists of a bent lever whose two arms are A C and B C, moveable about a fulcrum C. The fulcrum C is fixed to a stand which carries a graduated arc; over this arc the extremity B of the lever moves as C B turns round the fulcrum C. From the other extremity A, a scale-pan E is so suspended as to have its centre in all positions of the lever in the vertical passing through A.



A weight D is fixed to the arm C D so as to bring the centre of gravity of the whole lever and scale-pan to some point below the fulcrum C. the magnitude of this weight is so arranged that the extremity B of the lever shall point to zero on the graduated arc, when the scale-pan is empty and the lever in a state of equilibrium.

To graduate the arc, weights of 1, 2, 3, &c., pounds or ounces, or whatever denomination of weight the instrument is intended to indicate, are placed successively in the scale-pan; and the corresponding points of the arc over which B rests, are marked on the scale as 1, 2, 3, &c.

This balance is of great use for determining quickly the weights of bodies where extreme nicety is not essential.



To explain the graduation of this balance mathematically,

Let C be the fulcrum of the lever, A C and B C its arms; let G in B C be the centre of gravity of the whole lever, exclusive of the scale-pan and wire by which it is suspended. Let W represent this weight, and let C A $= a$ and C G $= b$. Also let S represent the weight of the scale-pan and wire by which it is suspended, P that of a body placed in the scale-pan.

Let θ be the angle C B makes with a line D C E drawn through C perpendicular to C F, the vertical passing through C; β the angle A C makes with D C E.

Also let $a =$ B C A be the angle the arms of the lever make with each other, and let $a' = 180 - a$, and let D and E be the points where the vertical lines passing through A and G cut the line D C E.

Then Prop. XXIV. $(P + S) \cdot CD = W \cdot CE$,

or $(P + S)\, a \cos\beta = W \cdot b \cos\theta$,

but $\beta = 90 - \theta + a' = 90 - (\theta - a')$; hence $\cos\beta = \sin(\theta - a')$.

Therefore $(P + S)a \sin(\theta - a') = Wb \cos\theta$,

or $(P + S)a \{\sin\theta \cos a' - \cos\theta \sin a'\} = Wb \cos\theta$,

and $\tan\theta \cos a' - \sin a' = \dfrac{Wb}{(P + S)a}$

or $\tan\theta = \dfrac{W \cdot b}{(P + S)a \cos a'} + \tan a'$.

Hence if we wish to graduate the arc for pounds, we must take P successively equal to 0, 1, 2, 3, &c., pounds, in which case $\tan\theta$ will be successively equal to $\dfrac{W \cdot b}{Sa \cos a'} + \tan a'$, $\dfrac{Wb}{(S + 1)\, a \cos a'} + \tan a'$, $\dfrac{Wb}{(S + 2)a \cos a'} + \tan a'$, $\dfrac{Wb}{(S + 3)a \cos a'} + \tan a'$, &c., where W and S represent the weight of the lever and scale-pan in pounds.

The Common Balance.—This instrument is popularly called the *scales*, or a *pair of scales*, and is perhaps more frequently used than any other for determining the weights of substances or goods. It consists of a lever supported on an axis or fulcrum equally distant from its two extremities; under each of these extremities a dish is suspended, in one of which the substance to be weighed is placed, and in the other the weights by which its value is determined.



The lever is so constructed as to be capable of moving on its axis in a vertical plane; and when a given weight is placed in one dish and a substance equal in weight to it in the other, after vibrating some little time, it assumes a horizontal position. When a slight excess of weight is added to either dish, the lever again vibrates and assumes such a position of rest that the extremity above the dish containing an excess of weight over the other, lies below the horizontal line. The lever is called the *beam*; and the two dishes, *scale-pans*.

There are three requisites in a balance :—

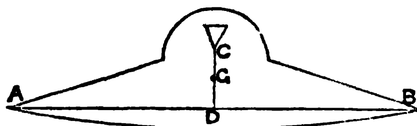
First.—When equal weights are placed in the scale-pans, the two extremities of the beam should rest in a perfectly horizontal line. This is called its *horizontality*.

Second.—On the slightest addition of weight to either scale, the beam should lose its horizontal position. This is called its *sensibility*, and is measured by the smallest weight which causes the beam to depart from its *horizontality*.

Third.—After any disturbance, the beam should assume a state of rest as speedily as possible: this is called its *stability*.

We shall now proceed to consider the mathematical conditions which must be satisfied, in order to obtain these requisites; for this purpose we must first confine our attention to the construction of the *beam*.

Let A B represent the beam, G its centre of gravity, and C the point on which its axis or fulcrum is supported.



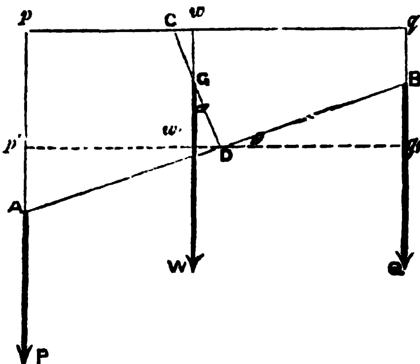
Join A B, and produce C G to meet A B in the point D.

To secure the *horizontality* of the beam, the points C and G must not coincide, for if they did, from the properties of the centre of gravity, we should have indifferent equilibrium, and the beam, either by itself or when loaded at its extremities with equal weights, would rest in every position in which the line A B might be placed.

When C does not coincide with G, by Prop. XIX., and the beam is at rest C G will assume a vertical position; hence in order that A B may be horizontal, C D must be perpendicular to A B. In order that this horizontal position may be that of equilibrium when equal weights are suspended from A and B, we must also have A D = B D.

The axis C is generally a prismatic piece of metal, which pierces and also is firmly fixed at right angles to the beam. This prism rests on one of its edges, technically called the *knife-edge*, on a plane or curved surface, so placed as a support on each side of the beam, that the edge or line of support about which the beam oscillates is horizontal and perpendicular to the plane in which it oscillates. The axis might be a cylinder or cone working in a socket, but the *knife-edge* is generally preferred, in order to avoid friction as much as possible.

The conditions of *horizontality* given above require, therefore, that the plane passing through the line of support and the centre of gravity of the beam shall be at right angles to, and also bisect, the line passing through its extremities, or the points from which the scale-pans are suspended.



To determine the other two conditions, retaining the same letters as before, let two unequal weights P and Q, represented in magnitude and direction by A P and B Q, be suspended from A and B.

Let θ be the angle A B makes with the horizontal line $p' q'$ drawn through the point D, and let $p q$ cut A P in p and A Q in q.

Through C draw $p q$ parallel to $p' q'$, and cutting A P in p, and A Q in q. Let W, represented in magnitude and direction by G W, be the weight of the beam and its axis, and let the line G W cut $p q$ in w, $p' q'$ in w'.

Also let $A D = D B = a$, $C G = b$, and $C D = c$.

Since C D is at right angles to A B, and θ is the angle A B makes with the horizontal line $p' q'$, it will also make the angle θ with the vertical line w G w'.

Also because $p q$ is parallel to $p' q'$ and P p', $w w'$ and $q q'$ are parallel to each other, $p w = p' w'$, and $q w = q' w'$.

Taking the moments of the three forces P, Q, and W, about the point C, we shall have, as a condition of equilibrium,

$$W \cdot Cw + Q \cdot Cq = P \cdot Cp.$$

But $Cq = Cw + wq = Cw + w'q'$

$= Cw + w'D + Dq'$

$= CG \sin\theta + GD \sin\theta + DB \cos\theta$

$= b \sin\theta + (c - b) \sin\theta + a \cos\theta$

$= c \sin\theta + a \cos\theta.$

Again $Cp = pw - Cw = p'w' - Cw$

$= p'D - w'D - Cw$

$= a \cos\theta - (c - b) \sin\theta - b \sin\theta$

$= a \cos\theta - c \sin\theta.$

Also $Cw = b \sin\theta.$

Substituting these values of Cq, Cp, and Cw in the equation $W \cdot Cw + Q \cdot Cq = P \cdot Cp$, we have

$$W \cdot b \sin\theta + Q(c \sin\theta + a \cos\theta) = P(a \cos\theta - c \sin\theta).$$

Dividing both sides by $\cos\theta$

$$W \cdot b \tan\theta + Q(c \tan\theta + a) = P(a - c \tan\theta);$$

Or
$$(Wb + Qc + Pc) \tan\theta = (P - Q)a;$$

And
$$\frac{\tan\theta}{P - Q} = \frac{a}{W \cdot b + (P + Q)c}.$$

Now the *sensibility* of the balance for a given difference of P and Q will be greater the greater the angle θ; also for a given value of θ, the sensibility will be greater the smaller the difference between P and Q. Since $\tan\theta$ is greater, the greater the angle θ, it follows that $\frac{\tan\theta}{P - Q}$ is a measure of the sensibility of the balance.

Hence the greatest sensibility will be attained when $\frac{a}{Wb + (P + Q)c}$ is as great, or $\frac{Wb + (P + Q)c}{a}$ as small as possible.

In making this calculation we have neglected the friction of the edge of the fulcrum on its supports, which will have a tendency to diminish the sensibility.

In instruments where great accuracy is required, such as chemical, philosophical, and assay balances, this friction is diminished as much as possible by making the knife-edge of the fulcrum of hard polished steel, and the support on which it rests a plane of polished agate. Having thus obviated the diminution of sensibility due to this friction, it appears from the above expression for the sensibility of the balance, that it will be greater, the greater a is and the less W, b, and c are.

We must, therefore, make the beam as light and as long as we possibly can, and the distances of G and D from C as small as may be consistent with other conditions.

The scale-pans and the cords, or apparatus by which they are suspended, must also be made as light as can be, compatible with the uses of the balance, as they increase the values of P and Q, whose sum is an element which diminishes the sensibility.

When equal weights act on A and B, the extremities of the beam, its stability is measured by its tendency to resume a state of rest, after its equilibrium has been disturbed. This tendency will depend upon the magnitude of the sum of the moments of the forces, about the point C, for any given position of the beam; *i. e.* for any particular value of θ.

The sum of the moments of the three forces P, Q, and W about C when A B is inclined at an angle θ to the horizontal, or C D at the same angle to the vertical, will be

$$W \cdot Cw + Q \cdot Cq - P \cdot Cp;$$
$$\text{or } W b \sin \theta + Q (c \sin \theta + a \cos \theta) - P (a \cos \theta - c \sin \theta).$$

When P and Q are equal, this expression becomes

$$W b \sin \theta + P c \sin \theta + P c \sin \theta;$$
$$\text{or } (W \cdot b + 2 P \cdot c) \sin \theta.$$

When θ is zero, $\sin \theta$ is also zero, and this expression becomes the same, or the sum of the moments is zero, which it must be in order that there should be equilibrium. But for any other finite value of θ, the magnitude of the *stability* will be greater, as $W \cdot b + 2 P \cdot c$ is greater; that is, as W, P, b and c are greater.

These conditions are contrary to those required for *sensibility*, which for any given value of a, demand that W, P, b and c shall be as small as possible.

Hence, while the arm of the balance remains of the same length, any increase of *sensibility* is made at the expense of its *stability*; but the quantities b, c, W and P remaining the same, we may increase the *sensibility* without injury to the *stability* by increasing a, or the length of the arm of the balance.

Where minute differences of weight are not matters of importance, and quickness of determination essential, as in weighing substances rapidly, which are not of great value in proportion to their weight, *stability* is of more importance than *sensibility*. The reverse, however, is the case where the substance is of great value in proportion to its weight, or where extreme accuracy is required.

In all that has been said, it must be remembered that the weight P includes that of the scale-pan, together with the apparatus by which it is suspended.

The scale-pan must be so suspended that its centre of gravity, as well as that of the weight or substance placed in it, may be exactly in the vertical line passing through the extremity of the beam, from which it is suspended. From the mathematical expressions given above for *sensibility* and *stability*, both depending upon P, it follows that the same balance will have different degrees of these qualities according as it is used for determining greater or less weights. The quantities of substances which are used in philosophical or chemical balances being generally very small, a balance which is very *sensible* when no weight is placed in the scales, will be of almost equal *sensibility* for every weight with which it is intended to be used.

A needle is usually fixed to the beam in the direction of the line C G, which points vertically upwards or downwards when the beam is in a horizontal position. A graduated arc or scale attached to the support of the balance to indicate the arcs described by the extremity of this needle, as the beam oscillates, is a very convenient addition. When G is very near C, the oscillations will be very slow; in this case, the equality of the weights in the scale-pans may be ascertained by means of the index-needle, before the balance comes to a state of rest. When the weights are equal, the extremity of the needle will describe equal arcs on both sides of the vertical line; when they are unequal, the scale which preponderates will be indicated by a greater arc being described on that side than on the other. This method renders *stability* of much less importance than sensibility.

Where great accuracy is required, the Jurors for Philosophical Instruments in the Great Exhibition of 1851, in their Report, recommended the disuse of this long index-needle, almost in contact with the graduated arc, and the substitution in its place of a graduated arc attached to one end of the beam. This arc is viewed through a fixed compound microscope, having a horizontal wire in the focus of the eye-piece; or by a mirror attached to the beam, in which the reflected image of a scale is viewed through a telescope.

In very accurate instruments the scale-pans are suspended from steel knife-edges resting on agate planes fixed to the extremities of the beam. The balance should be constructed as much as possible of brass, as steel and iron are apt to acquire magnetic properties. Palladium has been used for the construction of the beam, and platinum is to be preferred for the scale-pans. When glass scale-pans are used, care must be taken not to excite their electrical properties; a difference of half a grain may be produced by merely cleaning one of the glass scales with a dry silk handkerchief. In constructing the beam, care must be taken by making it hollow, or if solid by cutting out portions of it, to secure the greatest degree of lightness with the greatest length of arm, which is compatible with its rigidity, or the rentention of its form, under all the weights to which it is intended to be subjected.

All very sensitive balances have a contrivance by which the knife-edges which support the beam and scale-pans are lifted from the planes on which they play, whenever the instrument is not in use. The knife-edges are thus preserved from becoming blunt, or wearing the agate planes, and the beam is freed as much as possible from every strain which would tend to alter its shape.

In addition to all these precautions, the balance should be enclosed in a glass case, for its preservation from dust and injury: to prevent error from the action of currents of air, a window at the side affords the means of introducing weights and the substance to be weighed, without removing the case. A cup containing quick-lime, or some other powerful absorbent of moisture, should also be kept within the case.

Ramsden constructed a balance for the Royal Society of such extreme *sensibility*, that when weighed with 10 pounds, it turned with about the thousandth part of a grain.

From the above description it is manifest, that the construction of a perfect balance may be regarded as impossible, though one may be nearer than another to perfection. Borda invented a very simple method, by means of which very accurate results may be obtained by a balance sufficiently *sensitive* and well-constructed on its knife-edges, though the points of suspension of the scale-pans may not be equidistant from its fulcrum.

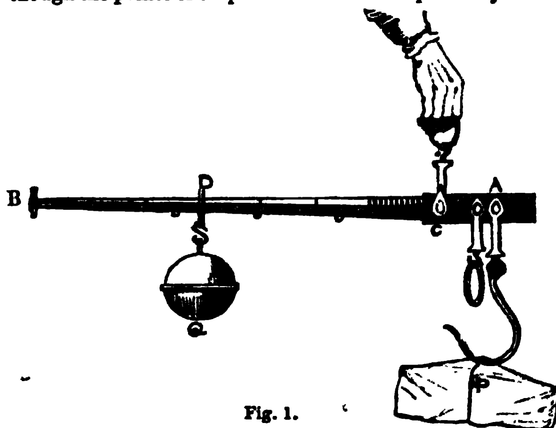


Fig. 1.

His ingenious device is to weigh the article whose weight is to be determined by weights or any other substance, such as sand, placed in the other scale: when equilibrium has been thus obtained, the article is removed and replaced by weights until equilibrium has been again restored. These latter weights determine that of the article; and thus any error arising from inequality of the arms of the balance is eliminated.

The Roman Steelyard or Balance.—This balance consists of an iron or steel lever A B, with unequal arms A C and B C resting on a fulcrum with a knife-edge at C, which plays on a pivot passing through a support held or suspended by a ring, as

shown in Fig. 1. The substance P whose weight is to be determined is suspended from the point A by a hook, while a constant weight Q, called the *counterpoise*, is attached to a ring D, which can be slipped along the longer arm C B. The edge of the arm C B is so graduated that the point D at which the counterpoise Q suspended from it balances P, determines the weight of the latter in pounds and ounces, or any other convenient denomination of weight for which the steelyard may have been graduated.

The steelyard is sometimes furnished with a second fulcrum and apparatus for its suspension, nearer to A than C; the position of the lever is then inverted, and another graduation on what was before the under edge of the arm B C, gives the weight of P for the new position of the fulcrum. The same instrument has thus two different ranges of weights for the same counterpoise, the fulcrum nearer to A being used for substances lying within a range of weights greater than that for which the other is graduated.

To Graduate the Roman Steelyard.—The Roman steelyard may be regarded as a heavy lever with unequal arms resting on a fulcrum C.

Let W be the weight of the lever and the hook or scale-pan by which P is suspended, G the position of their joint centre of gravity, Q the weight of the counterpoise, together with the ring and chain by which it is suspended from B C.

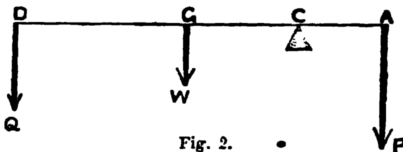


Fig. 2.

The steelyard may now be considered as a lever without weight, whose fulcrum is C (Fig. 2), acted on by three parallel and vertical forces, P, W and Q, at the points A, G, and D, represented in magnitude and direction by the lines A P, G W, and D Q.

When these forces produce equilibrium, taking moments about the point C, we have

$$P \cdot A\,C = W \cdot G\,C + Q \cdot C\,D.$$

Hence, transposing, we have

$$Q \cdot C\,D = P \cdot A\,C - W \cdot G\,C$$

$$\text{or } C\,D = \frac{P}{Q} A\,C - \frac{W}{Q} G\,C.$$

Now let D_1, D_2, D_3, D_4, &c. (Fig. 3), represent the distances from C at which weights of one, two, three, four, &c., pounds, or any other unit of weight for which the steelyard is to be graduated, suspended from A are respectively balanced by the counterpoise Q.



Fig. 3.

Substituting, therefore, for P the numbers 1, 2, 3, 4, &c., and D_1, D_2, D_3, D_4, &c., for D in the above equation, we have

$$C\,D_1 = \frac{A\,C}{Q} - \frac{W}{Q} G\,C$$

$$C\,D_2 = 2\frac{A\,C}{Q} - \frac{W}{Q} G\,C$$

$$C\,D_3 = 3\frac{A\,C}{Q} - \frac{W}{Q} G\,C$$

$$C\,D_4 = 4\frac{A\,C}{Q} - \frac{W}{Q} G\,C, \text{ \&c.}$$

In A C take a point B such that $B\,C = \frac{W}{Q}\,G\,C$, and make $B\,D_1 = \frac{A\,C}{Q}$, where Q represents the weight of the counterpoise, in the same unit which is chosen for the graduation, then make $D_1\,D_2 = B\,D_1$, $D_2\,D_3 = B\,D_1$, $D_3\,D_4 = B\,D_1$, &c. D_1, D_2, D_3, D_4, &c., will mark the points at which the counterpoise Q will balance weights of 1, 2, 3, 4, &c., pounds suspended from A.

For $C\,D_1 = B\,D_1 - B\,C = \frac{A\,C}{Q} - \frac{W}{Q}\,G\,C$.

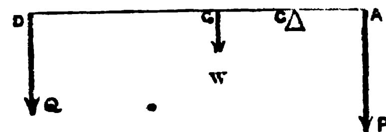
$$C\,D_2 = B\,D_2 - B\,C = 2\,B\,D_1 - B\,C = 2\,\frac{A\,C}{Q} - \frac{W}{Q}\,G\,C.$$

$$C\,D_3 = B\,D_3 - B\,C = 3\,B\,D_1 - B\,C = 3\,\frac{A\,C}{Q} - \frac{W}{Q}\,G\,C.$$

$$C\,D_4 = B\,D_4 - B\,C = 4\,B\,D_1 - B\,C = 4\,\frac{A\,C}{Q} - \frac{W}{Q}\,G\,C.$$

When the instrument is so constructed that the centre of gravity G of the steelyard and scale, or hook, lies in the vertical line passing through C, the quantity B C or $\frac{W}{Q}\,G\,C$ becomes zero, and the points of graduation are taken at equal intervals from C.

The Danish Steelyard or Balance.—This instrument differs from the Roman steelyard in these respects, that the counterpoise is fixed at one of its extremities, while the fulcrum is moveable. The edge of the steelyard is graduated, and the point at which the fulcrum is placed to cause the fixed counterpoise to balance the substance whose weight is to be determined, marks its value.



To Graduate the Danish Steelyard.—Let P, W, Q and the straight lines P A, G W, and D Q represent the same forces in this case, as in that of the Roman steelyard, in this instance, however, it is C and not D which is the moveable point.

Then taking moments of the forces as before, about the point C, we have

$$P \cdot A\,C = W \cdot G\,C + Q \cdot D\,C$$
$$= W\,(A\,G - A\,C) + Q\,(A\,D - A\,C).$$

By transposition $(P + W + Q)\,A\,C = W \cdot A\,G + Q \cdot A\,D$;

$$\text{or } A\,C = \frac{W \cdot A\,G + Q \cdot A\,D}{P + W + Q}.$$

Let C_1, C_2, C_3, C_4, &c., represent the points at which 1, 2, 3, 4, &c., pounds will be balanced by the counterpoise when the fulcrum is placed under these points. Hence substituting C_1, C_2, C_3, C_4, for C, and 1, 2, 3, 4, &c., for P in the preceding expression, we have

$$A\,C_1 = \frac{W \cdot A\,G + Q \cdot A\,D}{1 + W + G}$$

$$A\,C_2 = \frac{W \cdot A\,G + Q \cdot A\,D}{2 + W + G}$$

$$A\,C_3 = \frac{W \cdot A\,G + Q \cdot A\,D}{3 + W + G}$$

$$A\,C_4 = \frac{W \cdot A\,G + Q \cdot A\,D}{4 + W + G}.$$

The reciprocals of these quantities are in arithmetical progression, and therefore the distances of the points C_1, C_2, C_3, C_4, &c., from A will form an harmonical progression.

The Balance of Quintenz.—This balance, named after its inventor Quintenz, is frequently employed on railways for determining the weight of luggage, and affords a good example of a combination of levers. It is represented in Fig. 2. Fig. 1 is added for the purpose of showing its mechanism more clearly. A platform A B on which the body Q, which we wish to be weighed, is placed, is fixed firmly to an upright piece of framework B C. B C again is attached rigidly to an oblique piece D C fixed in D, as shown in Fig. 1, so that the whole portion A B C D forms one rigid and inflexible body. This rigid piece A B C D is supported at E by a knife-edge fulcrum, resting on the bar or lever F G, and at D by a rod H K, which is firmly fixed to D, and suspended by a hook at K from the lever L M N.

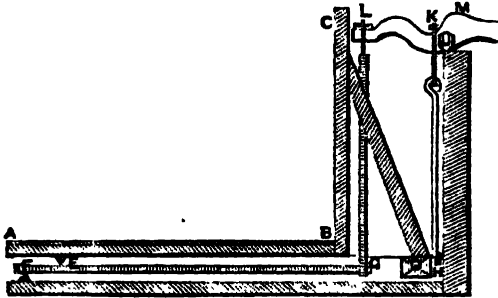


Fig. 1.

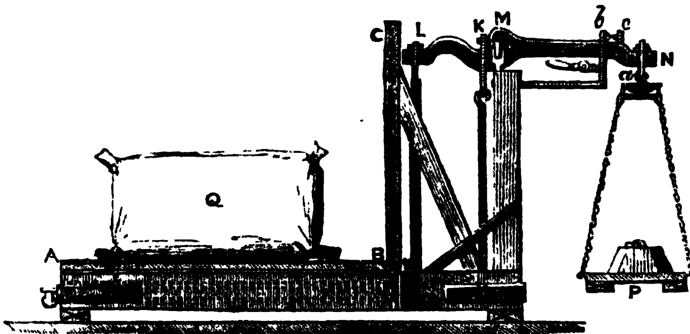


Fig. 2.

The whole weight, therefore, of Q and the framework A B C D rests on the points E and K.

The lever F G is supported by a fulcrum at one extremity F, while the other extremity G is suspended by a rod L G, hanging at L from the extremity of the lever L M N.

Lastly, the lever L M N is supported by a fulcrum M, which is fixed to the framework of the machine; the extremity N of this lever has a scale-pan suspended from it, in which weights may be placed.

The points E, L, and K are so chosen that $\frac{M\,L}{M\,K} = \frac{F\,G}{F\,E}$.

To avoid the consideration of the weights of the various parts of this machine, we will suppose that the weight of the scale-pan and the length of the arm M N of the lever L M N have been so chosen, as to produce an equilibrium of all the parts of the

machine, and to keep L M N in a perfectly horizontal position, when no weight is placed on the platform A B.

To determine the weight P, which must be placed in the scale-pan suspended from N to balance a substance of a given weight placed on the platform A B, we will examine the conditions of equilibrium for the several parts of the machine.

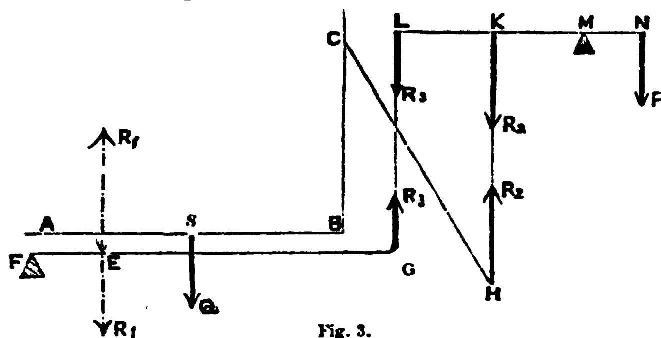


Fig. 3.

Let the weights P and Q be represented in magnitude and direction by N P and S Q (Fig. 3.)

The weight of the substance placed on the platform A B being equal to a single force S Q passing through the vertical which passes through its centre of gravity, will be balanced by the reaction of the pressure R_1 which it produces on the fulcrum E, represented in magnitude and direction by E R_1, and the reaction R_2 of the tension it produces on the rod H K.

The pressure R_1 of the fulcrum E on the lever F G is kept in equilibrium by the reaction of the pressure it exerts on the fulcrum F, and the reaction R_3 of the tension it produces on the rod G L.

Finally, the lever L M N, resting on the fulcrum M, is acted on by three vertical forces P, R_2 and R_3, acting on the points N, K and L.

For the equilibrium of the rigid body A B C H we have as a condition

$$Q = R_1 + R_2.$$

For the lever F E G taking moments about E we have

$$R_1 \cdot E F = R_3 \cdot F G.$$

$$\text{Or } R_1 = R_3 \cdot \frac{F G}{E F}.$$

Lastly, for the lever L M N taking moments about M we have

$$P \cdot M N = R_2 \cdot M K + R_3 \cdot L M.$$

$$\text{Or } P \cdot \frac{M N}{M K} = R_2 + R_3 \cdot \frac{L M}{M K}.$$

But $\frac{L M}{M K} = \frac{F G}{E F}$ by the construction of the machine.

Hence $P \cdot \frac{M N}{M K} = R_2 + R_3 \cdot \frac{F G}{E F} = R_2 + R_1 = Q.$

And $\frac{P}{Q} = \frac{M K}{M N}.$

If M N be taken ten times as long as M K, we have M N = 10 M K and Q = 10 P; and in this case a weight the tenth part of Q, when placed in the scale-pan, will produce equilibrium.

Instead of placing weights in the scale-pan P (Fig. 2), the arm of the lever M N may be graduated, and used as a Roman steelyard.

Before placing a body on the platform A B (Fig. 1) whose weight is to be determined, it is necessary to observe whether the arm of the lever M N is in a perfectly horizontal position. This is indicated by a horizontal pin *b* fixed to the framework of the instrument, being in the same line with a similar pin fixed to the arm of the lever. To bring these points into this position, small weights are added to or taken from a little cup *a*, placed under the point from which the scale-pan is suspended.

Roberval's Balance.—Many of the balances used in shops are constructed on the principle of this machine, which is interesting for its paradoxical character. It is apparently a lever, on the arms of which if two weights balance each other, they will still continue to do so from whatever points in those arms they may be suspended. The accompanying diagram will give an idea of the construction of this machine.

It consists of four bars, A B, C D, A C, and B D; A B being equal to C D, and A C to B D. These four bars are united together by four pivots, A, B, C, and D; they also rest upon two pivots or axes E and F fixed to an upright bar E F, resting on a firm base E H.

The holes for the pivots E and F are so placed in A B and C D that B E is equal to D F.

Lastly, two bars K L and M N are fixed firmly at right angles to A C and B D, so that K L and A C form one rigid piece and M N and B D another.

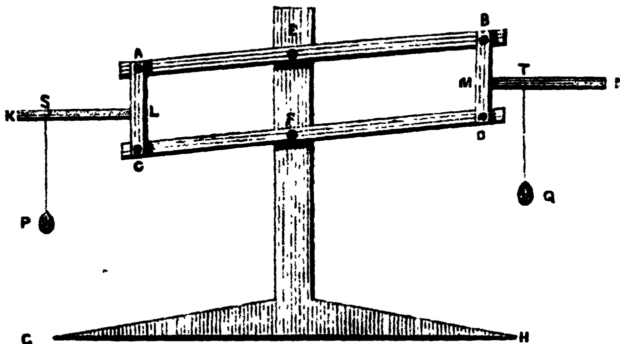


Fig. 1.

The six pivots A, B, C, D, E, and F, are so constructed that the bars may move freely about them, with as little friction as possible, in the vertical plane, while no lateral motion is permitted. From this construction it is evident that whatever position the framework A B C D may assume, when weights are suspended from the arms K L and M N, the four bars A B, C D, A C, and B D, will always form a parallelogram, and the arms K L and M N retain a horizontal position.

It is a peculiar property of this machine, that if two weights P and Q balance each other when suspended from two points S and T in K L and M N, they will also balance from whatever points in K L and M N they are suspended.

To show the properties on which this peculiar statical paradox depends, we shall neglect the weights of the various parts of the machine; or, which will come to the

same thing, suppose the weights of its various parts so chosen that it shall be in equilibrium for every position in which the framework A B C D can be placed, when no weights are suspended from the arms K L and M N. We also neglect the friction of the six pivots A, B, C, D, E, and F.

Let two weights P and Q, represented in magnitude and direction by S P and T Q (Fig. 2), be suspended from the arms K L and M N at the points S and T, and let us suppose that they balance each other.

Also let the distances A E and C F be represented by a, E B and F D by b. Since K L and A C are rigidly connected, the pressure of P on S will be transmitted by the bars K L and A C to the pivots A and C, where it will produce two pressures P_1 and P_2.

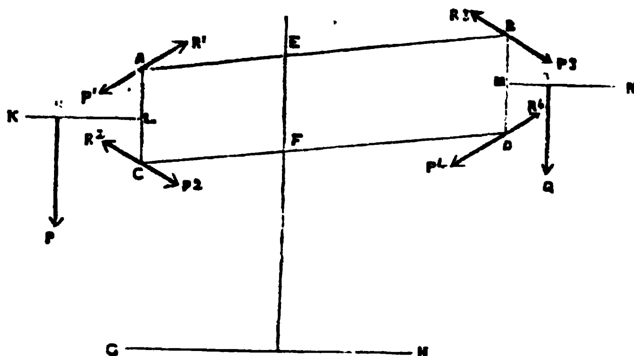


Fig. 2.

These pressures are indeterminate, both in magnitude and direction.

Let P_1, the unknown pressure on A, be represented in magnitude and direction by $A P_1$; and P_2 that on C by $C P_2$.

As the machine is supposed to be in a state of equilibrium, P_1 will be counteracted by the reaction R_1 acting in the direction $A R_1$, $A R_1$ and $A P_1$ being in the same line, and $R_1 = P_1$.

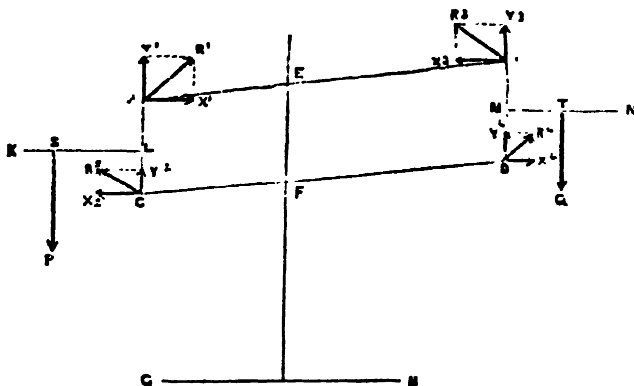


Fig. 3.

Similarly P_2 will be counteracted by the equal and opposite reaction R_2, represented in magnitude and direction by $C R_2$.

Again, because the bars B D and M N are rigidly connected, the pressure of Q on T

will be transmitted and produce two pressures P_3 and P_4 at B and D, which will be counteracted by equal and opposite reactions R_3 and R_4.

The bars K L and A C are kept in equilibrium by the three forces P, R_1, and R_2, acting at S, A and C in the directions S P, A R_1 and C R_2.

Resolving the forces represented by A R_1 and C R_2 into the vertical and horizontal forces X_1, Y_1, and X_2, Y_2, represented by A X_1, A Y_1, C Y_2, and C X_2 (Fig. 3).

We shall have by Prop. XVIII.

$$P = Y_1 + Y_2 \text{ and } X_1 = X_2.$$

The couple X_1 . AC whose tendency is to twist the rod A C is entirely destroyed by the reaction of the pivots at A and C, and the forces Y_1 and Y_2 will be equal and opposite to two forces which will exert a pressure on the extremities A and C of the levers A B and C D.

In a similar manner, by resolving the forces R_3 and R_4 into the horizontal and vertical forces X_3, Y_3, X_4, and Y_4, we shall have

$$Q = Y_3 + Y_4 \text{ and } X_3 = X_4.$$

The forces Y_3 and Y_4 being equal and opposite to two vertical pressures acting on the extremities B and D of the levers A B and C D.

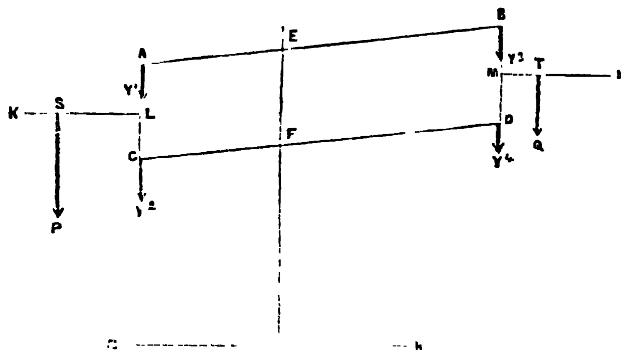


Fig. 4.

Finally, we have the lever A B resting on the fulcrum E, kept in equilibrium by the forces Y_1 and Y_3 (Fig. 4).

Hence $Y_1 \cdot a = Y_3 \cdot b$.

Also for the lever C D we have

$$Y_2 \cdot a = Y_4 \cdot b.$$

$$\text{Hence } (Y_1 + Y_2) \, a = (Y_3 + Y_4) \, b;$$
$$\text{But } Y_1 + Y_2 = P, \text{ and } Y_3 + Y_4 = Q;$$
$$\text{Therefore } P \cdot a = Q \cdot b.$$

Provided therefore that the pivots are strong enough to resist the lateral strains acting on them, the condition of equilibrium for the machine is that P multiplied by a shall be equal to Q multiplied by b. a result entirely independent of the quantities c and d, or of the distances S L and M T.

In constructing balances on this principle, a and b are taken equal to each other, in which case E and F bisect A B and C D, and P and Q are equal.

Wheel and Axle.—The second mechanical power is the wheel and axle; this machine in its simplest form consists of a circular wheel A B firmly fixed at right

angles to a cylinder C D E F so that both revolve together round a common axis G H.

The forces P and W are supposed to be applied by weights suspended from one extremity of the cord wrapped round the wheel or cylinder to which the other extremity is attached. The forces by this contrivance always act at right angles to the circumferences of the wheel and cylinder.

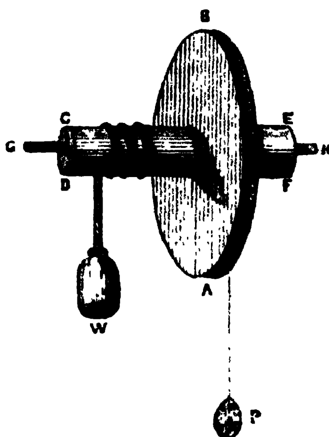


Fig. 1.

The cords are supposed to be perfectly flexible, inextensible, and destitute of weight. Their friction on the surface of the wheel and axle, as well as the magnitude of their diameters, is also neglected.

Fig. 2 represents a section of the wheel and cylinder perpendicular to their common axis.

Let A B be the radius of the wheel and A C that of the cylinder or axle. Since the forces P and Q always act at right angles to the circumference of the wheel and axle, the force P may be represented in magnitude and direction by B P at right angles to A B, the force W by C W at right angles to A C.

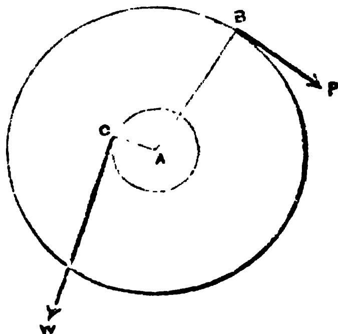


Fig. 2.

When these forces produce equilibrium, taking moments about A we have

$$P \cdot AB = W \cdot AC \text{ or } \frac{P}{W} = \frac{AC}{AB},$$

that is

$$\frac{\text{The force acting on the surface of the wheel}}{\text{The force acting on the surface of the axle}} = \frac{\text{radius of axle}}{\text{radius of wheel}}.$$

This machine is only a modification of the lever; for referring to Fig. 2 we may consider it in its position of equilibrium as a bent lever, whose arms are A B and A C and whose fulcrum is A.

When it is in motion, we may regard the wheel and axle as made up of a number of spokes; these spokes come successively into action as levers, and thus the advantage of an endless leverage is produced, and the power and weight each act constantly in a straight line instead of describing circular arcs as in the common lever.

Instead of the wheel, one or more bars are sometimes fixed at right angles to the axle, and these are often so disposed as to allow several men to act at once on the machine. When the axis of the axle is horizontal, a bar fixed at right angles to the extremity of the bar, forming what is called a winch, forms a convenient means for applying the force.

When the axis is horizontal, as in Fig. 3, the machine is called a *windlass;* when vertical, as in Fig. 4, a *capstan.*

Toothed Wheels.—The third mechanical power is the *toothed wheel,* and is extensively used in the construction of cranes, clock and watchwork, and almost every variety of machinery.

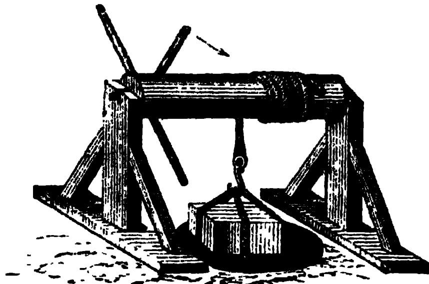


Fig. 3.

Toothed wheels consist of thin cylinders, having their circumferences indented or covered with projections called *teeth* or *cogs,* set at equal distances from each other.

If two such wheels have their axes placed in such a position that the surfaces of the wheels may be in the same plane and the edges of their teeth touch each other, as in Fig. 1, and one of the wheels be made to revolve round its axis, its teeth will act in succession on the teeth of the other, and cause it to revolve round its axis in an opposite direction.

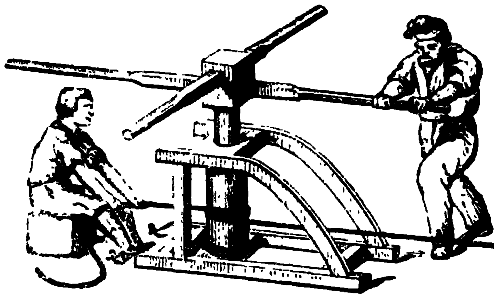


Fig. 4.

To determine the conditions of equilibrium for this machine, we will suppose A and A′ to be the axes about which the wheels revolve, and the weights P and W which produce equilibrium to be attached to the extremities of cords wrapped round, and having their other extremities fastened to, cylinders fixed perpendicularly to the wheels, and having a common axis with them.

Let A B and A′ B be the radii of these cylinders.

The weight P will communicate a tension to the rope B P, which will produce a pressure on the cylinder A B; this pressure will be communicated from the cylinder to the point C, where the tooth of the wheel C E is in contact with the tooth of the wheel C′ E′. In a similar manner, the pressure produced by W on the cylinder A′ B′ will be communicated to the same point.

These two pressures at C will act perpendicularly to the surfaces in contact, and when there is equilibrium they will destroy each other; consequently, they will be equal and opposite to each other. Let these pressures be represented by R, and their magnitudes and direction by C R and C R′.

From A and A′ draw AD and AD′ at right angles to CR, and join AA′ cutting CR in F. The wheel AE is kept in equilibrium by the force P acting in the direction BP, and R in the direction CR; and the wheel CE′ by the force W in the direction B′W and R in the direction CR′

Hence taking moments about A we have $P \cdot AB = R \cdot AD$
And taking moments about A′ we have $W \cdot A'B' = R \cdot A'D'$

Dividing one equation by the other $\frac{P}{W}\frac{AB}{A'B'} = \frac{AD}{A'D'}$, or $\frac{P}{W} = \frac{AD}{A'D'} \cdot \frac{A'B'}{AB}$

In the two triangles ADF, A′D′F, the angles at D and D′ are right angles, and the angle DFA = angle D′FA′; hence the triangles are similar, and therefore

$$\frac{AD}{A'D'} = \frac{AF}{A'F}$$

and substituting this value we have

$$\frac{P}{W} = \frac{AF}{A'F} \cdot \frac{A'B'}{AB}$$

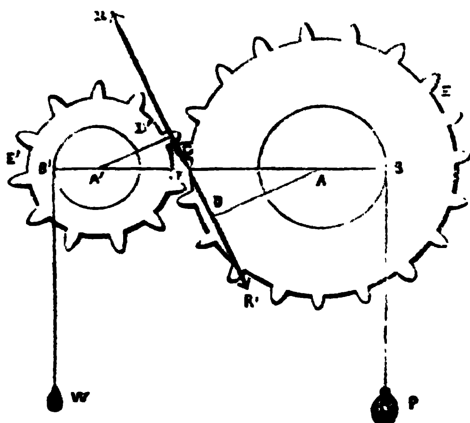


Fig. 1.

If we make $AB = A'B'$, the effect of the combination will depend upon the teeth of the wheels and their radii only, and then

$$\frac{P}{W} = \frac{AF}{A'F}$$

When the teeth are small in comparison with the radii of the wheels, AF and A′F will be nearly equal to these radii. And since the intervals between the teeth must be equal, in order that the wheels may work through a whole revolution, the number of teeth in each wheel will be proportional to their respective circumferences. Hence in this case

$$\frac{P}{W} = \frac{\text{radius of wheel CE}}{\text{radius of wheel CE}'} = \frac{\text{circumference of CE}}{\text{circumference of CE}'} = \frac{\text{number of teeth in CE}}{\text{number of teeth in CE}'}$$

The edges of the teeth which come in contact with each other are sometimes formed of curves, which are portions of the curve called the involute of a circle. In this case the point C retains the same position throughout the whole revolution of the wheel. The discussion of this property, as well as the best form of the teeth, requires a greater knowledge of the higher branches of geometry than can be assumed in an elementary work.

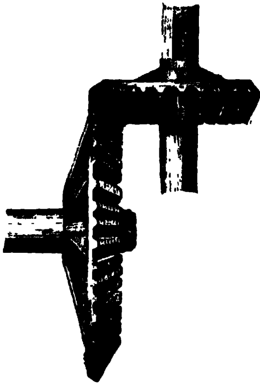
When the number of teeth in a wheel is small, the wheel is called a *pinion* and the teeth *leaves*.

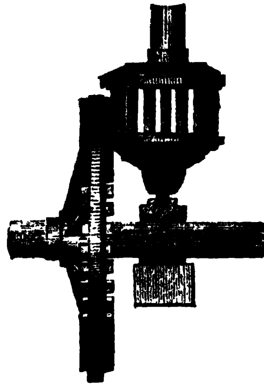
The axles about which the wheels revolve may be placed at right angles to each other, as in Figs. 2 and 3, or inclined at any angle to each other as in Fig. 4.

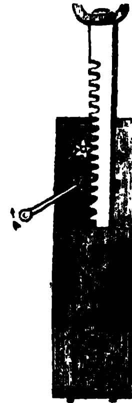
In Figs. 2 and 4 the pinions are placed on the surfaces of frustrums of cones, whose axes coincide with those of the wheels: the wheels are then called *bevelled wheels*. When the teeth project from the edges of the wheel, it is called a *spur* wheel; when they project from the surface of the wheel as in Fig. 3 it is called a *crown* wheel.

The teeth in which those of one of the wheels work may be placed along the edge

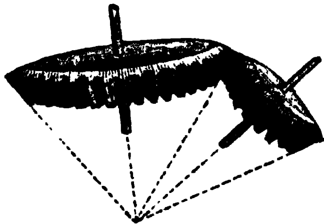
of a straight bar instead of the surface of a cylinder, as in Figs. 5 and 6; the bar being so confined as to allow it only to move in the direction of its length. The arrangement

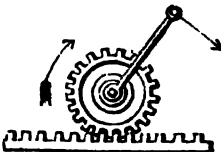

Fig. 2.

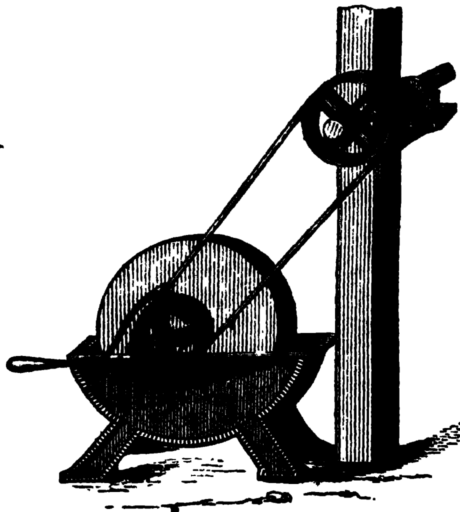

Fig. 3.


Fig. 5.

in Fig. 5, where the wheel is made to revolve by a winch, is called a *jack*, and is often employed for raising heavy weights a small height.


Fig. 4.


Fig. 6.


Fig. 8.

Wheels are sometimes turned by simple contact with each other, as in Fig. 7; and when they are at a distance from one another, as in Fig. 8, they may be made to act

on each other by a band, strap, or chain passing over and in close contact with a portion of the surfaces of both. In these cases the minute protuberances of the surfaces, or the friction they exert on each other, prevent the surfaces from sliding, and act as minute teeth. A band which slips, may frequently be made to act by chalking its surface.

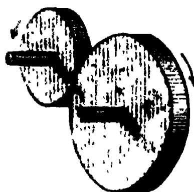


Fig. 7.

The Pulley.—The fourth mechanical power is the pulley. In its simplest form it consists of a solid circular wheel B C, having a smooth groove cut in its circumference. This wheel revolves freely round an axis or pivot passing through its centre at right angles to its surface, and having its extremities fixed to a wooden or metallic frame F G. This frame may be either fixed or moveable. The wheel is called the *sheaf*, *shiver*, or *rundle*; the axis the *gudgeon*, and the frame in which the axis is fixed the *block* of the pulley.

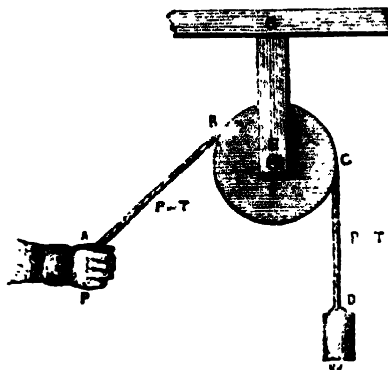


Fig. 1.

The force P is applied to one extremity of a cord passing freely over the grooved circumference of the sheaf, which moves with it round its axis, thus diminishing the friction of the cord, and the weight W, which is to be overcome, is attached to the other extremity of the cord.

Several pulleys may be combined together, forming what is called a *system*. The cords which pass over the sheaves are supposed to be perfectly flexible and inextensible; the friction of the cords, as well as that of the axis, and, when not specially mentioned, the weights of the blocks and sheaves, are not considered in our calculations.

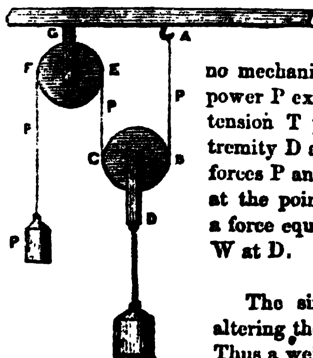


Fig. 2.

The single pulley, with its block fixed, affords no mechanical advantage of the power over the weight, for the power P exerted on the extremity of the cord communicates a tension T proportional to P throughout its length to its extremity D at which W acts. When there is equilibrium, these forces P and W must be both equal to the tension T of the cord at the points A and D, or the tension of the cord will produce a force equal and opposite to P at A, and equal and opposite to W at D.

$$\text{Hence } P = W = T.$$

The single fixed pulley affords a convenient means for altering the direction of the application of a force to a machine. Thus a weight which acts vertically downwards, may be made to exert an equal force in any direction we require, by a proper position of the pulley and the support to which its block is fixed.

Single Moveable Pulley.—Let a weight P be attached to one extremity of a cord

passing over the fixed pulley F E, round the moveable pulley C B, and having its other extremity fastened to a fixed beam at A, the block of the pulley F E being fixed to the beam at G in such a manner that E C and A B shall be parallel. The weight W is attached to the block D of the pulley C B. Neglecting the weight of the pulley C B, P will communicate a tension throughout the cord, and this tension will produce a force P acting in the direction C E at E, and another force P acting in the direction B A at B.

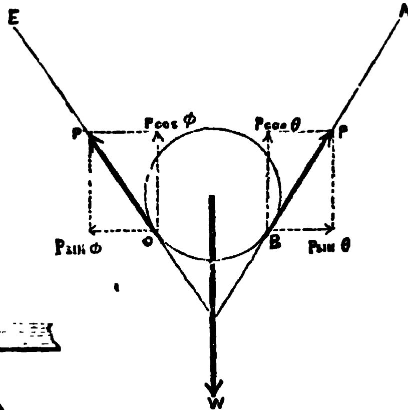


Fig. 3.

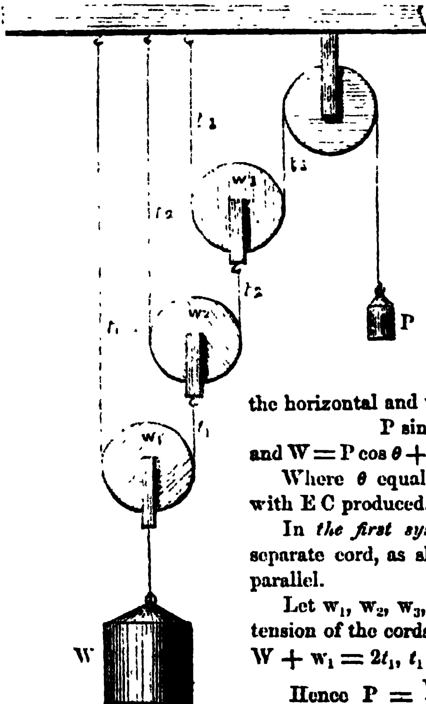


Fig. 4.

Hence when there is equilibrium, we shall have

$$W = P + P = 2P.$$

When the weight of the pulley is taken into account, this weight must be added to W. When the cords are not parallel, resolving the forces P acting at C, and P acting at B, into vertical and horizontal forces, as shown in Fig. 3, and equating the horizontal and vertical forces acting on B C, we have

$$P \sin \phi = P \sin \theta, \text{ hence } \phi = \theta$$

and $W = P \cos \theta + P \cos \phi = P \cos \theta + P \cos \theta = 2P \cos \theta$

Where θ equals half the angle A B produced makes with E C produced.

In *the first system of pulleys*, each pulley hangs by a separate cord, as shown in Fig. 4, so that the cords may be parallel.

Let w_1, w_2, w_3, be weights of the pulleys, t_1, t_2, t_3 the tension of the cords.

$W + w_1 = 2t_1$, $t_1 + w_2 = 2t_2$, $t_2 + w_3 = 2t_3$, and $t_3 = P$.

$$\text{Hence } P = \frac{w_3}{2} + \frac{t_2}{2} = \frac{w_3}{2} + \frac{w_2}{2^2} + \frac{t_1}{2^2} = \frac{w_3}{2} + \frac{w_2}{2^2} + \frac{w_1}{2^3} + \frac{W}{2^3}$$

If there had been four pulleys we should have had

$$P = \frac{w_4}{2} + \frac{w_3}{2^2} + \frac{w_2}{2^3} + \frac{w_1}{2_4} + \frac{W}{2^4}$$

Neglecting the weights of the pulleys, w_1, w_2, &c., for three pulleys we shall have $P = \frac{W}{2^3}$, for four $P = \frac{W}{2^4}$, and generally for n pulleys $P = \frac{W}{2^n}$.

When the cords are not parallel, we may see, from the case of the single pulley, that each tension, t_1, t_2, t_3, &c., must be multiplied by the cosine of the angle which its cord makes with a line drawn parallel to the direction in which W acts.

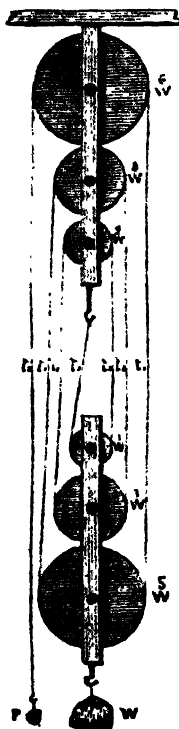


Fig. 5.

This multiplication will necessarily diminish the tension, as the cosine will always be a fraction less than unity.

In *the second system of pulleys*, the cord to which P is attached passes over all the pulleys as shown in Fig. 5. The pulleys are divided into two groups, the upper working in a common block which is fixed, and the lower in a common moveable block. The other extremity of the cord to which P is attached after passing over all the sheaves, is fixed to the upper block. W is attached to the lower block.

The weight or force P will communicate the same tension t_1 throughout the rope.

It is evident, therefore, that if we consider the ropes as parallel, and neglect the weight of the lower block and its sheaves when there is equilibrium, we shall have

$$W = 6P$$

and generally we shall have $W = nP$ where n is the number of the portions into which the cord is divided by the two blocks.

When the weights of the sheaves and blocks are taken into consideration, it is evident that the weight of the lower block and its sheaves must be added to W.

If the positions of the cords be not parallel, each portion must have its tension multiplied by the cosine of the angle to which it is inclined to the direction of W, as in the first system of pulleys.

In Fig. 6 we have represented a very powerful arrangement, having the pulleys in each block arranged as shown in the figure. The sheaves of the upper row in the upper block, as well as those of the lower row of the lower block, are all of the same diameter; those of the lower row of the upper, and upper row of the lower are equal to each other in diameter, though less than the former. The cord is omitted for the sake of clearness, but one extremity is fixed to the hook A below the lower block; it then passes in succession under each of the sheaves in the lower block, and over those in the upper in the order shown by the numbers attached to the sheaves in the figure: the power is applied to the extremity of the cord which passes over the pulley marked w_{20}.

As there are twenty portions of the cord between the two blocks, in this case $W = 20$ P.

This arrangement permits the portions of the cords to be nearly parallel, and has the additional advantage that the power P acts immediately over the weight W, so that the parallelism of the cords will not be deranged, as it would otherwise be liable to be; it has, however, serious practical defects.

If the weight W be raised by the power P a given height, say for instance one foot,

each portion of the cord between the two blocks must be raised the same height, which will necessarily cause P to descend 20 feet. From the arrangement of the cord it is evident that one foot of the cord will pass over the pulley w_1, two feet over w_2, three feet over w_3, &c., and 20 over w_{20}.

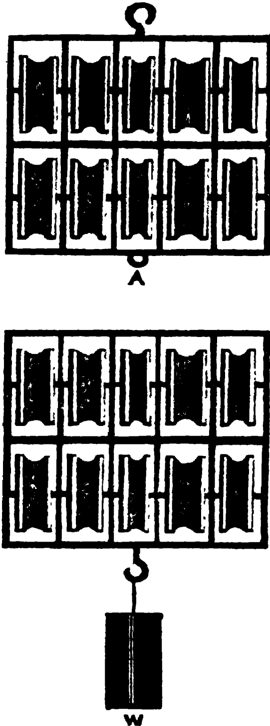


Fig. 6.

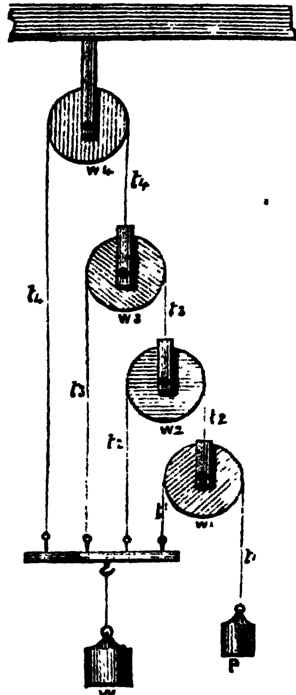


Fig. 7.

Now since the sheaves w_1, w_2, w_3, &c., are of the same diameter, and revolve each round its own axis, and twice as much cord passes over w_2 as w_1, three times as much over w_3 as w_1, and so on; it follows that w_2 will revolve twice as fast, w_3 three times as fast, as w_1, and so on for the other sheaves.

This inequality of motion leads to great inequality in the wear of the various parts of the machine. If we attempt to remedy this defect by fixing the sheaves in the same row to a common axis, parts of the cord must necessarily scrape or slide over the grooves of the sheaves instead of moving with them; and this will lead to great increase of friction. To remedy these defects, Mr. White contrived the pulley known by his name. The sheaves in each block were cut out of one piece of wood or metal, being formed of a series of parallel and circular grooves, each increasing in diameter. The diameters of the grooves were made to bear to each other the same proportions as the portions of rope which were to pass over them—those for the lower block being as the odd numbers 1, 3, 5, 7, &c., and those in the upper as the even numbers 2, 4, 6, &c.

By this arrangement all inequality of wear was supposed to be obviated, and the friction was reduced nearly to that of the two axles of the blocks. But this ingenious contrivance is found practically to fail, as the elasticity of the cord, which is supposed by the arrangement to be inelastic or inextensible, is subject to continual change by the moisture or dryness of the atmosphere, and by the friction of the cord upon the grooves.

In *the third system of pulleys* each cord passes over a pulley, and has one extremity attached to the weight W, as shown in Fig. 7, the cords are supposed to be parallel to each other.

Let w_1, w_2, w_3, w_4 represent the weights of the pulleys, t_1, t_2, t_3, t_4 the tensions of the cords passing over them.

Then $t_1 = P$.

$t_2 = 2t_1 + w_1 = 2P + w_1$.

$t_3 = 2t_2 + w_2 = 2^2P + 2w_1 + w_2$.

$t_4 = 2t_3 + w_3 = 2P^3 + 2^2w_1 + 2w_2 + w_3$.

Therefore $W = t_1 + t_2 + t_3 + t_4$.

$= (1 + 2 + 2^2 + 2^3) P + (1 + 2 + 2^2) w_1 + (1 + 2) w_2 + w_3$.

$= (2^4 - 1) P + (2^3 - 1) w_1 + (2^2 - 1) w_2 + (2 - 1) w_3$.

The same reasoning may be extended to any number of pulleys n, in which case

$$W = (2^n - 1) P + (2^{n-1} - 1) w_1 + (2^{n-2} - 1) w_2 + \&c. + (2 - 1) w_n.$$

When the weights of the pulleys are neglected, w_1, w_2, &c., w_n are equal to zero, and

$$W = (2^n - 1) P.$$

When the strings are not parallel, their respective tensions must be multiplied by the cosine of the angle they make with the direction of W, as in the preceding systems.

The Inclined Plane.—The inclined plane is the fifth mechanical power. It consists of a plane surface A B, which is supposed to be perfect in hardness and smoothness, and inclined at some angle a to the horizontal line. A heavy body, whose weight is W (Fig. 1), resting on the plane, and supported by a force P acting in some direction D E, constitutes the weight in this machine.

Let β be the angle D E makes with B A, and B C be drawn at right angles to A C. a is called the inclination of the plane, B C its height, A B its length, and A C its base. When there is equilibrium, D will be acted on by three forces, the force P acting

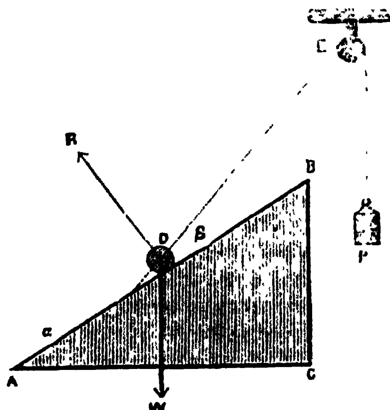


Fig. 1.

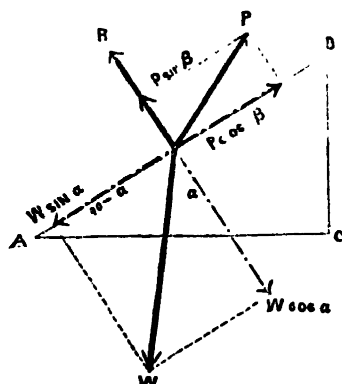


Fig. 2.

in the direction D E, the weight W acting vertically downwards, and the unknown reaction R produced by the pressure of the body D on the plane A B, which acts perpendicularly to the surface of the plane.

Let R, P, and W be represented in magnitude and direction by D R, D P, and D W (Fig. 2).

Resolving these forces parallel and perpendicular to the surface of the plane A B, as shown in Fig. 2, we have the following conditions of equilibrium:—

$$W \sin \alpha = P \cos \beta, \text{ or } \frac{W}{P} = \frac{\cos \beta}{\sin \alpha}.$$

The magnitude of R may be found from the equation

$$R + P \sin \beta = W \cos \alpha;$$

$$\text{or } R = W \cos \alpha - P \sin \beta = W \cos \alpha - W \frac{\sin \alpha \sin \beta}{\cos \beta}$$

$$= W \left(\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \beta}\right) = \frac{W \cos (\alpha + \beta)}{\cos \beta}.$$

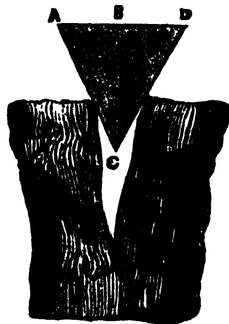
When P acts along the inclined plane in the direction D B, $\beta = 0$, and

$$\frac{W}{P} = \frac{1}{\sin \alpha} = \frac{A B}{B C} = \frac{\text{length of the plane}}{\text{height of the plane}};$$

$$\text{also, } R = W \cos \alpha = W \frac{A C}{A B} = W \times \frac{\text{base of the plane}}{\text{length of the plane}}.$$

Correctly speaking, the angles, &c., of Fig. 2 do not correctly belong to the inclined plane of Fig. 1, unless the centre of gravity of the body D touches the plane A B, but to an imaginary plane passing through the point A and the centre of gravity of D.

The Wedge.—The sixth mechanical power is the wedge, which may be regarded as two equal and similar inclined planes A B C, B D C, with their bases fixed together. It is used for cleaving substances, in which case the edge C is introduced into a cleft, and the surface A D struck by a hammer or mallet, so as to cause the wedge to enlarge the cleft and split the substance.



Considered as a statical machine, the power is a weight applied to A D sufficient to balance the pressures exerted by the substance into which the wedge is thrust on the sides A C and D C of the wedge, considering these surfaces as perfectly smooth.

The consideration of this machine is now omitted in many treatises, since "in the theory of the wedge, there are introduced so many conditions, which are perfectly inapplicable in practice, so many gratuitous assumptions and suppositions so inconsistent with practical truth, that the whole doctrine has little or no value. Nothing can more plainly demonstrate the inutility of the theory of the wedge tha . that, in this theory, the power is supposed to be a pressure exerted on the back of the wedge, which is supposed to be capable of balancing the effect of the resistance in producing the recoil of the wedge. In all cases where the wedge is practically used, the friction of its faces with the resisting substance is sufficient to prevent the recoil; so that, strictly speaking, no force whatever is necessary to sustain the machine in equilibrium; and to move it, pressure is never resorted to—inasmuch as the slightest percussion is far more effective. The only general theoretical principle respecting the wedge, which obtains always in practice is, that its power is increased by diminishing the angle CD."—*Mechanics, Society for the Diffusion of Useful Knowledge.*

The Screw.—The seventh mechanical power is the *screw*, and may be regarded as a modification of the inclined plane. If we take a triangular piece of paper A B C,

the angle at C being a right angle, in A C take a point A', and through A' draw A' B' parallel to A B. Then let the portion A A' B' B be blackened, and wrap the triangle A B C round a cylinder D E. Let now a groove be cut perpendicular to the surface of the

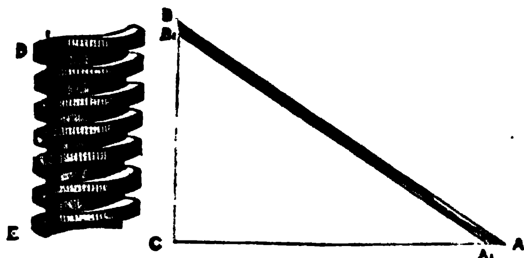


Fig. 1.

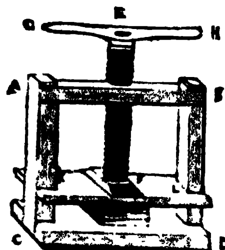


Fig. 2.

cylinder and following the direction of the dark spiral band, formed by the triangle A B C. The projecting portion between the groove is called the *thread* of the screw, and is evidently an inclined plane passing round the cylinder. The cylinder in which the groove is cut is called the *screw*. A hollow cylinder is now cut through a beam A B, and a spiral cavity, cut in the surface of the hollow cylinder, corresponding in magnitude to the thread of the screw, so that one may be regarded as the cast or mould of the other. This is called the *nut* of the screw.

If A B be firmly fixed to the upright beams A C, B D, and these again to a plane base C D, the screw E F can be made to pass through the nut by causing it to revolve round its axis E F by means of the arms E H or E G, and as it moves through the nut its axis will always be vertical, and its extremity F will press upon a plane K L so confined by the bars A C and B D in the frame A D C B as to have only a vertical motion. The pressure of the screw will thus be communicated by K L to any substance placed between K L and C D.

To obtain the conditions of equilibrium of the screw, we neglect the weight of the screw itself, and the friction of the surface of the thread on that of the nut.

To simplify, as much as possible, the problem, we first suppose the portion of the thread which, by the action of a force P_1 at right angles to the extremity G of the lever G E (Fig. 2) communicates a pressure R_1 to the surface of the nut with which it is in contact, reduced to a single point Q (Fig. 3), and the surface of the nut unfolded into the inclined plane M N O.

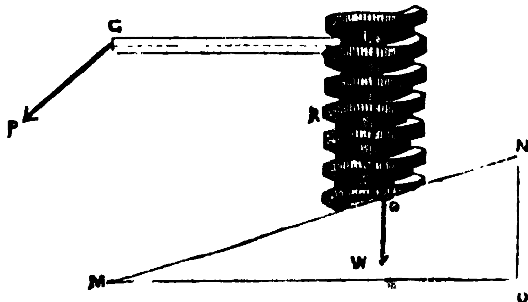


Fig. 3.

The resistance R_1 which the surface of the nut opposes to the motion of Q will be perpendicular to the surface M N of the inclined plane M N O, and the resolved portion

of this force in a vertical position will give the pressure W_1 which the screw will communicate to the substance placed between K L and C D.

Let a be the length of the arm G E, b the radius of the cylinder of the screw, α the angle of the plane M N O.

Resolving R_1 vertically and horizontally, we have the following condition of equilibrium:—

$$W_1 = R_1 \cos \alpha;$$

and taking moments about the axis of the screw,

$$a \,.\, P_1 = b\, R_1 \sin \alpha.$$

Now we may conceive the portion of the thread of the screw divided into a number of portions Q_1, Q_2, &c., Q_n, kept in equilibrium by forces P_1, P_2, &c., P_n, W_1, W_2, &c., W_n, and producing reactions R_1, R_2, &c., R_n on the surface of the nut.

$P_1 + P_2 +$ &c. $+ P_n$ will be the whole power P applied to the extremity of the arm G E, and $W_1 + W_2 +$ &c. $+ W_n$ will be the whole vertical pressure W which the screw will produce.

Then, as before, we shall have

$$\begin{aligned} W_2 &= R_2 \cos \alpha \text{ and } aP_2 = bR_2 \sin \alpha \\ W_3 &= R_3 \cos \alpha \quad\quad aP_3 = bR_3 \sin \alpha \\ \&c. &= \&c. \quad\quad\quad\quad \&c. = \&c. \\ W_n &= R_n \cos \alpha \quad\quad aP_n = bR_n \sin \alpha \end{aligned}$$

Hence, adding these equations, we have

$$(W_1 + W_2 + W_3 + \&c.\ W_n) = (R_1 + R_2 + R_3 + \&c. + R_n) \cos \alpha$$
$$a\,(P_1 + P_2 + P_3 + \&c. + P_n) = b\,(R_1 + R_2 + R_3 + \&c. + R_n) \sin \alpha;$$

and dividing these equations we have

$$\frac{W_1 + W_2 + W_3 + \&c. + W_n}{a\,(P_1 + P_2 + P_3 + \&c. + P_n)} = \frac{\cos \alpha}{b \sin \alpha} \quad \text{or} \quad \frac{W}{aP} = \frac{\cos \alpha}{b \sin \alpha};$$

$$\text{and } \frac{W}{P} = \frac{a}{b \tan \alpha} = \frac{2\pi a}{2\pi b \tan \alpha};$$

$$\text{or } \frac{W}{P} = \frac{\text{circumference of circle described by the extremity G of arm G E}}{\text{vertical distance between two threads of the screw}}.$$

For if NG (Fig. 3) be the vertical distance between two threads of the screw, MO will be equal to $2\pi b$ and $NO = MO \tan \alpha = 2\pi b \tan \alpha$.

Friction.—In our previous investigations we have supposed all our surfaces in contact with each other to be perfectly smooth. This perfect smoothness can never be attained in practice; the roughness, or want of smoothness, of two surfaces in contact with each other, opposes a resistance to their motion over or along each other which is called *friction*. From certain experiments it appears that friction may be due, in some measure, to the nature of the surfaces in contact with each other, and influenced by the molecular forces which the particles in contact may have with each other.

Experiments made with a number of different substances have led to the following laws:—

That the force of friction is proportional to the pressure acting on the surfaces in contact so long as the materials of the surfaces in contact remain the same, and act at right angles to the direction of the pressure.

That for the same pressure the friction is the same, whatever may be the magnitude of the surfaces in contact.

These laws are not strictly true, but are subject to considerable variation in certain

extreme cases, as when the pressures are very great and the surfaces in contact very small.

The friction of moving surfaces is also much less than that of the same surfaces in a state just bordering on motion.

For the state just bordering on motion the friction of two smoothly planed pieces of wood, the grain being in the same direction, is half the pressure; for the same the grain being in opposite directions, is one-fourth the pressure. The friction of two metallic surfaces is one-fourth the pressure; and of one surface metallic and the other wood, one-fifth the pressure.

This friction may be greatly diminished, by the use of lubricants, such as oil, tallow, black-lead, &c.

If the points in contact be mere lines, as in the case of the knife-edge of the fulcrum of a lever, this friction may be considerably reduced. Thus, the friction of wooden surfaces is diminished in this case from one-fourth to one-twelfth the pressure exerted by the surfaces in contact.

WALTER MITCHELL.

DYNAMICS.

SECTION I.—ACCELERATING FORCES.

Introduction.—Every *body*, or material particle, is necessarily in a state of either rest or motion. A body strictly at rest is regarded as acted upon either by no force at all, or else by forces which oppose and neutralize one another. Investigations respecting forces which, acting upon a body or upon a system of bodies, thus keep the whole at rest, belong to the first great division of Mechanical Philosophy—*Statics*. In this branch of the subject, the result of the several applied forces is *pressure* or *tension*, but no movement; in the second great division, now to be treated of, the result of the applied forces is *motion*: there may be pressure as well, but it is the motion or change of place of the body acted upon, that is the exclusive subject of *Dynamics*.

Of physical causes we know little or nothing: we observe motion or pressure, and we infer *force*—that is, some cause for the phenomena: but it is with the effects or phenomena alone that our observations and reasonings are concerned.

Ignorant, however, as we are of the essence of force, we cannot be under any doubt that it is the same in kind when operating in the production of motion as when operating in the production of pressure; for whenever, in the latter case, the resistance or obstacle is removed, motion necessarily ensues. But there is one consideration inseparably connected with that of motion, which has no place in statics: it is the consideration of *time*, an element which necessarily enters into the very idea of motion.

Nor can we conceive of a force which transmits its influence to a distant body—as the force of magnetism, or the force of gravity—as doing so independently of time. It is inconceivable that some interval does not elapse, however minute, between the cause and the effect: the transmission of a force through space must take time. A few years ago, the following question was proposed to the author of the present treatise:—A beam or bar uniformly strong—that is, resisting fracture in every part alike—is immoveably fixed at one extremity, from which it hangs vertically; to the other, or lower extremity, is then attached a weight indefinitely great: where will the beam break? The condition is that the beam has no tendency to break at one place more than at another, and yet the weight suspended to it is to be immeasurably great. The answer is, that time being required to transmit the force through the particles of the beam, the fracture takes place at the lowest part, the falling weight bringing with it only the lamina of material in immediate contact with it.

As the dynamical effect of force is motion, and as motion implies space passed

through, and the time of passage, it is clear that, as in all physical inquiries, measuring causes by their effects, the measure of force must in some way be compounded of space and time.

The simplest kind of motion is *uniform* motion, or that in which the moving body passes through equal spaces in equal intervals of time; and the simplest path the moving body can describe, is a straight path. We can fully examine motion of *this* kind without any reference to force at all; we can take the motion as we find it, without any inquiry as to its origination; and ascertain all we wish to know respecting it. In fact, a body so moving is not, during the motion, acted upon by any force at all. Conceive an isolated body at rest, all other bodies and forces in nature being removed: it is plain that that body will remain at rest, for there is nothing to disturb its condition. In like manner, conceive such isolated body to be moving uniformly in a straight line: it must continue to move uniformly, for there is nothing to disturb its condition: to suppose a force to act upon it at any point of its path, and yet for the uniform onward motion to remain the same, would be to suppose a cause without an effect. A body, therefore, thus moving uniformly onward, is not actuated by any force whatever. It is true it could not pass from rest to uniform motion in a straight line without a cause— an impulse, for instance—but nothing acts upon it afterwards, or during its uniform motion; otherwise that uniformity would be interfered with.

It is this simplest kind of motion that we shall first consider, and shall then proceed to that which is due, not to instantaneous impulse, but to *force* continuously acting upon the moving body. The proper method of measuring this force will shortly be explained.

Uniform Rectilinear Motion.—In appplied, as well as in pure mathematics, certain fundamental positions or postulates must at the outset be assented to. In Dynamics there are three such postulates: they are known as the three laws of motion.* The first of these is as follows:—

A material particle, if at rest, and unacted upon by any external force, will remain at rest. A material particle, if in motion, and unacted upon by any external force, will continue its motion uniformly, and in a straight line. This law is only a formal enunciation of the *inertia* of matter, by which is meant its incapability of altering, *of itself*, the state into which it is put by any external cause, whether that be a state of rest or a state of motion.

It is plain that the motion spoken of, being neither unchecked nor expedited, nor its direction any way interfered with, must be uniform in its *rate*, and rectilinear in its course—the course originally impressed.

The *rate* of a body's uniform motion is estimated by the extent passed over in some determinate portion of time—a second, a minute, an hour, &c. In dynamical investigations, the *second* is generally taken for the unit of time; and what in popular language is called *rate* of motion, is here called *velocity*; hence if a body moves uniformly over ten feet every second of time, we say that it moves with a velocity of ten feet, or that its velocity is ten feet "per second" being understood. Putting the symbol v for velocity, we should therefore have $v = 10$ feet. As a second is taken for the unit of time, so a foot is taken for the unit of length. It is common, however, to call the length of track described by a moving body the *space* passed through, and to represent this length by s; but it will be borne in mind that by s length only is meant.

The symbol employed for *number of seconds* is t: it is to be carefully observed that t does not stand for the concrete quantity *time*, but only for the *number* of seconds

* The three laws of motion are formally enunciated at the end of this treatise.

in that time; so that *length*, viz. *feet*, is the only concrete quantity represented by the symbols v, s, t, which are related to one another as follows, namely:—

$$v = \frac{s}{t},\ s = vt,\ t = \frac{s}{v}$$

So that any two of the three quantities concerned being known, the third may be immediately found. If, however, t is not reckoned from the commencement of the uniform motion, but only after a certain space s' has been passed over by the moving body, then s being the whole length of path from the commencement, the three equations will be

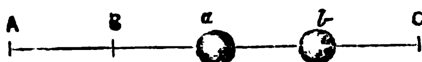
$$v = \frac{s - s'}{t},\ s = s' + vt,\ t = \frac{s - s'}{v}$$

Any one of these equations is sufficient for the entire theory of the motion of a body impelled by a single impulse, or influenced originally by any cause producing uniform motion.

It may, perhaps, be as well to remark here, that we speak of the path of a particle and of a body indiscriminately. In most works on Dynamics all mention of *body* is avoided in this part of dynamics, and the motion of a single particle only considered. But this exclusion of body as an assemblage of particles is, we think, as injudicious as it is unnecessary. There *seems*, it is true, more mathematical precision of language in speaking of the line (s) traced out by a moving particle, than of what is described by a bulky body; but let it be only understood that by the path of a body we mean the line traced out by its centre of gravity, and all grounds for depriving the mass of volume and reducing it to an indivisible isolated particle—of which indeed we can have no clear idea—will be removed. Throughout the present portion of dynamics, our investigations are entirely independent of mass or volume; and, consequently, to make a single physical particle the subject of those investigations to the studied and systematic exclusion of body, is to perplex and mislead the learner—in fact, to convey to him the notion that what he is learning is applicable only to geometrical abstractions—to physical nonentities, and not to the actual material substances around us.

We shall now give an example of the application of the formulæ just established.

Two bodies (a, b) animated by the uniform velocities v, v', set out simultaneously from the points A B, and move in the direction of the straight line A B continued: required the time of their coming together.



Suppose a overtakes b at the point C, then

$$\text{A C} = vt,\ \text{B C} = v't;$$

that is, calling A C, s, and A B, s',

$$s = vt,\ s - s' = v't.$$

$$\therefore vt - s' = v't\ \therefore t = \frac{s'}{v - v'}.$$

So that the abstract number denoting the units of time—that is, the number of seconds—will be found by dividing the space between the points of starting by the difference of the spaces denoting the velocities.

If uniform motion be the result of an impulse communicated to a body originally at rest, we may reasonably conclude that the velocity produced will be proportional to the intensity of the impulsive energy. For if a body receive a certain velocity in consequence of a certain impulse, it ought to acquire double that velocity if at any point of

its path that impulse be repeated in the same direction; but if the second impulse, like the first, take place at the starting point, it must unite with the first impulse, so that the consequence of a double intensity of impulse will be a double velocity in the body impelled, and in like manner a triple intensity will produce a triple velocity, and so on.

Variable Rectilinear Motion.—When a moving body does not pass over equal spaces in equal times, the motion is not uniform, but variable. The rate at which the body moves, or its velocity at any instant, is the space it *would* describe in a second of time, measuring from that instant, provided all force were then withdrawn, and the body left to itself. Its motion during this second—as no force acts upon it—would of course be uniform.

Hence velocity in general is measured as follows:—

If the motion be uniform, the velocity is measured by the space actually passed over in a second. If the motion be variable, the velocity at any instant is measured by the space that *would be passed over* in a second, if all force were withdrawn at that instant, and the body left to proceed with the motion it then has.

Accelerating Force.—A body may so move as to require equal accessions of velocity in equal times—that is, it may move so that its velocity at any time t being v, we may have

$$\begin{array}{llllll} \text{at the times } t, & t+1, & t+2, & t+3, & t+4, & \&c. \\ \text{the velocities } v, & v+v', & v+2v', & v+3v', & v+4v', & \&c. \end{array}$$

Or the velocity may, in like manner, be retarded, v' being negative. Under such circumstances the velocity is said to be uniformly *accelerated*, or uniformly *retarded*.

The cause of the uniform acceleration or retardation of a body's velocity, we call *force*; and measuring causes by their effects, we take the amount of this constant acceleration or retardation as the measure or representative of a constant force, continuously acting on the moving body.

The symbol for this force is f, which, viewed only in its effects, denotes merely the velocity *generated* (or destroyed) in a second of time. In the illustration above, for instance, $f = v'$.

It is of importance that the student do not attach any other meaning to this symbol f, than that here assigned to it. The nature of the occult influence called dynamical force —or, as it is more frequently named, *accelerative* force—nobody can explain In the present inquiry we have to do only with its effect; and this we see is merely augmentation (or diminution) of velocity; and therefore, like velocity itself, it is expressed by *space*, that is, by linear measure: it is not the *force* that is accelerated, but the *velocity*.

If the force be constant or uniform, the acceleration of the velocity is also constant or uniform, as supposed above; but if the force be variable, the acceleration of the velocity is also variable: the only office of accelerative force is to produce accelerative velocity, and it is only by this latter phenomenon that we become cognizant of its activity, and can estimate its intensity.

Although, as remarked at the outset, we are altogether unacquainted with the essence of the thing called force, yet we may have abundant means of ascertaining whether its effects are constant or variable; and it must be carefully borne in mind that, in speaking of a constant force, or of a variable force, we have reference solely to the constant effect, or the variable effect. The force itself, whatever it be, may in reality remain entirely unchanged; and yet, if it affect a body differently at different distances from its supposed seat of action, we should call it a *variable* force. For example, there is a force familiar to every body called *gravity;* and there can be no doubt that this force, like

the magnitude of the earth itself, remains unchanged: yet as we find that the higher above the surface a body be taken, the less it will weigh, and the less will its velocity be accelerated, we say—exclusively in reference to these variable effects—that gravity is a variable force. It may be observed, however, that as this variation of weight and acceleration is insensible at moderate distances from the earth's surface, in all the ordinary applications of dynamics to terrestrial matters, gravity may without error be regarded as a constant force.

It is of importance that the student have correct conceptions of the sense in which the terms velocity and force are employed in these inquiries, and also that he should clearly perceive that *space* is the only concrete quantity concerned in the present portion of dynamics. The two propositions following, contain the whole theory of the rectilinear motion of a body under the influence of a constant accelerating force.

1. If a constant accelerating force f act on a body now at rest, during the time t, producing in it at the end of that time a velocity v, then $v = ft$. For since f expresses the velocity generated in each second, it follows that in the t seconds, the velocity v will amount to $ft \therefore v = ft$.

2. If s be the space through which the body is moved from rest by the constant accelerating force f, in the time t, then $s = \frac{1}{2} ft^2$.

Suppose the time t to be divided into n equal parts; then since equal velocities are generated in equal times, the velocity generated in the time $\frac{t}{n}$ is the nth part of that generated in the time t, that is, by last proposition, it is $\frac{ft}{n}$. Consequently,

$$\text{at the end of the times } \frac{t}{n}, \frac{2t}{n}, \frac{3t}{n}, \quad \ldots \quad \frac{nt}{n} \text{ or } t$$

$$\text{the velocities will be } \frac{ft}{n}, \frac{2ft}{n}, \frac{3ft}{n}, \quad \ldots \quad \frac{nft}{n} \text{ or } ft \quad . \quad . \quad (A)$$

and at the *commencement* of the same intervals, the velocities will be

$$0, \frac{ft}{n}, \frac{2ft}{n}, \quad \ldots \quad \frac{(n-1)ft}{n} \quad . \quad . \quad . \quad (B)$$

If the several velocities (A) were uniform during the several equal intervals of time, then the whole space S described would be

$$S = \frac{ft}{n} \cdot \frac{t}{n} + \frac{2ft}{n} \cdot \frac{t}{n} + \frac{3ft}{n} \cdot \frac{t}{n} + \quad \ldots \quad + ft \cdot \frac{t}{n}.$$

And if, in like manner, the several velocities (B) were uniform during those intervals, the whole space S′ described would be

$$S' = 0 \cdot \frac{t}{n} + \frac{ft}{n} \cdot \frac{t}{n} + \frac{2ft}{n} \cdot \frac{t}{n} + \quad \ldots \quad + \frac{(n-1)ft}{n} \cdot \frac{t}{n}.$$

Now, it is obvious that the space s actually described, is intermediate between the two spaces S, S′; being less than the former, and greater than the latter. And this is evidently true, however short the several equal intervals, that is, however large the number n may be.

But the shorter the intervals be made, the closer do the values S and S′ approach to each other, and therefore to the intermediate value s, till at length, when the intervals are diminished down to zero, by n becoming infinite, all three values S, s, S′ unite and become identical.

The determination of s is therefore reduced to the following algebraical problem, namely,—to determine the value of S or of S′ when n becomes infinite.

Now the general expression for the value of S is

$$(1 + 2 + 3 + \ . \ . \ . \ + n)\frac{ft^2}{n^2}$$

$$= \frac{n(n+1)}{2} \cdot \frac{ft^2}{n^2} = \left(1 + \frac{1}{n}\right)\frac{ft^2}{2}$$

In like manner, the general expression for the value of S' is

$$\{0 + 1 + 2 + \ . \ . \ . \ + (n-1)\}\frac{ft^2}{n^2}$$

$$= \frac{n(n-1)}{2} \cdot \frac{ft^2}{n^2} = \left(1 - \frac{1}{n}\right)\frac{ft^2}{2}$$

When n is infinite, since in that case $\frac{1}{n} = 0$, each of these expressions becomes $\frac{1}{2}ft^2$: consequently $s = \frac{1}{2}ft^2$. It appears then, from these two propositions, that if a body at rest be acted upon by a constant and uniform accelerating force f, during t seconds, the velocity acquired and the space described will be expressed by the equations

$$v = ft \text{ and } s = \tfrac{1}{2}ft^2 \quad . \ . \ . \ . \quad (1)$$

From these two important equations we may proceed to deduce some inferences.

1. The space described in any time, reckoning from the commencement of the motion, is half the space that *would be* described in the same time if the body were to move from rest with a uniform velocity equal to the last acquired velocity.

For since by (1) $s = \frac{1}{2}(ft)t$, and $v = ft \therefore s = \frac{1}{2}vt$; but vt is the space described in t seconds with the uniform velocity v: hence the space s, actually described under the influence of a uniform accelerating force, is half the space that would be described in the same time if the body were to commence with the final velocity and move uniformly.

2. The spaces described in equal successive portions of time, from the commencement of the motion, are to one another as the odd numbers 1, 3, 5, 7, &c.

For let t in (1) be successively 1, 2, 3, 4, &c., then the spaces from beginning are $\frac{1}{2}f \cdot 1$, $\frac{1}{2}f \cdot 4$, $\frac{1}{2}f \cdot 9$, $\frac{1}{2}f \cdot 16$, &c.; therefore the spaces for the successive portions of time given by subtracting these spaces from the beginning, each from the following, are $\frac{1}{2}f \cdot 1$, $\frac{1}{2}f \cdot 3$, $\frac{1}{2}f \cdot 5$, $\frac{1}{2}f \cdot 7$, &c.; so that the spaces described from the beginning are as the squares of the times; and the spaces described in the successive equal portions of time are as the numbers 1, 3, 5, 7, &c.

3. By means of the two equations (1), any one of the four quantities f, t, v, s, may be eliminated, and an equation deduced involving only the other three; so that any one of the four things—force, time, velocity, and space—may be expressed in terms of any two of the others: thus, as the simplest algebraic substitutions show, we have the following equivalent expressions:

Expressions for v, $\quad ft, \ \sqrt{2fs}, \ \frac{2s}{t}$

" " s, $\quad \frac{1}{2}ft^2, \ \frac{1}{2}vt, \ \frac{1}{2}\frac{v^2}{f}$

" " t, $\quad \frac{v}{f}, \ \sqrt{\frac{2s}{f}}, \ \frac{2s}{v}$

" " f, $\quad \frac{v}{t}, \ \frac{2s}{t^2}, \ \frac{1}{2}\frac{v^2}{s}$

4. A uniform accelerating force is measured by twice the space described from rest in one second.

For putting $t = 1$, we have $f = \frac{2s}{t^2} = 2s$; so that if we can only measure the space through which a body, acted upon by a uniform accelerating force, moves from a state of rest, in one second, we can correctly determine the *uniform effect* of that force on the body: it will be double the space thus passed through. In other words, this double space will be the constant increment of the velocity at each succeeding second; that is, the constant *acceleration* of the velocity.

It is found by experiment that the attraction of the earth upon bodies near its surface—that is, the force of gravity—causes a body to fall from rest a distance of 16·1 feet in the first second of time. Consequently, the force of terrestrial gravity—which force is usually represented by g—is $g = 32·2$ feet: that is, this force, continuously soliciting a falling body, will accelerate its velocity 32·2 feet every second.

It may not be amiss to repeat here, before proceeding to practical illustrations, that where there is uniform velocity, in a straight line, there is no force. We should be compelled to admit this, as a necessary consequence of our evaluation of force in dynamics:—where there is no acceleration of velocity, there can be no force urging the body onwards in its path.

The student must all along remember that when we speak of a force acting on a body, we always refer to the mechanical condition of the body at the particular instant that has brought it to where we find it. The impulse that, acting on a body at rest, puts it in a state of uniform motion, acts only during the instant of passing from rest to rectilinear motion: it expires, as it were, in the act; so that, at whatever point in the path described, the moving body comes under our examination, we are compelled to say that no force—not even the force of impulsion—is acting upon it *then*.

We shall now give a few examples connected with accelerating force.

Examples of Accelerating Force.—1. A body moves from rest with a uniformly accelerated velocity, and after the lapse of 3 minutes 5 seconds is found to have passed over 400 feet: required the accelerating force.

Here the time, namely, t secs. = 3 mins. 5 secs. = 185 secs., and the space described, namely, $s = 400$ feet, are given to determine f. From the formula $s = \frac{1}{2}ft^2$ we have

$$f = \frac{2s}{t^2} = \frac{800}{185^2} = \frac{32}{37^2} = ·0234 \text{ feet.}$$

Consequently the accelerating force is such as to increase the velocity ·0234 feet every second.

2. How far will a heavy body fall in four seconds?

Using g for f, we have $s = \frac{1}{2}gt^2$, where $g = 32·2$, and $t = 4$, $\therefore s = 16·1 \times 4^2 = 257·6$. Hence the distance is 257·6 feet.

3. In what time will a heavy body descend 400 feet?

$$s = \tfrac{1}{2}gt^2 \therefore 400 = 16·1t^2 \therefore t = \sqrt{\frac{400}{16·1}} = 5 \text{ seconds.}$$

4. If a body be projected vertically upwards with a velocity of 400 feet per second, how high will it ascend?

It will ascend to that height from which if it were let fall it would acquire a velocity of 400 feet upon reaching the earth; therefore since (page 134).

$$s = \tfrac{1}{2}\frac{v^2}{g} \therefore s = \tfrac{1}{2}\frac{400^2}{32·2} = 248 \text{ feet.}$$

5. Through what height must a body fall so that the velocity acquired may be equal to that height?

$$s = \frac{1}{2}\frac{v^2}{g}, \text{ but } s = v \therefore 1 = \frac{1}{2}\frac{v}{g} \quad \therefore v = s = 2g = 64{\cdot}4 \text{ feet.}$$

6. Through what height must a body fall to acquire a velocity of 100 yards a second?—Ans. 139·75 feet.

7. A body is projected upwards with a velocity of 1500 feet per second: how high will it ascend?—Ans. 34875 feet.

8. What time will elapse, after projecting the body in the last example, before it again reaches the earth?—Ans. 93 seconds.

In the foregoing inquiries respecting the rectilinear motion of bodies under the influence of an accelerating force, we have supposed the force to move the body from *rest*. We are now to consider the case in which the force begins to act upon a body already moving in the direction of that force, or receiving an impulse in that direction. Suppose a body to begin its motion with a velocity v', and from the commencement to be acted upon by a uniform accelerating force f; then, as before, v being the velocity acquired in t seconds, and s the space passed through in that time, the acquired velocity will evidently be made up of the original velocity v' and the velocity ft communicated by the force f; for this force adds to the velocity f feet every second, or ft feet in t seconds, so that in t seconds the acquired velocity is $v = v' + ft$.

Again, in virtue of the initial velocity v' alone, the body would pass over the space $v't$ in t seconds; and in the same time f alone would cause it to pass over the space $\frac{1}{2}ft^2$: hence the combination of these is the whole space passed over; that is, it is $s = v't + \frac{1}{2}ft^2$; so that the formulæ for final the velocity and the space described are

$$v = v' + ft, \; s = v't + \tfrac{1}{2}ft^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

If the initial velocity v' opposes that communicated by f, then v' is to be taken with a sign opposite to that of f.

From these two equations t may be easily eliminated, and a third equation obtained involving only the remaining quantities; thus, squaring the first, we have

$$v^2 = v'^2 + 2\,v'ft + f^2\,t^2 = v'^2 + 2f\,(v't + \tfrac{1}{2}ft^2).$$

$$\text{Hence, by the second equation, } v^2 = v'^2 + 2fs \; . \; . \; . \quad (3)$$

As before, when gravity is the force, f is replaced by g.

Ex. 1. A body is projected vertically upwards, with a velocity of 480 feet per second; at what height will it be at the end of three seconds?

Here v' the velocity of projection being 480 feet, this velocity continued uniform for 3 seconds would carry it to the height of 1440 feet; but gravity so counteracts its upward motion as to draw it back through a space of $\frac{1}{2}gt^2 = \frac{1}{2} \times 32{\cdot}2 \times 3^2 = 144{\cdot}9$ feet: hence the height to which the body is suffered to ascend is only $1440 - 144{\cdot}9 = 1295{\cdot}1$ feet; that is, $s = v't - \frac{1}{2}gt^2 = 1295{\cdot}1$ feet.

2. From an elevated position a body is projected vertically upwards with a velocity of 80 feet: required its place at the end of 6 seconds.

$$s = v't - \tfrac{1}{2}gt^2 = 80 \times 6 - \tfrac{1}{2}\,32{\cdot}2 \times 6^2 = 480 - 579{\cdot}6 = -\,99{\cdot}6.$$

Hence, the place of the body is 99·6 feet *below* the place of projection.

3. With what velocity must a body be projected downwards from the top of a tower 150 feet high to arrive at the bottom in two seconds?

$$s = v't + \tfrac{1}{2}gt^2 \; \therefore v' = \frac{s}{t} - \tfrac{1}{2}gt$$

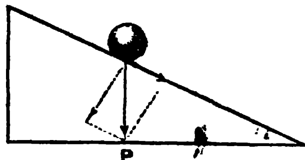
$$= \frac{150}{2} - 16{\cdot}1 \times 2 = 42{\cdot}8 \text{ feet.}$$

4. If a body be projected vertically downwards with a velocity of 20 feet, how far will it descend in 5 seconds?—Ans. 502·5 feet.

5. A body is projected upwards with a velocity of 100 feet: what will be its velocity when it has ascended 100 feet? (See form. 3.)—Ans. 51·6 feet.

6. In the preceding example find at what time the body is 100 feet from the earth, as well in its descent as in its ascent.—Ans. 1·2 sec. and 4·7 secs. after projection.

Motions of Bodies down Inclined Planes.—The fall of a body down an inclined plane is another instance of rectilinear motion under the influence of gravity. When a material substance is placed on an inclined plane, the force of gravity produces a certain vertical pressure P; if we resolve this pressure in two directions, the one along the plane and the other perpendicular to it, the former component will be P sin i, taking i for the inclination of the plane to the horizon; and to prevent the body from moving down, this is the force or pressure that must be counterbalanced.



As, therefore, P represents the pressure-force of gravity on the body in a vertical direction, and P sin i the pressure in the direction of the plane, and as, when motion takes the place of these statical pressures, the accelerating force of gravity is proportional to them, we shall have for the acceleration down the plane,

$$P : P \sin i :: g : g \sin i.$$

Hence the body is urged down the inclined plane by the constant force,

$$g' = g \sin i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

And, therefore, substituting this value of g' for f in the formulæ at page 134, those formulæ will then comprise the entire theory of motion down an inclined plane.

If l represent the length of the plane, and h its height, then $\sin i = \frac{h}{l}$: consequently the accelerating force down the plane is

$$g' = g\frac{h}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

And, therefore, the velocity acquired in descending down the whole length l, that is in descending through the space $s = l$, by the action of this force, must be (page 134)

$$v = \sqrt{2g'l} = \sqrt{2gh} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This expression we see involves only the height h of the plane, and is independent of its length l: hence the velocity acquired in descending down all planes of the same height is the same, and equal to the velocity acquired by falling vertically through that height.

But the velocities of two bodies, one falling through the perpendicular height, and one falling down the length of the plane, are respectively

$$v = gt \text{ and } v' = gt' \sin i \quad . \quad . \quad . \quad . \quad (4)$$

where t and t' are the respective times occupied in falling: these expressions are therefore equal, that is,

$$gt = gt' \sin i \therefore \frac{t}{t'} = \frac{\sin i}{1}.$$

So that the time of falling through the height is to the time of falling down the plane, as the sine of the plane's inclination to unity.

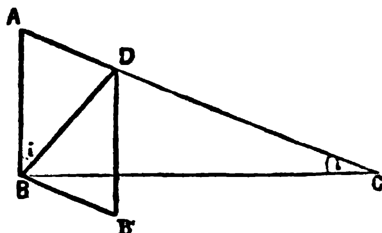
If we wish to know what extent of length down the plane a body will pass through,

while another falls through the whole height, then referring to the expression for the space (page 134), we have

$$\text{Vertical fall, } h = \tfrac{1}{2} g t^2 \text{; inclined fall in time } t,\ s = \tfrac{1}{2} g \sin i \,.\, t^2;$$

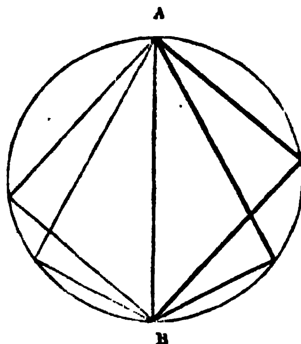
$$\therefore h : s :: 1 : \sin i, \text{ or } s = h \sin i.$$

If, therefore, from B we draw the perpendicular B D, the length A D will be that fallen through by one body moving down the plane A C, while another body falls through the height A B, because



$$A D = A B \cos A = A B \sin C.$$

If we draw the vertical D B', and B B' perpendicular to D B, then, by this theorem, the time of falling down the oblique line D B would equal the time of falling down the vertical D B', for D B = D B' cos D; but D B' = A B, since A B is a parallelogram, therefore the time of falling down the vertical A B is the same as the time of falling down either of the oblique lines A D, D B at right angles to one another: hence this remarkable property of the circle, namely:—If from the extremities A, B, of the vertical diameter A B, chords be drawn, as in the annexed diagram, a body would fall down either of them in the same time that it would fall through the vertical diameter A B.



The following are a few examples on the motion of bodies down inclined planes:—

1. If the length of an inclined plane be 60 feet, and its inclination to the horizon 30°, what velocity would a body acquire in falling down it for two seconds?

By the formula (4), since $\sin 30° = \tfrac{1}{2}$, we have $v = gt \sin i = 32{\cdot}2 \times 2 \times \tfrac{1}{2} = 32{\cdot}2$ feet per sec. Hence the velocity is the same as would be acquired by a vertical fall in one second.

2. How long would a body be in falling down an inclined plane whose length is 100 feet, and inclination 60°?

Substituting the acceleration $g \sin i$ for f, and l for s, in the expression for t, at page 134, and remembering that $\sin 60° = \tfrac{1}{2}\sqrt{3}$, we have

$$t = \sqrt{\frac{2\,l}{g \sin i}} = \sqrt{\frac{200}{32{\cdot}2 \times \frac{1}{2}\sqrt{3}}} = \sqrt{\frac{400}{32{\cdot}2 \times \sqrt{3}}} = \frac{20}{\sqrt{(32{\cdot}2 \times \sqrt{3})}} = 2\tfrac{1}{2} \text{ seconds.}$$

3. A body is projected up an inclined plane whose height is $\tfrac{1}{5}$th of its length, with a velocity of 50 feet. Find its place and velocity after six seconds have elapsed.

In this case the force $g\frac{h}{l}$ (equation 2) *retards* the motion of the body, and must therefore be considered as negative: hence, from equation (2) page 136, we have

$$s = v't - \tfrac{1}{2} g \frac{h}{l} t^2$$

$$= 50 \times 6 - 16{\cdot}1 \times \tfrac{1}{5} \times 36 = 6\,(50 - 16{\cdot}1) = 203{\cdot}4 \text{ feet.}$$

Also $v = v' - g \cdot \frac{h}{l} t$

$= 50 - 32 \cdot 2 = 17 \cdot 8$ feet, the velocity required.

4. If a body be projected up an inclined plane whose length is ten times its height, with a velocity of 30 feet, in what time will the velocity be destroyed, and the body begin to roll down?

The time is obviously the same as would be required to produce a velocity of 30 feet in a body falling from rest down the same plane: hence, substituting the acceleration $g \frac{h}{l}$ for f in the expression for t (page 134), we have

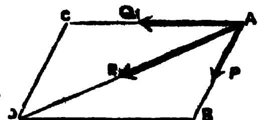
$$t = \frac{vl}{gh} = \frac{30 \times 10}{32 \cdot 2 \times 1} = \frac{300}{32 \cdot 2} = 9 \cdot 3 \text{ seconds.}$$

In the foregoing investigations, the student will observe that no allowance is made for the friction of bodies rolling down inclined planes; nor, whether bodies fall vertically, or obliquely as in this article, is the resistance of the air taken into account. These hindrances to free motion can be estimated only by practical and experimental researches, under various circumstances and conditions. The resistance of the air cannot be provided against, as is obvious; but in pieces of delicate machinery, many ingenious contrivances are resorted to, to diminish friction, and to render the departure from rigid mathematical theory as trifling as possible. The subject of friction will come under consideration in the treatise on PRACTICAL MECHANICS.

The Parallelogram of Velocities.—The forces considered in Dynamics are influences of the same kind as those considered in Statics; they merely exhibit their effects in a different manner. What in Statics produces pressure, would in Dynamics—that is, if the thing pressed were removed—produce motion: the effects in both cases being proportional to the causes:—a double pressure from the same body, like a double acceleration of velocity, implies a double force.

It may, however, be proper here to anticipate a difficulty which the student may possibly feel: he may perhaps reason thus:—"A weight of ten pounds produces a pressure ten times that of one pound, yet the one pound weight, if let fall, is accelerated just as much as the ten pound weight; whereas from what is here said, it would seem that the heavier weight ought to be accelerated ten times as much as the lighter, seeing that its pressure is ten times as great."

But this apparent difficulty can arise only from a wrong conception of what is stated above: we are not comparing the pressure of *different* bodies under the same force acting upon all their particles, but the pressure of the *same* body under different acting forces: for example, if the same body were acted upon, now by the force of gravity, and hereafter if the same body, or an equal body, were acted upon by a force only half that of gravity, then, as stated above, the effect in the former case, whether that effect be pressure or acceleration, would be double the like effect in the latter case. If it were otherwise, then the statical measure of force (pressure) and the dynamical measure (acceleration of velocity) could not each of them be a correct measure or expression of the intensity of what we call force.



It has been fully proved in STATICS (page 44), that if two forces P and Q act at A, and A B, A C represent their directions and intensities respectively, then will A D represent in direction and intensity their united effect.

The statical effect remains the same however long the forces or pressures act, for, *time* is no element of consideration in Statics. But the dynamical effect is that at the instant the body acted on by the forces is at A, the continuous action of the forces *afterwards* not entering into consideration.

The parallelogram of *velocities* is independent of the parallelogram of forces. It will be remembered that the velocity of a body at any point of its path is the space it *would* pass over in a unit of time, provided its rate or speed at that point were uniformly continued during the unit of time.

Suppose the body when at A were animated with a velocity that would alone carry it uniformly along A B to B, and with another velocity that would alone carry it uniformly along A C to C in the same time: these velocities may be considered as communicated by two simultaneous impulses in the directions A B, A C. the thing to be shown is, that the body would be carried uniformly along A D, and would arrive at D in the time spoken of.

For while the body is moving uniformly from A to B, conceive that the line A B, with the moving body on it, is carried parallel to itself, and with the second uniform velocity, up to C D: by the hypothesis, it will have arrived at C D in the same time that the body will have arrived at the extremity of the moving line: consequently, at the end of that time the body will be found at D. And that it will have arrived there with a uniform motion along the diagonal A D, will appear from considering that if any point of its path were out of that diagonal, the uniform relation of the component velocities, namely, A B : A C, would there be destroyed: that the diagonal is described with a uniform velocity,—or that equal portions of it are passed over in equal portions of the time, is plain, because the uniformly moving line A B passes over equal portions of the diagonal in equal portions of the time.

Since in the preceding figure, trigonometry gives for A D the expression

$$AD^2 = AB^2 + BD^2 - 2\,AB \cdot BD \cos ABD,$$

and since $-\cos ABD = +\cos BAC$, if the directions of the component velocities v, v' make an angle a with each other, then the resultant velocity V will be

$$V^2 = v^2 + v'^2 + 2vv' \cos a$$

From attending to the former part of the preceding examination, it will be seen that, however irregular the motion that would carry A to B, and however irregular the motion that would carry it to C, in the same time, the resultant of the two motions would necessarily carry it to D in that time, although not by the path A D, except the two component motions, during simultaneous portions of the time, are always as A B to A C. This truth will be of considerable importance in the next article on projectiles.

On the Motion of Projectiles.—If a body be projected obliquely upward or downward, the attractive force of gravity will cause it to take a curvilinear path. If the velocity of projection be considerable, and the body projected present much surface to the atmosphere, the resistance of the air will greatly modify the form of the curved path, and the range of the projectile; and it is no easy matter to determine one or the other. But if the body be supposed to move free from such obstruction (as in a vacuum), all the circumstances connected with the flight of a projectile can be readily ascertained, as also, with a close degree of approximation, even in actual practice, if the velocity of projection be but small.

Supposing atmospheric resistance to be removed, the path of the projectile will always be the conic section called a *parabola* (see PRACTICAL GEOMETRY, page 444), as we now proceed to show.

Path of Projectile a Parabola.—Let A B be the direction in which a body is projected from the point A, with a velocity v. If no other motion were impressed upon the body, it would move uniformly along A B with the original velocity v, and in t seconds would arrive at B, supposing $AB = vt$. But as gravity acts on the body from the commencement, this force alone, in t seconds, would draw the body down, along A C, to C, supposing $AC = \frac{1}{2} gt^2$ (page 134).

Consequently, completing the parallelogram B C, the body in t seconds is found at P, as proved at the close of last article. Now

$$CP = AB = vt, \text{ and } AC = \frac{1}{2} gt^2$$

$$\therefore \frac{CP^2}{AC} = \frac{2v^2}{g} \therefore CP^2 = \frac{2v^2}{g} . AC \quad . \quad . \quad . \quad (1)$$

This is what is called the *equation to a parabola*, A B parallel to C P being a tangent to the curve at the point A, and A C being parallel to the axis. The equation is usually written thus: $y^2 = ax$, where y is the ordinate C P of any point P, and x the abscissa A C, of that point, while a is any constant multiplier of the abscissa. The equation when A is at the *vertex* of the curve, is investigated at page 444 of the PRACTICAL GEOMETRY. It follows, therefore, that the projectile in its flight always traces out a parabola: the constant multiplier in its equation is $\frac{2v^2}{g}$. It is known from the theory of the parabola, that this constant multiplier is also 4 S A, S being the focus of the curve: it is further known, and is proved at the page just referred to, that the distance A S of any point A in the curve from the focus, is always equal to the distance of the same point from the *directrix*. Let this distance be called h, then

$$4 SA = \frac{2v^2}{g} = 4h \therefore v^2 = 2gh \quad . \quad . \quad . \quad . \quad (2)$$

But this value expresses the square of the velocity which a body would acquire, from the force of gravity g, by falling from rest, from the height h; consequently, the velocity of projection is equal to that which the body would acquire by falling from the directrix of the parabola, which it traces, down to the point of projection.

Velocity at any point of the Path.—It has just been shown that A being the point of projection, and v the velocity of projection, that velocity will be $v = \sqrt{(2g . SA)}$. But any point P of the path may be regarded as the point of projection, and the corresponding velocity as the velocity of projection, so that the expression (2) applies equally to any point in the curve traced, h being the distance of that point below the directrix, and v the velocity at the same point. Hence, the velocity of the projectile at any point of its course is the same as the velocity it would acquire by

falling vertically from the directrix down to that point. And it further follows, that at equal heights, in ascending and descending, the projectile will have equal velocities.

Greatest Height.—In order to ascertain the greatest height to which the projectile will ascend, we must of course know the velocity of projection, and the direction of that velocity. It will also be convenient to replace this velocity by two component velocities equivalent to it in effect:—the one a horizontal velocity, and the other a vertical velocity (see page 140). If a be the angle of projection, that is, the angle of elevation of the initial direction above the horizonal line, then the velocity v of projection will be equivalent to the horizontal velocity $v \cos a$, combined with the vertical velocity $v \sin a$.

Now, it is plain that gravity, which acts only in a *vertical* direction, cannot in any way disturb the *horizontal* velocity $v \cos a$, so that this velocity is the same at every point of the path. But the vertical velocity $v \sin a$, having the whole influence of gravity to check and oppose it, will be utterly destroyed and reduced to 0, when gravity has acted sufficiently long to impress on the body (supposing it left free to obey its solicitations) a downward velocity equal to the upward velocity $v \sin a$. The spaces through which a body must fall to acquire this velocity, or the height to which it must ascend to lose this velocity, is

$$s = \frac{1}{2}\,\frac{(\text{velocity})^2}{g} = \frac{1}{2}\,\frac{v^2 \sin^2 a}{g} = h \sin^2 a \text{ (by equa. 2)}$$

Hence, the greatest height to which the projectile discharged in a given direction can ascend, will be found by multiplying the height (h) through which a body must fall, to acquire the velocity (v) of projection, by the square of the sine of the angle (a) of elevation.

With the same projectile velocity v, the highest point to which the projectile can ascend under different elevations, will of course be that due to the elevation of 90°; and we see accordingly, that the multiplier $\sin^2 a$ is greater for this value of a, than for any other.

Time of Flight.—The time occupied in the flight of the projectile, that is, the time from discharging it till it falls to the horizontal plane passing through the point of departure, will of course be double the time in which the vertical velocity is destroyed, as the body must fall to the horizontal plane with the same vertical velocity with which it left it. The time in which, by gravity, the velocity $v \sin a$ would be generated, is

$$t = \frac{\text{velocity}}{g} = \frac{v \sin a}{g}$$

$$\therefore 2t = 2\,\frac{v \sin a}{g}, \text{ the time of flight.}$$

The time of flight, therefore, as might be expected, is the greater (with the same velocity of projection), the nearer the direction of projection approaches to the vertical; since $\sin a$ increases if a increases and does not exceed 90°.

Range of the Projectile.—The range is the distance on the horizontal plane, through the point of departure at which the projectile falls. It has just been seen that the time of flight is $2\,\frac{v \sin a}{g}$, and since the horizontal velocity $v \cos a$ is uniform all this time, we have only to find the space due to the velocity $v \cos a$ in the time $2\,\frac{v \sin a}{g}$, that is, we have

$$\text{Range} = v \cos a \times 2 \frac{v \sin a}{g} = \frac{v^2 \times 2 \sin a \cos a}{g} = \frac{v^2}{g} \sin 2a$$

Hence, with the same velocity v of projection, the range increases as sin $2a$ increases, and this it does as a increases from $a = 0$ up to $a = 45°$: therefore, with the same velocity of projection, the range at an elevation of 45° will be greater than at any other elevation whatever. And since the sine of an angle is the same as the sine of its supplement, that is, since sin $2a$ is the same as sin $(180° - 2a)$, it follows that the elevation $90° - a$ gives the same range as the elevation a: hence the ranges of projectiles, at any elevations above 45°, are the same as the ranges at elevations as much below 45°.

The length of the *maximum* range, that is, when $a = 45°$, is shown above to be $\frac{v^2}{g}$; v being the initial velocity, and this, as proved at page 141 (equation 2), is the same as $2h$: so that the length of the maximum range is equal to twice the height due to the velocity of projection.

If the range of a shot with a known elevation of the piece be ascertained, it will be easy to determine what the range would be with the same charge of powder at any other elevation. For calling the maximum range, or that due to the elevation $a = 45°$, R, and the range due to any other elevation (a') r', we have, from the result just deduced, $r = R \sin 2a'$; so that by help of the maximum range, any other range, with the same charge, may be easily found. Or if we know any range r, corresponding to the elevation a, then to determine the range r', corresponding to another elevation a', we have the two equations,

$$r' = R \sin 2a' \text{ and } r = R \sin 2a$$

$$\therefore \frac{r'}{r} = \frac{\sin 2a'}{\sin 2a} \quad \therefore r' = \frac{\sin 2a'}{\sin 2a} \cdot r.$$

The following are the principal results in the foregoing theory of projectiles in a non-resisting medium:—

$$\text{Time of flight, } t = 2\frac{v}{g} \sin a = 2 \sin a \sqrt{\frac{2h}{g}},$$

$$\text{Range, } r = \frac{v^2}{g} \sin 2a = 2h \sin 2a,$$

$$\text{Greatest range, } R = \frac{v^2}{g} = 2h,$$

$$\text{Greatest height, } H = h \sin^2 a.$$

Where g is 32·2 feet, a the angle of elevation, v the velocity of projection, and h the height from which a body must fall from rest to acquire that velocity.

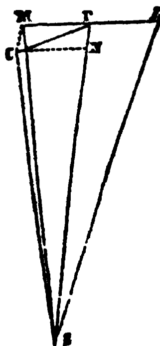
These results, however, ought to be regarded as but purely theoretical. In practice the resistance of the air is so great in high velocities, as to render them almost useless: the mathematical doctrine of projectiles, through a resisting atmosphere, is full of difficulties, and as the laws of resistance are as yet but insufficiently established, practical men must be guided chiefly by the results of actual experiment. The experiments in gunnery, by Dr. Hutton, as given in his "Mathematical Tracts," may be consulted with advantage.

Circular Motion—Centrifugal Force.—If a body move uniformly in the circumference of a circle, the force to which that motion is due must reside in the centre of that circle. This is only a particular case of the following more general proposition: namely, that if a body describe any curve, in virtue of a force continually diverting it from its wonted rectilinear path, that force must reside in a point such that, conceiving

a line to join that point with the moving body, this line, moving with the body, must sweep over equal sectional areas in equal times.

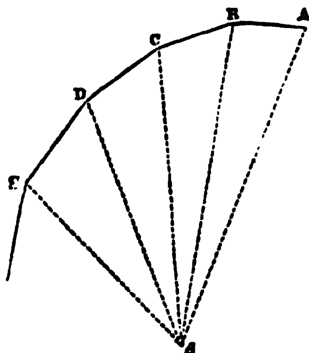
This may be readily proved by first establishing the direct proposition, namely—that if a body describe a curve about any centre of force, and a line be supposed to unite that centre with the body, equal areas will be described in equal times.

Let an attractive force at S act upon a body at the distance S B; and to simplify the inquiry, imagine at first that instead of acting continuously, the force exerts itself only at short regular intervals; that, in fact, it consists of only equal successive impulses.



Let A B be the rectilinear path described by the body during any one interval between two successive impulses. At B the impulse is repeated, and the course of the body is diverted into the new path B C, which is described during the next interval, at the end of which another impulse, in the direction C S, bends its path again, and so on. In the two intervals here considered, the triangular areas S A B, S B C, will have been described; and it will be easy to show that these areas, thus described in equal times, are equal. For if, when the body was at B, no fresh impulse had been given to it, it would have moved forward uniformly to M, describing B M, the continuation of A B, equal to A B; in which case the triangular areas S A B, S B M, would, of course, have been equal: it remains to show that the latter of these is also equal to the triangular area S B C.

The impulse from S, if it had acted alone on the body when at B, would have brought it to some point N in B S, in the prescribed interval of time; and its uniform motion along A B, if undisturbed, would in the same interval have brought it to M. In virtue of these two uniform motions, the one from B to N, and the other from B to M, in the same time, the actual motion of the body is along the diagonal B C of the parallelogram M N. Hence M C being parallel to B S, the two triangles B M S, B C S, upon the same base B S, are between the same parallels, and are therefore equal; therefore the triangle B C S is equal to the triangle A B S. It follows, therefore, that in the equal times, equal triangular spaces have been described.



In the same manner as it has now been shown, that the triangular area S A B, described in the first interval of time, is equal to that S B C described in the second equal interval, so may it be shown that the triangular area described in the third interval is equal to that described in the second, and so on. Hence these triangular areas, as figured in the margin, being all equal, it follows, uniting any number of them in one area, that equal areas are thus described in equal times.

As this is true, however small the individual intervals of time between the impulses may be, it is true when these intervals are insensibly small, or when the successive impulses follow in one continuous force, and the lines A B, B C, &c., unite in one continuous curve. Hence, when a body is constrained to move in a curve, by the continu-

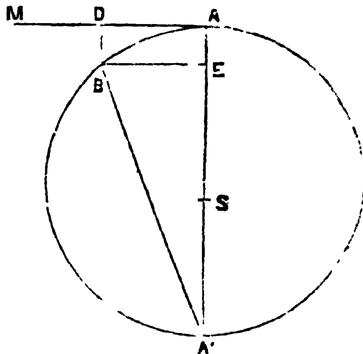
ous action of a central force, equal areas are described about the centre of force in equal times.

Conversely, if equal areas are described about a point in equal times, the centre of force governing the motion of the body must be at that point.

Suppose the two equal areas S A B, S B C, to be described about S in equal times (Fig. at page 144): then if the force acting on B were not in the direction B S, M C, parallel to the direction of the force, would not be parallel to B S; that is, two triangles S B C, S B M on the same base B S, though *not* between the same parallels, are equal, which is impossible (Euc. 40 of I.).

It follows from this that, if a body under the influence of a continuous force, move uniformly in the circumference of a circle, the force must be at the centre of that circle, because equal sectional areas are described in equal times, since equal arcs are.

The force by which a body moving in a curve is drawn at any instant towards the centre of attraction S, is called the *centripetal* force at the distance the body is at that instant: the opposing force with which the body tends to fly off from the centre, and proceed in a rectilinear path, is called the *centrifugal* force. In circular motion these forces are everywhere exactly equal; for neither, at any point of the circular path, prevails over the other, the distance from the centre of force S being everywhere the same. Let the arc A B be described in one second: draw B E perpendicular to A S; then in one second the body, originally at A, will have fallen from its wonted straight path A M, a distance = A E towards the attractive force at S: hence, 2 A E expresses the intensity of that force (page 135) acting on A. Join B A′; then since the arc A B differs insensibly from its chord (for the time of describing it may be regarded as minute as we please), we may regard A B A′ as a right-angled plane triangle, since the angle B is in a semicircle; therefore (Euc. 8. VI.)



$$AE : AB :: AB : AA'$$

$$\therefore AE = \frac{AB^2}{AA'} = \frac{AB^2}{2AS} \therefore 2AE = \frac{AB^2}{AS}.$$

Now 2 A E represents the accelerating force at S, or, taken in an opposite direction, it represents the centrifugal force f, and A B represents the velocity v in the curve; consequently the centrifugal force $f = \frac{v^2}{r}$, where r = radius.

If as usual π be made to stand for the number 3·14159, &c., the whole circumference of the circle will be $2\pi r$; therefore, calling the whole time of describing the circumference—that is, the periodic time, t—and remembering that the uniform velocity v is equal to the whole space divided by the whole time, we have

$$v = \frac{2\pi r}{t}, \quad f = \frac{4\pi^2 r}{t^2}.$$

In the foregoing reasoning we are required to take t so many *seconds;* but the unit of time may be regarded as much smaller than this as we please; and thus the error of

confounding the arc A B with its chord may be entirely removed, so that we may conclude rigorously that—

The centrifugal force in a circle varies as the radius divided by the square of the time of describing it: thus, for another circle we should have $F = \frac{4\pi^2 R}{T^2}$;

$$\therefore f : F :: \frac{r}{t^2} : \frac{R}{T^2};$$

where f, F stand for the centrifugal or centripetal forces at the respective centres of circles of radii r, R, and in which the periodic times are t, T. If $R = r$, that is, if the circles are equal, the centrifugal forces F, f are inversely as the squares of the times T, t.

An interesting application of these results is to the determination of the centrifugal force at different places on the surface of the earth, from knowing the time of one rotation on its axis. But in studying this application, the student will not fail to observe that the circular path of a body, on the surface of the rotating earth, is not described under the same circumstances as the circular path is considered to be described in the present article. *Here* there is no solid matter interposed between the circulating body and the central force; there is nothing to prevent its falling to the centre at any point of its orbit; it is kept always at the same distance from the centre, simply because the force pulling it towards that centre is exactly balanced by that driving it from it; the centrifugal force is just sufficient to deprive the body of all weight or pressure towards the centre. With the earth it is different: if a perforation be bored in the earth, beneath a body on its surface, the body will fall down it; because the centrifugal force, driving it from the centre, is less than the attracting, or centripetal, force pulling it the other way.

Centrifugal Force at the Earth's Surface.—The earth, by its diurnal rotation, carries round, with a uniform velocity, every point on its surface in 86164 seconds. At the equator the radius R is about 20922000 feet; therefore the centrifugal force F at the equator is

$$F = \frac{4\pi^2 R}{T^2} \times \frac{4\pi^2 \times 20922000}{86164^2} = \cdot 111245 \text{ feet}.$$

As this force opposes the force of gravity, it follows that if the earth had no rotation on its axis—that is, if no centrifugal force existed—gravity at the equator, instead of being what it really is, namely, $g = 32{\cdot}088$ feet, would be $G = g + {\cdot}111245$, and thus the weight of a body there would be a $\frac{\cdot 111245}{32 \cdot 088}$ part more than it actually is.

The ratio of G to F being

$$32{\cdot}199 : {\cdot}1112 \text{ or } 289 : 1 \text{ nearly};$$

$$\therefore F = \frac{G}{289} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

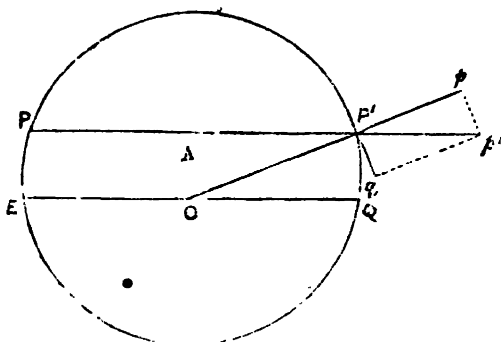
Now every parallel to the equator being carried round in the same time T, as the equator itself, by representing the centrifugal force in the parallel whose latitude is l, and radius r, by f, we shall have

$$\frac{f}{F} = \frac{r}{R} \therefore f = F \frac{r}{R} = \frac{G}{289} \cos l \quad . \quad . \quad . \quad (2)$$

Since, as is evident, $r = R \cos l$.

The force of gravity is not diminished by the whole of the centrifugal force, except at the equator; because in any parallel P A P′ this force acts, not in the direction P′ p

wholly opposed to the force of gravity, but in the direction $P'p'$. If, therefore, we decompose the force $P'p'$ in the perpendicular directions $P'p$, $P'q$, the former component, being directly opposed to the force of gravity, and the latter component being a force acting tangentially, and therefore urging the particle P towards the equator, we shall have



Force opposing gravity,
$P'p = P'p' \cos pP'p' = f \cos l.$

Hence (2) the expression for the diminution of gravity, in consequence of centrifugal force, at latitude l, is

$$\text{Diminution of equatorial gravity G in lat } l = \frac{G}{289} \cos^2 l.$$

Consequently the amount of diminution varies as the square of the cosine of the latitude.

The other component $P'q$ of the centrifugal force at P, being tangential, tends to drive the particles of the revolving body from the region of the poles to that of the equator, and to cause the body to assume the figure of an oblate spheroid, which is the figure that the earth has assumed: the expression for the tangential force is

$$\text{Tangential force, } P'q = f \sin l = \frac{G}{289} \sin l \cos l = \frac{G}{578} \sin 2l,$$

which therefore varies as the sine of twice the latitude. The force thus tending to accumulate matter about the equatorial regions is obviously greatest at lat 45°, for there $2l = 1$.

From the foregoing principles we may readily determine the time in which the earth must perform its diurnal rotation, in order that the centrifugal force at the equator may be exactly equal to the force of gravity there; that is, in order that a body at the equator may have no weight.

Let T' represent the time of rotation corresponding to which the centrifugal force would be equal to that of gravity G; then the time at present being T, since the centrifugal forces in the same circle are inversely as the squares of the times of describing that circle (page 146), we have equation (1)

$$G : \frac{G}{289} :: T^2 : T'^2;$$

$$\therefore T' = \frac{T}{\sqrt{289}} = \frac{T}{17}.$$

Hence, if the diurnal rotation of the earth were performed in the 17th part of the time really occupied—that is, if it were to turn round seventeen times as rapidly—bodies at the equator would lose all their weight; and would, therefore, if placed at a small distance above the surface, remain suspended without any support. If the rotation were more rapid than this, no body could remain on the surface: everything would be repelled from it by the centrifugal force.

It has been demonstrated, however, that a spheroid, if fluid, and of the same density as the earth, could not remain in equilibrium if it were to rotate in a shorter time than 2h. 25m. 36s. (See "Airy's Tracts," second edition, p. 150.)

From what has now been shown, it will be observed that the superior accumulation of matter about the equatorial regions of the earth arises from two circumstances, or the rather it is a twofold effect of the same cause. The centrifugal force generated by the diurnal rotation, acting in planes perpendicular to the axis, and on all the particles in each plane, and on the remoter particles of the plane more intensely than on the nearer, necessarily occasions the surface of the earth about the equator to recede more from the axis of rotation than the surface nearer to the poles. If the earth were not a rotating body, and an external influence opposed to that of gravitation were to diffuse itself like a belt round the equatorial regions, the matter under this belt would yield to the influence, and bulge out; and supposing the entire surface to be fluid, the waters of the polar regions would become proportionately depressed. But in the actual case the bulging out occasioned by the direct centrifugal force at the equator, exceeding the direct centrifugal force at every other parallel of latitude, is further increased by all these parallels contributing indirectly to the enlargement of the equator, as well as directly to the enlargement of itself; for, as shown above, the equator is the only one of the rotating circles which expends none of the force upon it in a tangential direction. Every other circle—every parallel of latitude on either side, contributes tangentially its own supply of matter towards the equatorial regions, this supply, however, becoming less and less as the parallels recede from that of 45°.

On account of the solid materials of which a portion of the earth is composed, and the particles of which resist separation by their cohesion, the effects of centrifugal force are in some degree counteracted; but it is the general opinion of philosophers, that the present solid parts of the earth were once in a fluid or semi-fluid state—in fact, in a state of fusion; and that the outer crust has become solidified by cooling.

But those portions of the earth's surface which still remain fluid, observably yield to the tangential influence of the centrifugal force. There is a tendency, as observation shows, in the waters north and south of the equator to flow towards that circle; and as they flow from parts where the velocity of the diurnal rotation is less than the velocity at the equator, the streams from high latitudes are gradually left more and more behind in their lateral approach to the equator, where the diurnal velocity is greatest. As this velocity is towards the east, we may therefore expect to meet with great westerly currents in the open seas north and south of the equator, which is conformable to experience.

SECTION II.—MOVING FORCES.

In the foregoing section of the present elementary treatise we have confined our attention exclusively to motion, unconnected with any considerations in reference to the quantity of matter moved. And although we have repeatedly used the term *body*, yet we have paid no regard to bulk or weight. The forces with which we have been dealing have, indeed, throughout been supposed to be analogous to the force of gravity, which gives the same velocity, in the same circumstances, to a single particle of matter as to the largest volume. We propose now to enter upon a few inquiries in which it will be necessary to take into account quantity of matter; and as preliminary to these,

some explanation must be given of the sense in which certain terms, hereafter to be employed, are to be understood.

Every person has a tolerably clear notion of *weight*: if two substances, however differing in bulk or volume, have the same weight at the same place, we infer that they contain the same quantities of matter; this is briefly expressed by saying that they are equal in *mass*. Yet mass and weight are not to be regarded as expressing the same thing; if any mass, or quantity of matter, were weighed at the pole, and then the same mass weighed at the equator, these weights would be different; though of course the transportation of the matter, from the pole to the equator, could not produce any change in its constitution. (See *Properties of Matter*, p. 12.)

A solid body may be considered as made up of particles or atoms, the aggregate of which constitute its volume; these particles may be packed (so to say) more closely in one body than in another of the same volume; so that the former may contain a greater quantity of matter than the latter; that is, a greater number of particles. Referring to this more compact constitution of the body, we say that it is more *dense* than the other body. *Density*, therefore, has reference to the quantity of matter in a given bulk; and if of two bodies of equal bulk, one be found to weigh, at the same place, m times as much as the other, the former is said to have m times the density of the other; it contains m times the quantity of matter, and is therefore m times the mass of the other.

The proper way, therefore, to represent the mass of any volume V, with a view to a correct comparison with other volumes, is this: Mass $=$ Density $\times$ Volume; or, in symbols, $M = D V$. And it is to be observed that density is employed only comparatively, not absolutely; as when we say that lead has about seven times the density of water, we mean that about seven times as much matter is packed into a given volume of lead, as into an equal volume of water; so that the mass of a cubic inch of lead is seven times the mass of a cubic inch of water.

As already observed, the mass of a body is proportional to its weight at the same place: the dynamical effect of this weight (W), or rather its dynamical measure, is $M g$, where M is the mass and g the force of gravity acting on it. And, in like manner, if a force f analogous to, but different in intensity from gravity, were to act on the mass M, the dynamical measure of the effect would be $M f$, which is called the *moving force*; so that in the equation $F = M f$, M is the mass, or quantity of matter moved, f is the accelerating force acting on it, and F is the moving force, the dynamical measure of the effect. In statics this would be pressure or tension.

The product of the mass by the velocity with which it moves, is called the *momentum* of the body; as the velocity increases, therefore, so does the momentum. In uniform velocity, the momentum is constant: in uniformly accelerated velocity, additional momentum is generated every second; the increments of momentum being constant, like the increments of velocity. In the preceding expression for F, namely, $F = M f$, f is no other than the constant increment of velocity generated in one second; consequently the momentum generated in one second expresses, in fact, the moving force F; just as the velocity generated in one second represents the accelerating force f.

This latter force, as sufficiently seen in the preceding section, is all we have to attend to in investigating the motion of any single body subjected to its influence. As it acts upon all the particles of the body alike, the consideration of the number of these particles, or the mass of the body, would be superfluous. But when, instead of a single isolated body, we have to examine into the motions of a system of bodies, connected together by any ties, as cords, rods, &c., or mutually acting upon one another in

any way, then, as it is easy to perceive, the quantity of matter in each body becomes an important item of consideration.

From what has now been said, the student will be prepared to give his assent to the following expressions, where V represents volume, and D density :—

The mass $M = D V$;

The moving force $F = M f \therefore f = \frac{F}{M}$ the accelerating force,

The momentum $= M v$.

But in order that he may attach an intelligible meaning to these symbols and relations, and may not mistake their proper signification, we shall here exemplify their interpretation in a particular case.

Suppose the volume of a body, that is, its magnitude, to be 10 cubic feet, and that in each cubic foot there is three times the quantity of matter that there is in a cubic foot of some standard substance—say pure water. Then representing the density of water by unity or 1, the quantity of matter in the body before us, that is, the mass M, will be

$M = D V = 3 \times 10 =$ the mass of 30 cubic feet of pure water;

that is to say, there is as much matter in the proposed 10 cubic feet as there is in 30 cubic feet of pure water.

If this mass move with a uniform velocity of 12 feet per second, the momentum of it, or the force of the blow with which it would strike an obstacle, would be

$M v =$ the momentum of 30 cubic feet of pure water, or of 30 cubic feet of a substance of the same density as water, moving at the same rate,—*viz.*, 12 feet per second.

If the velocity of the moving mass, instead of being uniform, is constantly accelerated, the constant acceleration being $f = 6$ feet per second, then there is an acceleration or accumulation of momentum; and as, when uniform, the momentum is $M v$, so the constant accumulation of it, called the moving force, is

$M f =$ the moving force (or constant accumulation of momentum) of 30 cubic feet of a substance of the same density as water accelerated 6 feet per second.

When the velocity of the moving body is uniform, there is no moving force, that is, there is no accumulation of momentum, because there is no accumulation of velocity. If motion be not the result of the force f acting on the body, the effect must be pressure or weight.

The preceding explanation of the sense in which the symbols are to be understood, may be varied a little as follows :—Let the quantity of matter in a cubic foot of pure water be taken for the unit of mass, and the density of pure water for the unit of density.

Let the momentum of a cubic foot of pure water, or of a substance of equal density, moving with a uniform velocity of one foot per second, be taken for the unit of momentum. Let the accumulation of momentum, or the constant quantity of momentum generated in one second in a cubic foot of pure water, or in the same volume of a substance of equal density, moving with an accelerating velocity of one foot per second, be taken for the unit of moving force. Then V representing the *number* of cubic feet in the volume of any body B, M the *number* of units of mass, v the *number* of feet per second in the velocity, and f the *number* of feet per second of acceleration, we shall have

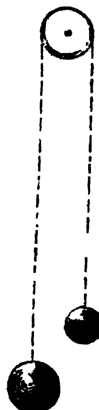
$M = D V$, the number of units of mass in the body B

$M v$, the number of units of momentum

$M f$, the number of units of moving force.

We shall now proceed to some applications of the theory of moving forces.

Applications of Moving Forces.—1. Two heavy bodies whose masses are M, M′, are connected together by a string which passes over a fixed pulley: required the circumstances of the motion.



Let M be the greater of the two masses; then M — M′ is the mass which, acted upon by gravity, causes M to descend and M′ to ascend. The moving force F, therefore, to which the motion is due, is $F = (M - M')\,g$; and therefore the accelerating force f, which, as above, is equal to F divided by the entire mass moved, is

$$f = \frac{M - M'}{M + M'}\,g.$$

And since the velocity generated in t seconds is ft, and the space passed over $\frac{1}{2}ft^2$ (page 134), we shall have, for the velocity of M downwards, or of M′ upwards in t seconds,

$$v = \frac{M - M'}{M + M'}\,gt, \text{ and for space, } s = \tfrac{1}{2}\,\frac{M - M'}{M + M'}\,gt^2.$$

These expressions determine the velocity with which M descends at the end of t seconds, and the space through which it will have fallen in that time.

Tension of the String.—If the string were fastened to the pulley, so as to prevent motion, M, acted upon by the accelerating force of gravity g, would exert upon it a pressure or tension Mg; but this is diminished in consequence of a part of the accelerating force, namely f, acting upon M, and wholly expended in producing motion: hence the pressure or tension suffered by the string is only the difference between the two pressures, namely:

$$T = M(g - f) = M\left(g - \frac{M - M'}{M + M'}\,g\right) = \frac{2\,M\,M'}{M + M'}\,g.$$

This, therefore, is the expression for the tension of the string. With regard to the other mass, M′, not only is the accelerating force of gravity g, acting downwards upon M′, wholly exerted in stretching the string, but the upward accelerating force f in addition: hence the pressure or tension due to M′ is

$$T = M'(g + f) = M'\left(g + \frac{M - M'}{M + M'}\,g\right) = \frac{2\,M\,M'}{M + M'}\,g,$$

the same as before; as of course it ought to be.

Pressure on the Axis of the Pulley.—What is tension as respects the string, becomes, of course, pressure when acting on the pulley. As the tension of the string is on each side the same, namely, T, as determined above, therefore the pressure on the pulley is twice this, namely:

$$\text{Pressure on axis of pulley} = \frac{4\,M\,M'}{M + M'}\,g.$$

It is scarcely necessary to remind the student that any mass multiplied by g, in other words, the moving force due to gravity, is mere weight when exerted statically.

Thus, calling the weights of the masses M and M′, W and W′ respectively, we shall find that the foregoing expression for T is the same as $T = \dfrac{2\,W\,W'}{W + W'}$.

For the expression at first given may evidently be written thus, namely:—

$$T = \frac{2\,Mg \cdot M'g}{Mg + M'g}.$$

Suppose, for instance, the weights hanging to the string were 7 lbs. and 5 lbs.

respectively; then the tension of the string would be $\frac{2 \times 7 \times 5}{7 + 5}$ lbs. $= 5\frac{5}{6}$ lbs.; that is, a weight of $5\frac{5}{6}$ lbs. would, if suspended to a fixed string, produce the same tension as is actually produced in the moving string. In like manner, the axis of the pulley sustains a pressure equal to $11\frac{2}{3}$ lbs. As the masses are proportional to the weights, we may, in a similar way, get a numerical expression for the accelerative force f—that is, by substituting the weights for the masses in the above value of f. Thus, taking the weights as here assumed, we have

$$\text{Acceleration, } f = \frac{7 - 5}{7 + 5} 32{\cdot}2 = 5{\cdot}36 \text{ feet per second,}$$

the accelerating force that gives motion to the system being $\frac{1}{6}$th of the accelerative force of gravity.

As already noticed, the moving force Mg, which combines quantity of matter with the force acting upon it, is *pressure* or *weight* when motion is not the result of the action of the force on the mass. The symbol g, therefore, which stands for acceleration in Dynamics, should stand for pressure or weight in Statics, as in the expression for T given above, namely,

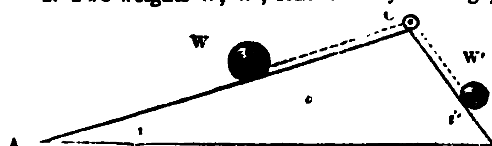
$$T = \frac{2\, Mg \,.\, M'g}{Mg + M'g} = \frac{2\, W W'}{W + W'}$$

The learner, therefore, will not fall into the mistake of regarding such an expression as $W = Mg$ equivalent to the statement, that "weight is equal to mass multiplied by 32·2 feet." This would, of course, be absurd. The symbol g here stands for "the weight impressed by gravity on the unit of mass," whatever that unit may be selected to be. The unit of mass being assumed or fixed upon by common consent, then, in the above expression, M is the *number* of these units in the body referred to, and g the weight of one of them.

The unit of mass is usually taken equivalent to a cubic foot of distilled water, at the temperature 60° of Fahrenheit's thermometer. The weight of this unit is 1000 ounces avoirdupois; and the weight W of any body whatever, containing in it M times the quantity of matter contained in one cubic foot of distilled water, will always be $W = M \times 1000$ ounces; and the same would obviously be the case if the unit of mass were anything else, g representing always the weight of that unit.

Whenever, in any mechanical inquiry, the investigation involves the mixed consideration of motion and statical pressure, although the motion and the pressure may be equally due to the same thing, and the same symbol (g for instance) be employed indifferently for the one or other of these different effects, yet, as the *effect* is all that we intend the symbol to represent, it is plain that its signification in the one case must be different from its signification in the other. And it is of importance that the student always bear this in remembrance: he will find some further reference to the matter in the treatise on HYDROSTATICS.

2. Two weights W, W', connected by a string passing over a small pulley C, are placed upon the two inclined planes C A, C B: it is required to determine the circumstances of their motion.



Let, as before, M, M' represent the two masses. The moving force of W, in the direction W A, is $Mg \sin i$; and the moving force of W', in

the direction W′ B, is $M'g \sin i'$: these oppose one another, so that the moving force F, to which the motion is due, is the difference of them, namely :—

$$F = (M \sin i \backsim M' \sin i')g.$$

And consequently the accelerating force f, which is equal to F divided by the sum of the masses moved, is

$$f = \frac{M \sin i \backsim M' \sin i'}{M + M'} g.$$

This therefore is the expression for the accelerating force which urges W down the plane C A, and which, consequently, draws W′ up the plane C B.

And since the velocity generated in t seconds is ft, and the space passed over $\frac{1}{2}ft^2$, the velocity acquired by W downwards, or by W′ upwards, in t seconds, is

$$v = \frac{M \sin i \backsim M' \sin i'}{M + M'} gt$$

and the space described is $$s = \frac{M \sin i \backsim M' \sin i'}{2(M + M')} gt^2.$$

If the two planes were vertical, the problem would become identical with the last; we should then have $\sin i$ and $\sin i'$ each equal to 1; and we see that the expressions for v, s would be those already deduced.

If only one of the planes were vertical, the problem would be converted into this, namely, to determine the circumstances of the motion when one body W, hanging vertically, draws another body W′ up an inclined plane. In this case, $\sin i = 1$; and we have only to put this in the preceding expressions, in order to obtain the necessary particulars of the motion.

Again, if one of the planes were vertical and the other horizontal, the problem would be to determine the motion when W, hanging vertically, draws W′ along a horizontal plane. In this case, $\sin i = 1$, and $\sin i' = 0$; so that the accelerating force would be

$$f = \frac{M}{M + M'} g.$$

Tension of the String.—For the tension of the string, under the original con ditions, we have, if W be the descending weight,

$$T = M\left(g \sin i - \frac{M \sin i - M' \sin i'}{M + M'} g\right)$$

$$= \frac{M\,M'}{M + M'} (\sin i + \sin i')\, g$$

which becomes the same as the expression for T in the last problem, when $\sin i$ and $\sin i'$ are each of them 1. By substituting W for Mg, and W′ for $M'g$, in each of the foregoing formulæ numerical expressions will be obtained, as in the former problem.

3. Two weights W, W′ are attached, the latter to a wheel, and the former to its axle: to determine their motions.

Let M be the mass of W acting at the axle, and M′ the mass of W′, acting at the rim of the wheel: let also R be the radius of the wheel, and r that of the axle.

Then the moving forces $M'g$, Mg acting at the distances R, r from the centre of motion, if free, would have the respective effects $M'g \cdot R$, and $Mg \cdot r$, which connected as they are, oppose each other. And if f', f be the actual accelerations of W′, W, the moving forces, at the distances R, r from the centre of motion, are actually $M'f' \cdot R$, and $Mf \cdot r$; so that

$$M'g \cdot R - Mg \cdot r = M'f' \cdot R + Mf \cdot r \quad . \quad . \quad . \quad . \quad (1)$$

The accelerations f', f must be to each other as the radii R, r, the velocities themselves being always in this constant ration, since the wheel and axle both turn in the same time; therefore $f' = \frac{R}{r} f$, therefore

$$(M' \cdot R - Mr)\, g = (M' \frac{R^2}{r} + Mr)\, f$$

$$\therefore f = \frac{M' Rr - Mr^2}{M' R^2 + Mr^2}\, g$$

the accelerating force of the ascending weight W.

And since $f' = \frac{R}{r} f$ we have

$$f' = \frac{M' R^2 - M\, Rr}{M' R^2 + Mr^2}\, g$$

for the accelerating force of the descending weight W'

The velocity, in t seconds, of the former, is

$$v = ft = \frac{M' Rr - Mr^2}{M' Rr + Mr^2}\, gt$$

and the space, $\qquad s = \frac{1}{g} ft^2 = \frac{1}{2} \frac{M' Rr - Mr^2}{M' R^2 + Mr^2}\, gt^2$

And for the latter we have

$$v' = \frac{M' R^2 - M\, Rr}{M' R + Mr^2}\, gt$$

$$s' = \frac{1}{2} \frac{M' R^2 - M\, Rr}{M' R + Mr^2}\, gt^2$$

The student will perceive that the first member of the equation (1), expressing the difference between the moving forces that *would* act if the weights were free, is the whole amount of moving force in actual operation; which whole amount is obviously expressed by the second member of the equation.

Impact or Impulse.—In this article we propose to consider the circumstances of the motions of bodies moving in certain straight lines from the effects of impulse, and impinging against one another.

Let any specified quantity of matter be considered as the unit of mass: if it be projected by any given impulse, it will move with a constant velocity proportional to the intensity of that impulse; this velocity may therefore be taken to represent the magnitude of the impulsion. If we take two such units of mass, and two such impulses act on them simultaneously, and in the same direction, the relative positions of the moving masses will be always preserved; so that if the two units be blended into one mass, and the two impulses be thus made to unite and form a double impulse, the same velocity will be impressed on the compounded mass. And it is plain that if M such units be blended into one mass, which receives an impulse M times the intensity we have supposed to be applied to the single unit, the same velocity would still be impressed.

Consequently, when any body whose mass is M moves from the effect of an impulse, the correct expression for the intensity of that impulse must be Mv; the mass multiplied by the velocity, and which, as before stated (p. 149), is the *momentum* of the mass: hence the momentum measures the intensity of the impulse.

Collision of Bodies: Direct Impact.—When two bodies moving by impulsion in the same straight line come into collision, the shock is called direct impact.

Suppose the bodies which thus impinge to be entirely inelastic, or of such materials that they are blended by the impact into one mass, our object will be, from knowing the intensity of each separate impulse—that is, the momentum of each body—to determine the momentum of the blended mass; and thence the circumstances of its motion. The following example will illustrate how the necessary particulars are to be ascertained :—

Example.—Two inelastic bodies, whose masses are M, M', move in the same straight line, with velocities v, v': required the velocity of their united masses after impact.

The momentum of M is Mv, and that of M', $M'v'$ the combination of these—namely, $Mv + M'v'$, is the momentum of the united mass $M + M'$.

But if v be the velocity of this mass, its momentum must be $(M + M')v_1$: hence, to determine the velocity v_1 after the collision, we have the equation

$$(M + M')v_1 = Mv + M'v'.$$

$$\therefore v_1 = \frac{Mv + M'v'}{M + M'} \quad . \quad . \quad . \quad . \quad (1),$$

the velocity required.

This result is on the supposition that the bodies are moving in the same direction, or that one of them, M', overtakes the other; but if they are moving in opposite directions so that the two meet, then the expression for v_1 will be

$$v_1 = \frac{Mv \sim M'v'}{M + M'} \quad . \quad . \quad . \quad . \quad (2).$$

If Mv exceed $M'v'$, then, after the collision, the mass will move in the direction in which M was proceeding; but if $M'v'$ exceed Mv, it will move in the direction in which M' was proceeding, as is obvious.

If $Mv = M'v'$, then the collision, at meeting, will destroy the motion of each, and bring both bodies to a stand still, since v_1 will then be 0.

If one of the masses M' be at rest, then since $v_1 = 0$, the velocity after impact will be

$$v' = \frac{Mv}{M + M'} \quad . \quad . \quad . \quad . \quad (3).$$

In all cases, the momentum after impact must be the sum of the momenta before impact, regarding those which act in opposite directions as having opposite signs; so that the momentum lost by one body, by the collision, is exactly equal to the momentum gained by the other. In the case (3) above, the momentum of the whole mass, after impact, being $(M + M')v_1 = Mv$, and the momentum gained by M', which was at rest, being $M'v_1$, this must be the momentum lost by M. This loss of momentum in M shows that the mass M' opposes a resistance to the communication of motion, and $M'v_1$ expresses the value of that resistance, which, as the mass was at rest, can be due only to the inertia of the body.

Let us now suppose that the bodies, instead of being perfectly inelastic, are perfectly elastic; and that, as in the former case, they move so as to impinge at some point in the common line described by their centres of gravity.

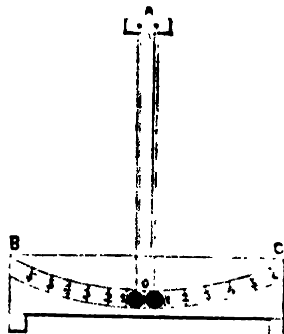
Elastic bodies are such as yield to the force of impact, and undergo a compression, and therefore a change of figure: the elasticity is that inherent force which the body exerts to recover its original form.

If the restoring force which the body exerts to recover its original figure is equal to the impressing force at the point of impact, so that the original form of the body is

perfectly restored, and in the same length of time that it took to alter that form, the elasticity is said to be perfect elasticity.

In the case of perfect elasticity, whatever velocity one body loses during the action of the compressing force, it afterwards loses just as much more during the action of the restoring force; and whatever velocity the other body gains during the compression, it gains as much more during the restitution. For, at the end of the time occupied in the compression, there is the same communication of momentum as in the case of inelastic bodies: at that instant the bodies, then in the closest union, must have equal velocities. The force of restitution, exactly equal to that of compression, then acts; and another effect, the opposite to that of collision, takes place: the collision brings the bodies together, the force of elasticity drives them asunder.

No bodies in nature are perfectly elastic, or completely inelastic. The most elastic substance at present known is glass. Newton, by a contrivance similar to that represented in the margin, determined a very close approximation to the defect from perfect elasticity of several substances. If two balls of the same substance, and in every other respect equal, were suspended from A by slender threads, and each let fall from equal distances, measured from the vertical, on the graduated arc B C, they would, after collision at *o*, each return to the point in the arc from which it started, if the elasticity were perfect. With ivory balls, the elasticity bore to perfect elasticity the ratio of 8 to 9; with glass balls, the ratio was 15 to 16. From recent experiments, of a different kind, Mr. Eaton Hodgkinson has determined the ratios to be, for ivory ·81, and for glass ·94; the latter ratio differing but very little from that deduced by Newton. The fractional or decimal part that the elasticity is of perfect elasticity in any substance, is called the *modulus of elasticity* of that substance.



Example.—Two perfectly elastic bodies, whose masses are M, M′, moving with velocities v, v', strike with direct impact. it is required to determine their velocities afterwards. While the compression continues, the bodies move as one mass; and therefore with the velocity

$$v_1 = \frac{Mv + M'v'}{M + M'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

So that M will lose the velocity $v - v_1$, and M′ will lose the velocity $v' - v_1$; and these express the velocities communicated, but in opposite directions, by the force of restitution. Hence the velocities, after impact, will be

$$\text{of the mass M,} \quad v - 2(v - v_1) \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{and of the mass M}', \quad v' - 2(v' - v_1) \quad . \quad . \quad . \quad . \quad (3)$$

Or, substituting for v_1 its value (1), we have

$$\text{velocity of M,} \quad v - 2\frac{M'(v - v')}{M + M'} \quad . \quad . \quad . \quad . \quad (4)$$

$$\text{velocity of M}', \quad v' - 2\frac{M(v' - v)}{M + M'} \quad . \quad . \quad . \quad . \quad (5)$$

If each mass be multiplied by the velocity it has after impact, the sum of the products will be $Mv + M'v'$; which is also the sum of the momenta before impact: hence the sum of the momenta after impact is the same as the sum of the momenta before

impact. If we subtract (3) from (2), the remainder will be $v' - v$: hence the difference of the velocities is the same both before and after impact.

If $M = M'$, that is, if the bodies are perfectly equal in mass, as well as perfectly elastic, the expressions (4) and (5) show that, after impact, the bodies will exchange their velocities, M' moving with the velocity v, and M with the velocity v'. Hence, if one of the bodies be perfectly at rest, the other which strikes it will impart to it its entire velocity, and will itself rest in its place.

If the elasticity be imperfect, the foregoing formulæ will require modification. Let e be the modulus of elasticity of the impinging bodies, that is, let the force of restitution after compression be only the eth part of that of compression: then the *additional* velocity lost by M from the action of this force, instead of being equal to that lost directly,—namely, $v - v_1$, will be only $e\,(v - v_1)$; and the additional velocity lost by M', only $e\,(v' - v_1)$; so that the velocities after the impact will be

$$\text{of the mass } M, \quad v - (1 + e)\,(v - v_1) = v - (1 + e)\,\frac{M'\,(v - v')}{M + M'}$$

$$\text{and of the mass } M', \quad v' - (1 + e)\,(v' - v_1) = v_1 - (1 + e)\,\frac{M\,(v' - v)}{M + M'}$$

The momenta after impact will be, for M, the first of these expressions multiplied by M; and for M', the second multiplied by M': the original momenta are for the former, Mv, and for the latter $M'v'$. Hence, the momentum lost by one of the bodies, by the impact, is gained by the other: for, as the above expressions show, this momentum is

$$\text{for } M, \quad (1 + e)\,\frac{M\,M'\,(v - v')}{M + M'}$$

$$\text{and for } M', \quad (1 + e)\,\frac{M\,M'\,(v' - v)}{M + M'}$$

which are the same in value, but opposite in signs.

When the bodies are perfectly elastic, it has been seen above, as an immediate consequence of equations (2) and (3), that the sum of the momenta before impact is the same as the sum of the momenta after impact; and also that the difference of the velocities of the two bodies must be the same after impact as before; that is, calling the velocities after impact V_1 and V', we must have

$$M\,V + M'\,V' = Mv + M'v'.$$

$$\text{Also} \qquad V - V' = v' - v.$$

$$\therefore \quad M(V - v) = M'(v' - V'),$$

$$\text{and} \qquad V + v = v' + V'.$$

Now, if we multiply these two last results together, we shall have the equation

$$M(V^2 - v^2) = M'(v'^2 - V'^2).$$

$$\text{Consequently} \qquad M\,V^2 + M'\,V'^2 = Mv^2 + M'v'^2.$$

We may therefore conclude that when the bodies are perfectly elastic, the sum of the products of each body into the square of its velocity is the same both before and after impact.

A particular name is given to the product of a mass into the square of the velocity with which it moves—it is called the *vis viva*, or the *living force*; so that in the collision of perfectly elastic bodies there is no loss of *vis viva* occasioned by the impact. The consideration of the force thus called the *vis viva* enters largely into certain inquiries connected with the motion of fluids.

1. A ball whose elasticity is e, strikes a perfectly hard plane,—the raised edge of a

billiard-table, for example: required the motion of the ball after the impact. Let A B be the straight line described by the centre of the ball before impact, and call the angle A B D′, α; then, v being the velocity along A B C, let it be represented by B C. The components of this velocity—the one perpendicular to the plane, and the other parallel to it—are B D, B E. The component B E is not modified by the impact; but B D, by the force of restitution, is converted into B D′: hence, compounding the velocities B E, B D′, the path, and velocity v_1, after impact, will be denoted by B C′.

The velocity B D′ is $e \times$ B D $= ev \cos \alpha$.

The velocity B E is $v \sin \alpha$:

because the velocity parallel to the plane is unaltered by the impact,

$$\therefore v_1^2 = (ev \cos \alpha)^2 + (v \sin \alpha)^2 = v^2 (e^2 \cos^2 \alpha + \sin^2 \alpha)$$

$$= v^2 (e^2 \cos^2 \alpha + 1 - \cos^2 \alpha) = v^2 \{ 1 - (1 - e^2) \cos^2 \alpha \} \quad . \quad . \quad . \quad . \quad (1)$$

For the direction B C′, after impact, we have

$$\cot \alpha' = \frac{\text{B D}'}{\text{D C}'} = \frac{ev \cos \alpha}{v \sin \alpha} = e \cot \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the elasticity be perfect, that is if $e = 1$, then, from (1) and (2), $v_1 = v$; and $\alpha' = \alpha$ hence, in the case of perfect elasticity, the ball will rebound with the velocity with which it struck; and the path it takes will make an angle of reflection equal to the angle of incidence.

But if the elasticity be imperfect, that is, if e be less than 1, then, from (1), v will be less than v; and, from (2), cot α' will be less than cot α; and consequently α' will be greater than α: hence, in the case of imperfect elasticity, the velocity after impact will be less than the velocity before impact; but the direction after impact will make a greater angle with the perpendicular than the direction before impact.

2. The annexed diagram represents two bodies, A and B, impinging upon each other at the point o; C D, representing in magnitude and direction the velocity of the former, and C′ D′ in magnitude and direction the velocity of the latter. Let $m\,n$ be the line perpendicular to their surfaces at o. By the resolution of velocities, A may be regarded as animated by two velocities, of which one, C E, is parallel to $n\,m$, and the other, C F, perpendicular to the same line. In like manner, B may be regarded as animated by the velocity C′ E′, parallel to $n\,m$, and the velocity C′ F′ perpendicular to $n\,m$.

If A and B, at the instant of contact, were animated solely by the velocities C F, C′ F′, they would merely slide one past the other, and would experience no impact or shock: the shock they do suffer is therefore due solely to the velocities C E, C′ E; and even then, that any shock may take place, the former velocity must exceed the latter.

The intensity of the impact is the same, therefore, as if the two velocities C E, C′ E′, alone existed; and the two bodies will move as in the case of direct impact, only that these motions will be combined with the unchanged velocities C F, C′ F′: we shall

therefore only have to compound the velocity of A in the direction *n m*, after the direct impact spoken of, with the velocity C F, perpendicular to *n m*, in order to obtain the magnitude and direction of A after the shock; and in a similar way are the magnitude and direction of the velocity of B to be found.

NOTE.—It must be noticed by the student, that what has here been taught respecting the collision of bodies, concerns only their rectilinear motions—their actual advance in space. Unless the line of direction in which two bodies strike, pass through the centre of gravity of each, rotation, as well as translation in space, will invariably be the result: the motion of translation is all that is sought to be determined in discussions on the collision of bodies; and it can be proved that this progressive motion is not in the slightest degree modified by the rotation of the impelled body. More advanced principles have fully established the following general proposition, viz.—When a body is acted upon by any impulsive forces, of which the resultant does not pass through the centre of gravity, the body will have in consequence a double motion:—1, the centre of gravity will move as if the forces were immediately applied to it; and 2, the body will rotate as if this centre were absolutely fixed. But want of space compels us to bring the present introductory treatise to a close · its object has been merely to present to the young student a clear and perspicuous development of the fundamental principles of Dynamics, and not to carry him forward into those higher and more imposing applications of those principles, which necessarily demand a knowledge of much more recondite mathematical theories; and for which no provision has been made in the present series of treatises.

The student who has carefully mastered what is here delivered—and who moreover shall have acquired a familiarity with the Differential and Integral Calculus—will, we hope, find his study of the more advanced dynamical researches somewhat facilitated by the previous perusal of this elementary tract. Before concluding it, however, it is proper to give a formal enunciation of what have been called Newton's Three Laws of Motion: these, as already observed at page 130, are certain dynamical axioms, or postulates, assuming principles of too fundamental a character to admit of being rigorously established either by abstract reasoning or by experimental proof. . In the foregoing treatise we have, in general, tacitly taken these for granted, in several special topics of inquiry: we have preferred this course, to the usual custom of stating the three laws of motion in all their generality at the outset of the subject, because we think that their meaning and applicability cannot be clearly understood and perceived, till some familiarity with the language of Dynamics, and with a few of its more elementary problems, has been acquired.

The Three Laws of Motion.—There are various forms of expression in which Newton's three laws are delivered by different writers on Dynamics: the following enunciation of them will perhaps be found as intelligible as any.

First Law.—A body once at rest, will remain at rest, unless acted upon by some external force; and if moving in any direction, will continue to move in that direction. unless acted upon by some external force.

Second Law.—When a force acts upon a body in motion, the effect of this action is the same, in magnitude and direction, as if it acted on the body at rest.

Third Law.—This was stated by Newton as follows:—"Action and reaction are equal and contrary;" that is to say, A cannot act mechanically upon B, without A itself being reacted upon equally, but in an opposite direction.

The conjoined effect of velocity, and the moving mass, is *momentum*, as defined at

page 149, and this momentum is the dynamical evidence of the "action" referred to in the law, which therefore merely affirms that whatever momentum a body communicates in any direction, *that* momentum it loses in that direction; or, which is the same thing, it receives an equal momentum in a contrary direction.

A great deal of abstract argument, and of mechanical contrivance and experiment, have been employed to demonstrate the truth of these positions; we have neither space nor inclination to enumerate them: some things *must* be taken for granted, as fundamental or primitive principles, in every department of science; reflection and common sense, exercised in the examination of such first principles, will usually produce a stronger amount of conviction of their truth than persuasive arguments or approximative experiments: their verisimilitude, which renders it hard even to imagine a contravention of them, must be accepted instead of rigid proof: and when it is known that the results of the remotest investigations, all primarily resting on the truth of these assumptions, are in every instance verified by actual experience, the original or *prima facie* probability of their correctness becomes elevated into absolute certainty.

We may remark in conclusion, that the fundamental principles of Dynamics, and the strict mathematical theories and deductions founded on them, find their fullest verification only in the movements of the heavenly bodies. Terrestrial mechanics is encumbered with many considerations operating as hindrances and drawbacks to the rigid application of those theories. Machines of human contrivance perform their functions through the intervention of rods and bars, wheels and pinions; and thus the consideration of *friction*, an obstacle that sometimes largely modifies the purely mathematical results, becomes imperatively necessary in practical mechanics. But the machinery of the heavens, without any physical ties to hold it together, goes on in obedience to an Almighty Immaterial agency; and thus the phenomena exhibited are in the completest harmony with the accurate deductions of mathematical science.

The modifying influence of friction, in the ordinary mechanical arrangements, will be examined into in the treatise on PRACTICAL MECHANICS.

The following particulars will often be found of service in dynamical inquiries:—

Force of gravity in the latitude of London 51° 31′ 8″ N, 32·1908 feet.
Force of gravity in the latitude of Paris 48° 50′ 14″ N, 32·1820 „
Force of gravity near the Equator, latitude 0° 1′ 34″ S, 32·0881 „
Force of gravity at Spitzbergen, latitude 79° 49′ 58″ N, 38·2526 „
Length of the pendulum beating seconds in the latitude of London, 39·139 inches.
Equatorial diameter of the earth, 7925·465 geographical miles.
Polar diameter of the earth, 7898·972 „
A French metre = 39·3708 English inches.
A French gramme = 15·434 grains.

HYDROSTATICS.

HYDROSTATICS is the science which treats of fluids when kept in a state of rest or equilibrium under the action of mechanical forces or pressures. Fluids are distinguished from solid bodies by obvious and remarkable peculiarities; and, in consequence of these differences, the statics and dynamics of the latter become only partially applicable to the former; so that the statics of water, and the dynamics of water, require a distinct and separate consideration. These two divisions of the general science of mechanics, thus applied exclusively to water, and such like fluids, are called respectively *Hydrostatics* and *Hydrodynamics*.

To define a *fluid* would be to describe exactly what it is—its internal constitution, as well as the phenomena peculiar to that constitution, which it presents to our senses. But this is beyond our power: we know that, in common with all material existences connected with this earth, it has weight; but that, unlike solid bodies, it is without that cohesion of parts by which a solid preserves its shape in whatever position it be placed, and in virtue of which it is moved, as a whole, by a force applied only to a part.

It is this absence of cohesion from among the constituent particles of a fluid, that renders those particles so freely moveable among themselves, that causes them so readily to yield to any impression, and to obey the slightest effort to separate and detach them, as also to admit of the passage of solid bodies through them. It is the want of cohesion, too, that causes a fluid to change the figure it is made to assume, when supported in a vessel, as soon as any of that support is removed. A fluid presses laterally as well as vertically; and, in consequence of the lateral pressure, tends to spread itself horizontally when left unconstrained.

In all these particulars it differs entirely from a solid body: in *this* all the particles mutually cohere, and, in the absence of violence, maintain their relative positions. In a fluid, on the contrary, they are entirely free to interchange situations, and to move among themselves.

In this general statement of the characteristic differences between a solid and a fluid, it is to be borne in mind that we refer only to fluids such as we find them on this earth—fluids acted upon by the force of gravitation, and therefore having weight. If we conceive a vessel of water to be suspended in space, and gravity and every other force to be removed, then the fluid having no weight, there would be no pressure on the vessel in any direction; so that the water would preserve the shape of the vessel, even though the latter were removed. In the absence of all force there would be nothing to disturb the original arrangement of the component parts of the fluid mass.

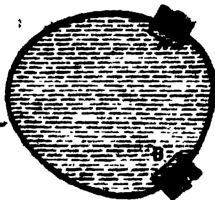
Two Kinds of Fluids.—Fluids are divided into compressible fluids and incompressible fluids: the former are those which by pressure may be forced to contract into

smaller space—such a fluid is the air we breathe, and every kind of gas. The incompressible fluids are those which cannot by pressure be reduced to smaller bulk. Whether in strictness any such fluid actually exists, is more than we can say; because, however it might resist compression from the mechanical forces at our command, it would be presumptuous to conclude that compression was impossible. This, there is reason to believe, is not the case with any fluid: water was formerly thought to be incompressible, but by great mechanical force it has in a slight degree been actually reduced to smaller compass. Still this, and the other common liquids, are so little compressible, that no practical error can arise from treating them as incompressible fluids.

They are also classed under the heads of elastic fluids and inelastic fluids: the former are such as yield to compression, and upon the pressure being removed expand to their former bulk. Air and gases are elastic fluids. Water, and liquids generally, are of course regarded as inelastic (in this sense), as they do not admit of compression in any degree worth notice.

Fluid Elasticity.—The term elasticity, as applied to fluids, is not precisely the same in meaning as when applied to solids. A solid body is elastic when, having yielded to the force of a blow, an equal force, called the force of restitution, is exerted in the opposite direction, restoring the body to its original figure: there is no condition as to whether or not what is called the force of compression actually causes the body to contract into smaller compass. But as regards fluids, elasticity refers to their contraction by pressure into smaller bulk, and their subsequent expansion to the original volume when the pressure is withdrawn. It is in this sense that the elasticity of fluids is to be understood. But taking the term with the signification attached to it in the mechanics of solid bodies, fluids may certainly be considered as elastic: a quantity of water poured down from any height on a hard substance will to a certain extent rebound, and disperse itself in spray; and so likewise will it do if poured upon water itself. A flat stone, or an oyster-shell, thrown very obliquely on the surface of a pool of water, will also rebound, and even a cannon-ball will do the same; but in none of these instances is the fluid compressed into smaller bulk.

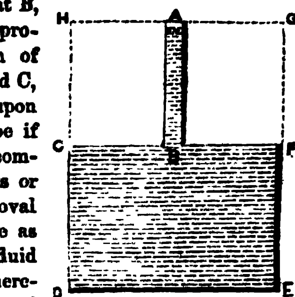
Transmission of Pressure.—The fundamental property of water and all other fluids is, that a pressure applied to any part of its surface is transmitted equally in all directions throughout the entire volume of the fluid: this is called the principle of equal pressure, and may be proved in various ways; for instance, if a vessel be perforated all round at any depth, and glass tubes, however bent, be inserted in the apertures, and then water or any other liquid be poured into the vessel, we find that when the level of the perforations is reached the water presses into the tubes; and however high we raise the level of the water in the vessel, to the same level it always rises in each tube—thus showing that, on the same extent of surface at the same depth, there must be the same pressure.



Again: let a vessel of any shape be filled with water, and to any two equal perforations in its sides let pistons A, B be fitted; let the proper amount of pressure be applied to the piston A to keep the fluid in its place, and also the proper amount of pressure to the piston B. Then it will be found that any additional pressure applied to one of the pistons is transmitted to the other; so that equal additional pressures must be applied to both pistons to keep the fluid in its place, or to prevent one of the pistons from being forced outwards.

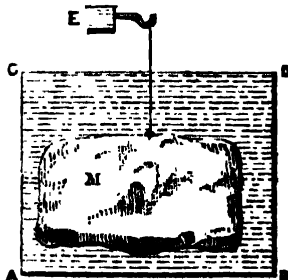
And if the vessel were covered with such pistons, the same pressure will be transmitted to each of the others that is applied to any *one* of them.

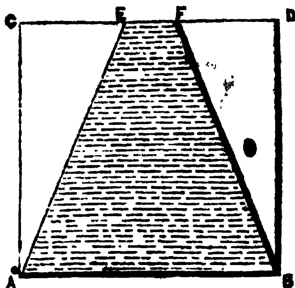
This very remarkable property of fluids is perfectly general, having place whatever be the contour or external form of the vessel: if, for instance, the form be that in the margin, namely, A B C E F, the downward pressure at B, arising from the weight of the water in the tube A B, produces an equal upward pressure upon every portion of surface, equal to the base of the tube, between B and C, and between B and F. Also the pressure of the fluid upon the base D E of the vessel is the same as it would be if the vessel were enlarged to the form H E, and the compartments H B, G B filled up with the fluid, the sides or partitions B C, B F being removed; for before the removal of these sides the upward pressure on B C is the same as the downward pressure on it, from the weight of the fluid afterwards introduced into H B; the two pressures therefore neutralize one another, and thus there is no additional pressure sustained by the bottom of the vessel D E: in like manner is there no additional pressure from the additional fluid in the compartment G B.



This is a property of fluids so extraordinary, that it has been called the *Hydrostatic Paradox;* namely, that the quantity, and therefore the weight of the fluid, may be indefinitely increased, and no increase of pressure be sustained by the bottom of the vessel, the pressure on the bottom being due entirely to the *height* of the fluid, and quite independent of its other dimensions.

Suppose in any vessel C B, a solid body M, as a mass of lead, were suspended, then if a fluid be poured in to fill up the empty space about M, the pressure on the bottom of the vessel will be the same as if the mass M were removed, and its place supplied by additional fluid: the pressures on the sides too of the vessel must remain unaltered, whether the mass M be of lead or of the fluid; for the mass, whatever it be, so that it be incompressible by the surrounding fluid, can not in any way modify the pressures exerted on the bottom and sides of the vessel when filled wholly with the fluid. It may be observed here, that not only is the *pressure* on the bottom of the vessel the same when the fluid in it is only the border of fluid surrounding M, as when, M being removed, it is quite full, but the *weight* of the entire vessel and contents is the same in both cases, though M be firmly supported by the beam E—for only so much of M is thus supported as is equal to the excess of its weight above the weight of the fluid which fills up the space occupied by M upon the removal of that body. The pressure of a fluid on the base of a vessel is no indication or measure of the weight



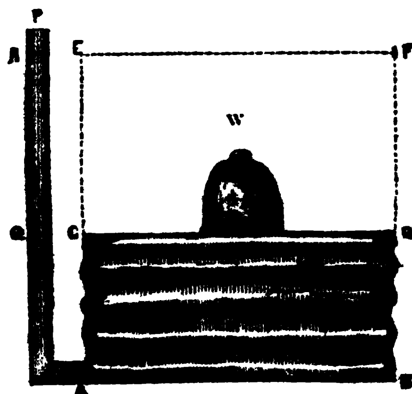


of the fluid: the pressure on the base A B is the same whether the vessel filled with the fluid be C A B D, or E A B F, though the weights are of course different.

It thus happens that by enlarging the base of a vessel, and narrowing the upper part, or by narrowing the upper part only, we may cause the pressure of the contained fluid on the base to *exceed* its weight in any given ratio:—for instance, if the vessel be of the form of a cone, standing on its base, the pressure on the base will be three times the weight of the fluid itself. For the pressure on the base will be the same as if that base supported a cylinder of fluid instead of a cone of the same height, and we know that the content of the cylinder is three times the content of the inscribed cone.

Paradoxical as the statement may seem, that the pressure on the base is three times the whole weight of the fluid, the fact may be readily explained on the principle of the equal transmission of fluid-pressure. If the hollow cone be of heavy metal, and of sufficient weight to be completely water-tight when merely placed on the base, then, however great this weight of metal, if only the cone be sufficiently high, and water be poured into it through an orifice at the top, the upward pressure of the fluid will act with greater and greater intensity against the interior surface, till at length the cone will be seen to rise, forced upwards by the superior pressure, and the water will escape. This will be the subject of a problem hereafter. Equal pressures upward and downward have no effect on the weight.

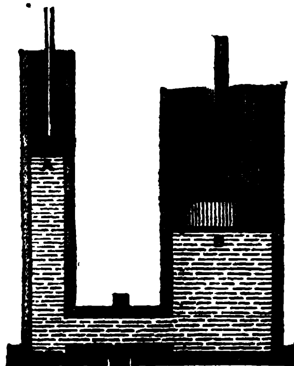
Hydrostatic Paradox.—But what is more emphatically called the hydrostatic paradox is this:—A B, C D are two stout boards connected together, like the boards of a pair of bellows, by a water-tight leathern band: if water be introduced into the enclosure through the pipe P or otherwise, the upward pressure of the fluid will separate the boards; and upon stopping the further supply of water, the fluid will stand at the same level both in the tube and in the receptacle C B; so that the small portion of water in the tube up to Q, balances or keeps in equilibrio the large body of water C B. If now a heavy weight be placed on the board C D—a weight equal to that of a mass of water that would fill up the space E D above the board—then the additional small portion of water filling the tube up to R would keep that weight supported.



If, instead of water, the cavity Q B be inflated with air, by a person standing on C D, and blowing into the tube P Q with his mouth, the same effect will take place; the person may thus easily raise himself higher and higher, and may therefore literally be said to be blowing himself up. Such a contrivance is called the *hydrostatic bellows*.

All the phenomena hitherto enumerated are necessary consequences of the transmission of fluid-pressure in all directions. In the illustration just given, the downward pressure of the slender column of water P Q is transmitted as an upward pressure upon every area equal to the section Q of the entire surface C D; and thus the downward pressure of the body of water E D, or of the equivalent weight W, is counterbalanced.

Bramah's Hydrostatic Press.—The hydrostatic bellows is little more than a philosophical toy for illustrating in a popular and striking manner that peculiar and important property of fluids by which pressure on a small surface is communicated undiminished to every portion, equal in area, of a large surface. Without the aid of levers or pulleys or wheels, or other such mechanical contrivances for accumulating and concentrating force, this fundamental property of water enables us to command as great a pressing force as we please at the expense of as little applied power as we please. One of the most interesting and useful exemplifications of this is furnished by Bramah's hydrostatic press, a machine the principle of which will be sufficiently understood from the annexed diagram, without entering into minute details of its construction. Water communicates with two cylinders of metal, as in the figure: one of these cylinders is of considerably larger section than the other. Water-tight pistons A, B being fitted to both, any force or pressure applied downwards to the smaller A, acts upwards upon the larger B with an intensity as many times the applied pressure as the area of it is contained in the area of B; so that by diminishing the diameter of A, or by increasing the diameter of B, we may make the upward pressure upon B as many times the downward pressure upon A as we please. Suppose, for instance, the diameter of the cylinder A is half an inch, and that of the cylinder B one foot; then as the areas of circles are proportional to the squares of their diameters, we shall have



$(\frac{1}{2})^2 : 12^2 ::$ applied pressure $::$ resulting pressure $=$ 576 times the applied pressure;

so that a pressure of 1 cwt. applied to the piston A will produce a pressure of 576 cwt., or nearly 29 tons, upon the piston B. It is plain that by means of a lever, applied to the piston-rod A, the downward pressure may be increased to any required amount; and, in fact, by increasing the disparity in the diameters of the cylinders, and using only a moderate leverage on A, the transmitted pressure may be made as great as the metal cylinders can sustain. Something of this kind, no doubt, takes place in nature: water finds its way down the chinks and crevices of rocks and mountains, settling in whatever cavity within there may be to receive it. This cavity in time becomes filled, and a number of slender and irregular columns of water reaching from the reservoir to the surface, the upward and lateral pressure of the reservoir becomes at length greater than the rocky receptacle can sustain, a rupture takes place at the weakest part, and devastation is spread around. On a small scale, this effect is actually produced artificially in mining; where water-pressure is sometimes thus introduced for the purpose of blasting rocks.

It may not be altogether unworthy of notice here that the Creator has provided a remarkable preventive for these destructive effects of fluid-pressure when exerted through a high column. The sap of trees, extending from the roots to the height of 80 or 100 feet, if it gravitated like the column of water in a Bramah's press, would rupture the trunks of the largest trees; but when fluid is introduced into *very narrow* tubes, an upward force, called capillary attraction, acts on the fluid in opposition to its

downward pressure; and it is this force which sustains the sap in trees, and neutralizes the downward pressure of the fluid.

Explanation of Terms.—In the treatise on Dynamics (page 149) the meaning of the term *mass* was explained, and the distinction shown between it and weight. The weight or pressure produced by a mass M under the influence of gravity may be denoted thus, namely, $W = Mg$. But in applying this notation, care should be taken to preserve consistency of meaning between the two members of the equation. M should be regarded as so many units of mass, just as in Dynamics t is regarded as so many units of time: the unit of mass may be arbitrarily chosen; and whatever it be,

$$\text{one unit of mass} \times g = \text{weight of the unit};$$

so that here g is not to be regarded as the symbol for the accelerating force of gravity, but for the *weight-force*—the force which gives to the mass M its weight or pressure: and employing M as an abstract number, namely, the number of units of mass, we have

$$1 \times g = \text{weight of the unit of mass};$$

that is, in the present inquiry, g stands for the following effect—namely, the weight impressed by gravity on the unit of mass. This is a definite and perfectly intelligible measure of the effect of gravity as a statical force; with the acceleration produced by it we have here no concern; and when, as in the current works on this subject, it is said, "let M represent mass, and g the accelerating force of gravity, and we shall have $W = Mg$," the language is calculated to mislead the student into the supposition, that the mass of a body is the 32nd part of its weight, which is of course an absurdity: the term accelerating force should never be employed in any statical inquiry, as it is unintelligible without reference to motion: the effect of the influence so called is, in statics, continued but stationary pressure or weight, and nothing else.

In order to estimate the mass or quantity of matter in any body, it is necessary—as in all other cases of measurement—to have reference to some conventional standard, as the unit of measure; accordingly, by general consent, the mass-unit is the quantity of matter contained in the volume-unit (a cubit foot, or cubic inch) of distilled water at a certain temperature. Consequently, g stands for the weight of a cubic foot or inch, as may be agreed upon, of distilled water: as the cubic foot weighs just 1000 ounces, this is the unit to be preferred, so that the expression $W = Mg$ implies that if we take the number of cubic feet in a body of distilled water containing the same quantity of matter as (and therefore of the weight of) W, then 1000 oz. multiplied by that number will give W, the multiplying *number* being all that is represented by M.

If a body contain D times as much matter as a body of distilled water of equal volume, the former is said to have D times the density of water, so that the density of water being taken $= 1$, that is, being taken for the unit of density, D will denote the density of the body: hence, if V be the volume, or rather the *number* of cubic feet in the body, DV will be the number of cubic feet in its equivalent, as to quantity of matter, of water; and therefore

$$W = Mg = DVg \quad . \quad . \quad . \quad . \quad (1)$$

Also if a body weigh S times as much as a body of distilled water of equal volume, the *specific gravity* of the former is said to be S. By the specific gravity, therefore, of any substance, is simply meant the ratio of the weight of any volume of it, to the weight of an equal volume of distilled water.

Both the density and specific gravity of any substance are expressed by the same abstract number: thus the density of mercury as compared with distilled water is 14, its specific gravity also is 14; but the density refers entirely to the mass, the specific

gravity to the weight: as there is 14 times as much matter in a cubic foot of mercury as in a cubic foot of distilled water, the density of the former is 14 times that of the latter; and since, as a consequence of this superior density, a cubic foot of mercury is 14 times as heavy as a cubic foot of distilled water, the specific gravity of mercury is 14 also.

Hence, W and V denoting weight and volume as before, we have

$$W = SVg \ (g = \text{one thousand ounces}) \quad . \quad . \quad . \quad . \quad (2)$$

Interpretation of Symbols.—From the explanations now given, it will be perceived that the symbols to be hereafter employed have the following significations, namely:—

$M =$ the *number* of cubic feet in a body of water containing the same quantity of matter (and therefore of the same weight) as the body proposed.

$V =$ the *number* of cubic feet in the body proposed.

$D =$ the *number* by which a volume of water must be multiplied to give the same quantity of matter as is contained in an *equal* volume of the proposed body.

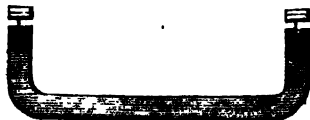
$S =$ the *number* by which a volume of water must be multiplied to give the same weight as an *equal* volume of the proposed body. Hence, D and S are the same abstract numbers, but the former refers to quantity of matter, and the latter to weight.

$g =$ 1000 oz. avoirdupois: the weight or pressure communicated by gravity to a cubic foot of distilled water, at a temperature of 60° Fahrenheit.

All these symbols, except the pressure or weight g produced by gravity, are abstract numbers: the symbol W is of course the concrete quantity *weight*, as in common language. And weight—including under this term both pressure and tension—is the only concrete magnitude with which we have to do in Statics.

Fluid Pressures.—1. When a fluid is at rest, its upper surface is horizontal.

Let P, Q be any two points in the upper surface of a fluid at rest, and the vertical lines P A, Q B be drawn, A B being a horizontal line of particles of the fluid. Then as A B is in equilibrio, the pressures on the extremities A, B in the horizontal direction must be equal; but the pressure on A in the direction A B is the same as the vertical pressure of the column of particles P A, and the pressure on B in the direction of B A is the same as the vertical pressure of the column of particles Q B. But these pressures are equal; therefore the column of particles P A is equal to the column of particles Q B; that is, the points P, Q are equally distant from the horizontal line A B, and are therefore themselves in a horizontal line; and in the same way is it shown that *any* two points on the upper surface are in a horizontal line, and therefore that the entire surface is a horizontal plane. It is evident, too, that if A B be the upper surface of a fluid sustaining a lighter fluid that does not mix with it, the separating surface A B will be horizontal; for, from what is shown above, every point of A B is pressed alike.



It must be observed, that what are called vertical lines, are lines perpendicular to the surface of the earth; and the upper surface of a fluid, here shown to be perpendicular to these verticals, will therefore be a surface parallel to that of the earth—a spherical surface: but it is customary to call but a small portion of such a large surface a plane.

It is upon this property of fluids that the value of levelling instruments depends; that is, instruments which serve to show whether or not any two points are at the same horizontal level.* The common level consists of a bent tube (as in previous page), open at the ends, which are turned up. The tube is nearly filled with a fluid, generally mercury, which supports two floats bearing sights, with a slender wire across, the wires being at equal distances from the floats. When held in the hand, the two surfaces bearing the floats are necessarily horizontal, however the tube itself may be inclined; and consequently the two wires are always on the same horizontal plane; also whatever other objects, seen through the sights, may be on the same level as the wires, must likewise be in the same horizontal plane, or on the same level.

2. The pressure perpendicular to a surface immersed in a fluid is equal to the weight of a column of the fluid whose base is the area A of the surface, and whose altitude is the perpendicular depth of the centre of gravity of the surface.

For let the vertical length of any linear column of particles pressing on the surface be a_1, and the point of the surface pressed be P_1. Regarding this point as a small area, we have for the pressure or weight of the column, SgP_1a_1, S being the specific gravity of the fluid. In like manner, for another vertical column of length a_2, pressing on another point P_2, we have SgP_2a_2; and so on. Hence, the whole pressure perpendicular to the surface is

$$Sg\,(P_1a_1 + P_2a_2 + P_3a_3 + \quad . \quad . \quad . \quad .)$$

But if G be the depth of the centre of gravity of the assemblage of points P_1, P_2, P_3, &c., that is, of the proposed surface A, then by Statics,

$$P_1a_1 + P_2a_2 + P_3a_3 + \quad . \quad . \quad . \quad . \quad . = (P_1 + P_2 + P_3 + \quad . \quad . \quad .)\,G$$

$$\therefore \text{Pressure perp. to the surface} = SAG\,.\,g$$

where AG is the volume V of a column of fluid of base equal to the area A and height G; so that $W = SVg$ (page 167). Hence, if a given area A, immersed in a fluid, revolve round its centre of gravity, the pressure perpendicular to its surface must be the same in every position. Also if the area be a rectangle, the pressure upon it, when it forms the bottom of a vessel, will be double the pressure upon it when it forms one of the vertical sides; so that the pressure upon the four sides of a cubical vessel filled with liquid, is equal to twice the pressure on the base, that is, to twice the weight of the fluid.

If the sides of a vessel filled with fluid are all vertical, the entire pressure on the sides is equal to the weight of a column of the fluid whose base is the rectangle formed by developing the sides into a plane, and whose height is half that of the fluid.

By means of the preceding proposition, it is easy to find the amount of pressure sustained by a rectangular dam, or by a pair of flood-gates. If we multiply the area of the dam, or flood-gate, by half the depth of the water, we shall have the volume of water the weight of which will be the pressure. For example: let the water be 8 feet deep, and the breadth of the flood-gate 6 feet; then the area of the surface pressed is 48 feet: hence,

$$48 \times 4 = 192 \text{ cubic feet of water} = 12000 \text{ pounds} = 5\tfrac{5}{14} \text{ tons.}$$

Since the centre of gravity of a straight line is at its middle point, if two straight lines a_1, a_2 be placed vertically in a fluid, the upper extremities of each being on the surface, then pressure on a_1 : pressure on a_2 :: $a_1 \frac{a_1}{2} : a_2 \frac{a_2}{2}$

$$:: \; a_1^2 : a_2^2$$

that is, the pressures are as the squares of the lengths. But if the lines are inclined to

the surface of the fluid at the angles α_1, α_2 respectively, the perpendicular depths of the centre of gravity are

$$\tfrac{1}{3}a_1 \sin \alpha_1, \text{ and } \tfrac{1}{3}a_2 \sin \alpha_2.$$

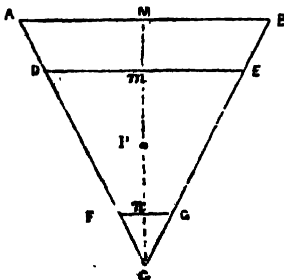
The pressures are therefore as

$$a_1 \times \tfrac{1}{3}a_1 \sin \alpha_1 : a_2 \times \tfrac{1}{3}a_2 \sin \alpha_2,$$

or as
$$a_1^2 \sin \alpha_1 : a_2^2 \sin \alpha_2;$$

that is, as the squares of the lines into the sines of the angles of inclination, or as the squares of the lines themselves if the inclinations are equal.

If a triangle be immersed at any inclination in a fluid, with its vertex downward and its base horizontal, and at the surface of the fluid, then the pressures on any two lines D E, F G, the one as distant from the base as the other is from the vertex, will be equal.

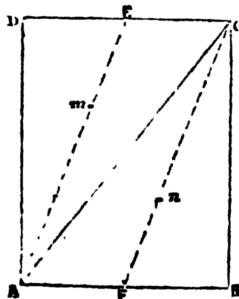


For draw C M bisecting A B, and therefore bisecting the parallels D E, F G in m and n.

Now the pressures on D E, F G are as D E . Mm : F G . Mn; that is, as D E . Cn : F G . Cm. But Cn : Cm :: F G : D E; hence the pressures are as D E . F G : F G . D E; that is, they are equal.

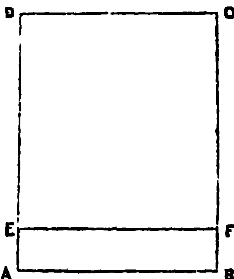
If the triangle be reversed, C being at the surface of the fluid, the student may easily prove that the pressures on any two parallels are as the squares of those parallels, or as the squares of their depths.

3. If one of the sides of a vessel filled with fluid be a rectangle, and if this rectangle be divided by a diagonal into two triangles, then the pressure on one triangle will be double that on the other.



For let A B C D be the rectangle, and A C a diagonal. Let A E bisect D C, and C F, A B; and take Em = $\tfrac{1}{3}$EA, and Fn = $\tfrac{1}{3}$FC: then m and n are the centres of gravity of the two triangles. The depths of these centres are to one another as Em : Cn; that is, as $\tfrac{1}{3}$: $\tfrac{2}{3}$, or as 1 : 2; and as the areas of the two triangles are equal, therefore the pressures upon them are as 1 : 2.

4. Problem.—To divide the preceding rectangle, by lines parallel to the base, into n rectangles, so that the pressure on each may be the same.



Let E B be the lowest rectangle; then, since the centre of gravity of a rectangle is its middle point, the depth of the centre of gravity of A C is $\tfrac{1}{2}$DA, and of that of E C, $\tfrac{1}{2}$DE. The pressures, therefore, on these rectangles are as

$$DA \times \tfrac{1}{2}DA : DE \times \tfrac{1}{2}DE, \text{ or as } DA^2 : DE^2.$$

But the pressure on A C is divided into n equal pressures, and the pressure on E C into $n - 1$ equal pressures: hence

$$DA^2 : DE^2 :: n : n - 1.$$

$$\therefore DE = \sqrt{\frac{n-1}{n}} \cdot DA.$$

$$\therefore \text{EA} = \text{DA} - \text{DE} = \text{DA} \left\{1 - \sqrt{\frac{n-1}{n}}\right\}$$

If, for instance, the rectangle is to be divided into two rectangles, so that the pressure on each may be the same—that is, half the whole pressure—then since $n = 2$, $\text{DE} = \frac{1}{\sqrt{2}}\text{DA} = \frac{1}{2}\text{DA}\sqrt{2}$.

It appears from this proposition, that in constructing a flood-gate, or an upright embankment, to resist the pressure of water against it, it is unnecessary to make the material equally strong throughout; the proper degree of strength being secured at the bottom, where the pressure is greatest, the strength upwards may be diminished in the proportion of DA^2 to DE^2; that is, the strength may decrease as the square of the depth decreases.

PROBLEM.—To determine the pressure upon the internal surface of a hollow sphere filled with fluid.

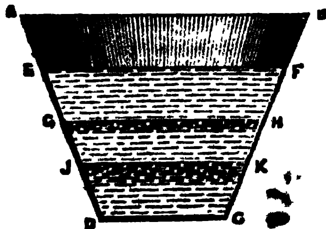
The centre of gravity of the surface pressed is at the centre of the sphere, so that the distance of the centre of gravity from the surface of the fluid is the radius r. The area of the surface of the sphere is $4\pi r^2$, where π stands for 3·1416: hence, the pressure is the same as that of a cylindrical column of the fluid of base $4\pi r^2$ and altitude r.

If S be the specific gravity, the weight of this column of the fluid is $4\pi r^3 Sg$; but the volume of the sphere is $\frac{4}{3}\pi r^3$, and therefore the weight of the contained fluid is $\frac{4}{3}\pi r^3 Sg$. Hence the pressure on the internal surface of the sphere is three times the weight of the contained fluid.

If a cone have its base equal to the surface of the sphere, and its altitude equal to the radius of the sphere, the pressure on the base of the cone will be the same as that on the surface of the sphere, when both are filled with the same fluid (page 164).

PROBLEM.—To determine the pressure on the horizontal base of a vessel containing different fluids.

Let EF, GH, JK, be the surfaces of the different fluids; these surfaces are all horizontal (p. 167). Let p, p_1, p_2, p_3, be the perpendicular depths of the several layers of fluid JC, GK, EH, AF, and S, S_1, S_2, S_3 their respective specific gravities · then the pressures of these several layers on the base DC will be,



$$\text{DC}\,.\,pSg,\ \text{DC}\,.\,p_1S_1g,\ \text{DC}\,.\,p_2S_2g,\ \text{CD}\,.\,p_3S_3g$$

and consequently the whole pressure on the base will be

$$\text{DC}\,(pS + p_1S_1 + p_2S_2 + p_3S_3)g$$

that is, the pressure on the base is found thus: multiply the area of the base by the sum of the products of the perpendicular thickness of each fluid into its specific gravity: the number of cubic feet in the result will be the number of 1000 ounces of pressure.

If the depths of the several layers of fluid are all equal, then the pressure on the base will be $\quad \text{DC}\,.\,p\,(S + S_1 + S_2 + S_3)\,g$

so that if we multiply the area of the base by the perpendicular height of one of the layers of the fluid, and the product by the sum of the specific gravities, the result will be the number of 1000 ounces of pressure.

Example.—A cylindrical vessel is filled with mercury to half its height, and the remainder is then filled with water. Supposing the specific gravity of mercury to be 14, required the pressure on the base, and on the concave perpendicular surface.

Let a be the height of the cylinder, and r the radius of its base: then the area of the base of the cylinder will be πr^2, and since $S = 14$, and $S_1 = 1$, we shall have for the pressure on the base

$$\pi r^2 \cdot \tfrac{1}{2}a\,(14 + 1)\;1000 \text{ oz.} = \frac{15}{2}\,\pi r^2 a \times 1000 \text{ oz.}$$

The pressure on the concave surface of the lower half of the vessel is the same as if the water in the upper half were removed, and a column of mercury $\frac{1}{14}$th as high substituted in its stead, making the height of the entire column of mercury $(\frac{1}{2} + \frac{1}{28})a$. Hence, the area of the lower half of the concave surface being $2\pi r\frac{1}{2}a$, and the depth of the centre of gravity of it $\frac{1}{4}a + \frac{1}{28}a = \frac{2}{7}a$, we have for the pressure on that surface

$$2\pi r\tfrac{1}{2}a \cdot \tfrac{2}{7}a \cdot 14g = 4\pi ra^2 \times 1000 \text{ oz.}$$

The pressure on the upper half of the surface is $2\pi r\frac{1}{2}a \cdot \frac{1}{4}ag = \frac{1}{4}\pi ra^2g$: therefore the whole pressure on the concave surface is $4\pi ra^2g + \frac{1}{4}\pi ra^2g = \frac{17}{4}\pi ra^2 \times 1000$ oz.

Or the pressure on the concave surface may be found thus:—remembering the fundamental principle, that a pressure exerted on the surface of a fluid is transmitted in all directions.

Pressure on lower half of cylinder by the mercury alone,

$$2\pi r\tfrac{1}{2}a \cdot \tfrac{1}{4}a \times 14g$$

Pressure of the water on the surface of the mercury,

$$2\pi r\tfrac{1}{2}a \cdot \tfrac{1}{2}a \times 1g$$

Pressure of the water on the upper half of the cylinder,

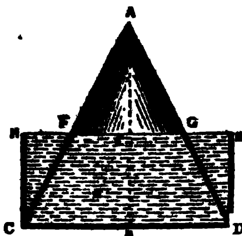
$$2\pi r\tfrac{1}{2}a \cdot \tfrac{1}{4}a \times 1g$$

Sum of the pressures, $\quad \dfrac{17}{4}\pi ra^2g = \dfrac{17}{4}\pi ra^2 \times 1000$ oz.

In a similar manner may the united pressures of layers, equal or unequal, of different fluids be always ascertained: the bottom layer exercises its own pressure, and this is increased by the weight of the whole superincumbent mass: the second layer in like manner exercises its own pressure, increased by the weight of the mass above *it;* and so on.

PROBLEM.—A hollow cone rests with its base on a smooth horizontal plane, and water is poured in at the top. How high will the water rise before it lifts the cone off its support and escapes?

As the water rises in the cone, it exercises an upward pressure on its slant sides, which increases as the perpendicular height of the fluid increases; and as soon as this upward pressure becomes equal to the downward pressure or weight of the conical shell, the equilibrium is just maintained, and no more water, that is, no more upward pressure, can be sustained by the cone, which will therefore be lifted up, and give egress to the water: we have therefore to find the height of water, the upward pressure of which is just equal to the weight of the cone.



The upward pressure, thus just balanced by the weight of the cone, would be equally balanced if a cylinder on the same base, and of the same

altitude as the water, were to surround the cone, and the vacant space to be filled with the fluid. Thus if B E were the height of the water, and if a cylinder H D were to surround the cone, and the space H F C to be filled with water, the downward pressure or weight of the water in this space would replace the weight of the cone, so that the equilibrium would remain undisturbed if the cone were withdrawn; we have therefore to determine the height B E of the cylinder, so that the weight of water contained between it and the cone may be exactly equal to the weight of the cone.

Let A B $= a$, B C $= b$, A E $= x$: then the volume of the cone A C D is $\frac{1}{3}\pi b^2 a$; and the volume of the cone A F G, being to that of A C D as x^3 to a^3, is $\frac{1}{3}\pi b^2 a \cdot \frac{x^3}{a^3}$: hence the volume of water in the frustum F D is

$$\text{volume of water in F D} = \tfrac{1}{3}\pi b^2 a \left(1 - \frac{x^3}{a^3}\right)$$

$$\text{weight of the same} \quad = \tfrac{1}{3}\pi b^2 a \left(1 - \frac{x^3}{a^3}\right) g \quad . \; . \; . \; (1)$$

the specific gravity of the fluid being 1.

Also, if the cone were removed, the volume of water in the cylinder H D would be

$$\text{volume of water in cylinder} = \pi b^2 (a - x)$$

$$\text{weight of the same} \quad = \pi b^2 (a - x)\, g \quad . \; . \; . \; (2)$$

Hence, subtracting (1) from (2) we have for the weight of water between the cylinder and cone, that is, for the upward pressure of the water in the cone on its sides,

$$\text{upward pressure on cone} = \pi b^2 g \left\{ (a - x) - \tfrac{1}{3}a\left(1 - \frac{x^3}{a^3}\right) \right\}$$

If, therefore, the weight of the cone be w, we shall have to solve the cubic equation

$$\pi b^2 g \left\{ (a - x) - \tfrac{1}{3}a\left(1 - \frac{x^3}{a^3}\right) \right\} = w$$

$$\text{or} \qquad x^3 - 3a^2 x + 2a^3 = \frac{3a^2 w}{\pi b^2 g}$$

$$\text{or} \qquad \left(\frac{x}{a}\right)^3 - 3\left(\frac{x}{a}\right) + 2 = \frac{3w}{\pi b^2 a g}$$

The upward pressure of the fluid compels an equal pressure downwards on the base; the water in the cavity between the cone and cylinder is just sufficient to balance the upward pressure, or to replace the resistance of the sides, the pressure on the base remaining undisturbed; and it is thus that the base of the cone supports not only the water in it, but also an amount of pressure equal to the weight of the additional water between the cone and cylinder.

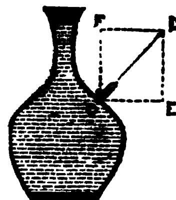
In any vessel containing fluid, where all the *vertical* pressures are downwards—that is, where the sides do not any of them incline inwards—the sum of the vertical pressures must be equal to the weight of the fluid. For every vertical line of particles presses downwards with the weight of those particles; so that the whole vertical pressure is the whole weight of the fluid.

The pressure is, therefore, the same as it would be if the fluid were to become solid, and the sides of the vessel to be removed; and the effect is the same as if the entire pressure or weight were concentrated in the centre of gravity.

The horizontal pressure at any depth is of course the same all round the vessel at that depth, the pressure of every horizontal line of particles being equal to the pressure of a vertical line of particles reaching from the surface to the horizontal line. The downward vertical pressure on the bottom of a vessel can exceed the weight of the fluid

contained in the vessel only when one or more of its sides, or a portion of a side, inclines inwards, occasioning an upward pressure, which reacts downwards on the base, the excess of the pressure on which, above the weight of the fluid, is just equal to this additional pressure; and the vertical pressure on the bottom can fall short of the weight of the fluid in the vessel only when a side, or portion of a side, inclines outward, occasioning a downward pressure on that side. The whole downward pressure on the sides, together with that on the base, makes up the weight of the fluid.

Take, for example, the case of a common decanter: the pressure at any point C is in the direction of the straight line perpendicular to the surface. This may be decomposed into two pressures, of which one C E is horizontal, and the other C F vertical: this last pressure being directed upwards in the figure, if the point C had been near to the bottom, it would have been directed downward, on account of the curvature there being in a contrary direction. If we conceive the pressure at every point of the interior surface of the decanter to be decomposed in like manner into a horizontal and vertical pressure, there will be a series of horizontal pressures like C E, and a series of vertical pressures like C F. The horizontal components mutually destroy one another; otherwise the decanter would tend to move horizontally, which is not the case. Of the vertical components, some are upward pressures and the others downward pressures: they may, therefore, be replaced by a single vertical force, which will act upward or downward, according as the component vertical forces upward or downward prevail. (STATICS, p. 72.) If this simple resultant pressure act upward, the pressure on the bottom of the decanter will exceed the weight of the liquid, because the downward pressure on the bottom, *minus* this upward pressure, must be equal to the weight: on the contrary, if the resultant be a downward pressure, then because the downward pressure on the bottom, *plus* this other downward pressure, is equal to the weight, the pressure on the bottom will be less than the weight.



PROBLEM.—To determine the resultant of all the pressures of a fluid upon the surface of a body immersed in it.

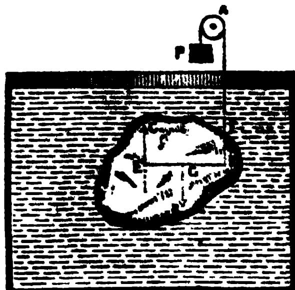
Instead of the immersed body C, conceive the fluid it displaces to become solidified: the surrounding pressures will keep the solidified fluid at rest, just as if it were in its original state. But for these pressures, the mass would fall downwards in virtue of its weight: the resultant of the pressures, therefore, just balances the weight, and acts in a direction opposite to that of gravity; that is, vertically upwards through the centre of gravity of the mass. And as the surrounding fluid exerts the same pressures, whatever be the body whose surface is pressed, it follows that the resultant of the pressures on the surface of any solid body is equal in intensity to the weight of the fluid displaced by it, and is directed vertically upwards through the centre of gravity of the fluid displaced.

When the solid floats at rest, the weight of the fluid displaced is equal to the weight of the solid; and the centre of gravity of the solid and that of the displaced fluid are in the same vertical line.

For the pressure of the fluid on the portion of surface immersed, and the weight of the solid, are the only forces acting: hence, as the body is at rest, the resultant of the pressures on it must be equal and opposite to the downward pressure, or weight, of the body, and act in the same vertical straight line. But since, as just shown, the

resultant of the pressures (equal to the weight of the displaced fluid) is directed upwards through the centre of gravity of the fluid displaced, and the weight of the solid is directed downwards through the centre of gravity of the solid, it follows that the resultant of the pressures, that is, the weight of the fluid displaced, is equal to the weight of the solid, and that the centres of gravity of both are in the same vertical line. If, instead of floating on the surface, the body be kept from sinking by a force, acting along a string, just sufficient to keep the body at rest, the conditions of equilibrium may be found as follows:—

Let G be the centre of gravity of the body suspended in the fluid by the string F A: let E C, G B be verticals through the centre of gravity of the displaced fluid and through the centre of gravity of the body. Let also



W = weight of the body, and W′ = weight of fluid displaced
T = tension of the string
V = volume of the fluid displaced
and D = density of the fluid.

The body is kept at rest by the forces $W' = DVg$, acting in the direction E C; W, acting in the direction G B; and T, acting in the direction F A. These three forces must therefore be all in one plane; and T must be $= W - W'$. Through G draw E G perpendicular to E C, F A; then as the body is at rest, the moments of the forces W, W′ to turn the body about K in opposite directions are equal (STATICS, p. 60).

$$\therefore W \cdot GK = W' \cdot EK = DVg \cdot EK.$$

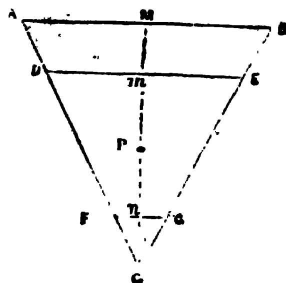
Hence the weight acting over the pulley A upon the body at F just sufficient to keep the body from sinking is $W - W'$, and in order that it may be kept from turning, there must be the condition $W \cdot GK = W' \cdot EK$. Should the body be lighter than the fluid, and tend to float instead of to sink, then the force on F to prevent its rising will of course be $W' - W$; the other condition to prevent turning remaining the same.

As noticed at page 163, if the vessel be full before plunging the body into the fluid, the quantity of the fluid which the immersion of the body causes to run over will occasion no diminution of the weight of the vessel and contents, nor yet any modification of the pressures on the bottom and sides: for the body merely fills the place of the bulk of fluid which its immersion drives out of the vessel. The circumstances as to the weight and pressures are the same as if the fluid, that originally occupied the space now filled by the body, had become solidified while at rest in the vessel. The weight P in the above diagram, and which measures the tension of the string, measures the excess of weight in the body above the weight of the water it displaces.

The Centre of Pressure.—The pressures of a fluid against the different points of a plane surface may be regarded as a system of parallel forces, acting perpendicular to the plane: the resultant of these forces is therefore perpendicular to the plane, and the magnitude or intensity of the resultant has already been shown (page 168) to be equal to a column of the fluid whose base is the surface pressed, and whose altitude is equal to the depth of the centre of gravity of the surface before the level of the fluid. The centre of pressure is that point of the surface to which if a single force equal and opposite to the resultant of the pressures were applied the plane would be kept at rest.

PROBLEM.—To find the centre of pressure of a fluid on a triangle whose base is horizontal and at the surface of the fluid.

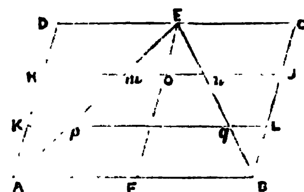
Let A B C be the triangle, and draw C M to bisect the base A B; also let D E, F G be two lines parallel to A B, and drawn so that the distances M *m*, *n* C may be equal. These lines being horizontal are uniformly pressed throughout, so that the centre of pressure on each is at its middle point, and, as already proved at page 169, the pressure on one is the same as the pressure on the other. Consequently we may regard the extremities *m*, *n* of the line *m n* as pressed by equal forces: the resultant of these is therefore equal to the sum of both applied to the middle point P, which point is evidently the middle point of C M.



Whatever two lines be taken equidistant from M and C, the point of application P of the resultant remains the same; and as the whole pressure on the triangle may be regarded as made up of all these linear pressures, it follows that the resultant pressure must pass throught P, which is therefore the centre of pressure on the triangle.

PROBLEM.—To find the centre of pressure of a fluid on a parallelogram, one of whose sides coincides with the surface of the fluid.

Let A C be the parallelogram, and draw E F bisecting the opposite sides D C, A B. The centre of pressure is necessarily in E F, as the pressures on each side of it are equal. Draw E A, E B, as also horizontal lines H J, K L, &c. Then the pressure on one of these lines, as H J, is to the pressure on A B as E *o* to E F, that is, as *m n* to A B. Hence representing the pressure on A B by the line A B, the lines *m n*, *p q*, &c., will correctly represent the pressures on H J, K L, &c. Consequently if the fluid were all removed, and a pressure equal to that originally on A B be applied to that line, and also a pressure to every line *p q*, *m n*, &c., in the triangle, the pressures being always proportional to the lengths of these parallels, the parallelogram in which the pressed triangle is inscribed will still be at rest. But the resultant of all the pressures, thus uniformly diffused over the triangle, must pass through the centre of gravity of the triangle. Hence the centre of gravity of the triangle is the centre of pressure of the fluid on the parallelogram; and consequently the centre of pressure on the parallelogram, one of whose sides is at the surface of the fluid, is on the bisecting line E F, and at a depth equal to two-thirds the depth of the opposite or lowest side of the parallelogram.



Again: suppose the upper side of the parallelogram to be below the surface of the fluid but parallel to it, let H J, for instance, be the upper side of the parallelogram, and D C the surface of the fluid. Then, as shown above, the pressure on the parallelogram H C may be replaced by a pressure uniformly diffused over the triangle E *m n*; and the pressure on the parallelogram A C, by the extension of the pressure on E *m n* uniformly over the triangle E A B: hence the pressure on the parallelogram A J may be replaced by a pressure uniformly spread over the trapezium A *m n* B. Consequently the centre of pressure of the parallelogram A J is the centre of gravity of the trapezium A *m n* B.

As, in the first case, when the upper side of the parallelogram is at the surface of

the fluid, the centre of the pressure is always on the same line E F, and at the same depth, however close the middle point E is to the extremities D, C—that is, however slender the parallelogram may be—it follows that the centre of pressure of a straight line E F, having one extremity at the surface of the fluid, is at ⅔rds the length of E F below E. The centre of pressure being the point about which all the pressing forces balance, it is evidently the point most in need of support, or where the opposing force should be more especially applied to resist the pressure of the fluid on the surface, and thus to prevent rupture. The staves of vats and casks, which may be regarded as so many rectangles, should be each more especially strengthened at one-third of their lengths from the bottom.

Specific Gravities of Bodies.—When bodies are compared together, having the same specific gravity, any volume of one must of course have the same weight as an equal volume of each of the others, so that the weights of such bodies are to one another as their volumes. The volume of any irregular body may be ascertained by the bulk of water it displaces by being immersed (though more conveniently from its weight and specific gravity); or, for the purpose of comparing different volumes, we may merely observe the height to which the water rises in the two cases in a cylindrical vessel, upon the immersion of the bodies. The following is an example.

Example.—A mass of gold immersed in a cylinder containing water, caused the surface to rise a inches; a mass of silver of the same weight caused it to rise b inches; and a mass still of the same weight, but composed of gold and silver, caused it to rise c inches. What was the proportion of gold and silver in the compound mass?

Let x be the volume of the gold, and y that of the silver: then as the volumes immersed are as the elevations of the surface caused by the immersions, we have

$$c : a :: x+y : \frac{a}{c}(x+y) \text{ the volume of the mass of gold}$$

$$c : b :: x+y : \frac{b}{c}(x+y) \quad \text{,, ,, ,, ,, silver.}$$

The weights of these masses being equal, let W be the weight of each; then the weights being as the volumes when the specific gravities are the same,

$$\frac{a}{c}(x+y) : x :: \text{W} : \frac{c}{a}\frac{\text{W}x}{x+y} \text{ the weight of gold in the compound}$$

$$\frac{b}{c}(x+y) : y :: \text{W} : \frac{c}{b}\frac{\text{W}y}{x+y} \quad \text{,, ,, silver ,, ,,}$$

$$\therefore \frac{c}{a}\frac{\text{W}x}{x+y} + \frac{c}{b}\frac{\text{W}y}{x+y} = \text{W}$$

$$\therefore bcx + acy = ab(x+y)$$

$$\therefore (bc - ab)x = (ab - ac)y$$

$$\therefore x : y :: a(b-c) : b(c-a)$$

It is probable that in some such way as this Archimedes solved the problem proposed to him by Hiero, King of Syracuse, who having ordered a crown of gold to be made, suspected that the crown furnished to him was a mixture of gold and silver, and wished the truth to be ascertained without injuring the workmanship.

As the weight of one volume is to that of another of different specific gravity as the product of volume and specific gravity of the former to the product of volume and specific gravity of the latter; therefore, if S, S′ represent the specific gravity of

gold and silver respectively, the weights of x and y will be the one to the other as Sx to $S'y$; therefore, by the foregoing proportion,

$$\text{wt. of the gold } (x) : \text{wt. of the silver } (y) :: a(b-c)S : b(c-a)S'$$

$$\text{or} \qquad :: \frac{a(b-c)}{b(c-a)} \cdot \frac{S}{S'} : 1$$

Consequently the weight of the compound is

$$\left\{\frac{a(b-c)}{b(c-a)} \cdot \frac{S}{S'} + 1\right\} \times \text{wt. of the silver} = W.$$

$\therefore$ the weight of the silver in the compound is $W \div \left\{\frac{a(b-c)}{b(c-a)} \cdot \frac{S}{S'} + 1\right\}$

PROBLEM.—Having given the volumes and specific gravities, or the weights and specific gravities of several bodies, to find the specific gravity of the compound.

1. Let the volumes V_1, V_2, V_3, &c., be given with the specific gravities S_1, S_2, S_3, &c.; and let S be the specific gravity of the compound; then since $W = SVg$, we have for the weight of $V_1 + V_2 + V_3 +$ &c.

$$(S_1V_1 + S_2V_2 + S_3V_3 + \&c.)\, g$$
$$= S\,(V_1 + V_2 + V_3 + \&c.)\, g$$
$$\therefore S = \frac{S_1V_1 + S_2V_2 + S_3V_3 + \&c.}{V_1 + V_2 + V_3 + \&c.}$$

2. Let the weights W_1, W_2, W_3, &c., with the specific gravities S_1, S_2, S_3, &c. be given, S being the specific gravity of the compound as before: then since $V = \frac{W}{Sg}$, we have for the volume of $W_1 + W_2 + W_3 +$ &c.

$$\frac{W_1}{S_1g} + \frac{W_2}{S_2g} + \frac{W_3}{S_3g} + \&c.$$
$$= \frac{W_1 + W_2 + W_3 + \&c.}{Sg}$$
$$\therefore S = \frac{(W_1 + W_2 + W_3 + \&c.)\, S_1\, S_2\, S_3\, \&c.}{W_1\, S_2\, S_3\, \&c. + W_2\, S_1\, S_3\, \&c. + W_3\, S_1\, S_2\, \&c.}$$

If m equal volumes are mixed, the specific gravity of the compound is

$$S = \frac{S_1 + S_2 + S_3 + \&c.}{m}$$

If m equal weights are mixed, the specific gravity of the compound is

$$S = \frac{mS_1\, S_2\, S_3 \ldots \ldots S_m}{S_1\, S_2 \ldots S_{m-1} + S_2\, S_3 \ldots S_m + \&c.}$$

When there are only two bodies to be compounded, then for equal volumes, $S = \frac{S_1 + S_2}{2}$; and for equal weights, $\frac{2S_1S_2}{S_1 + S_2}$, the former value being an arithmetic, and the latter an harmonic mean between the specific gravities of the two substances.

PROBLEM.—To determine the volume of any substance, however irregular, of known specific gravity.

Let S be its specific gravity, and W its weight in ounces, then since

$$W = SVg \;\therefore\; V = \frac{W}{1000\,S\text{ oz.}} \text{ cubic feet.}$$

In a similar way may the capacity of an irregular vessel be ascertained. Let the weight of the water that will fill the vessel be w ounces, then the capacity or volume of the vessel will be $\frac{w}{1000}$ cubic feet. If the result is to be in cubic inches instead of in

cubic feet, the divisor 1000 must be replaced by the multiplier 1·728, because $\frac{1728}{1000} = 1·728$: so that w being the number of the avoirdupois ounces in the weight, the volume or capacity will be $V = 1·728 \frac{w}{S}$ cubic inches.

Should w be the number of troy, instead of the number of avoirdupois ounces, then since

$$1 \text{ oz. troy} : 1 \text{ oz. avoirdupois} :: 480 : 437·5$$

$$\therefore 1 \text{ oz. avoirdupois} = \frac{437·5}{480} \text{ oz. troy} = ·911458 \text{ oz. troy}$$

$$\therefore V = \frac{1·728}{911458} \frac{w}{S} \text{ cubic inches} = \frac{1}{·52746} \cdot \frac{w}{S} \text{ cubic inches.}$$

The divisor ·52746 is the troy weight in oz. of a cubic inch of water, as is obvious:—it is the value of w when $V = 1$ and $S = 1$. The troy weight of a cubic inch of water in grains is 253·1808 grains.

It is in general easier to ascertain the weight of a minute body than to measure accurately its dimensions; and thus the following is the method usually recommended for finding the diameter of a small sphere of known specific gravity and ascertained weight in grains troy. Let d be the small diameter and w the weight in grains troy of the sphere, and S the specific gravity of the substance of which it is formed; then the volume of a sphere whose diameter is 1 inch being ·523598 inches, we have for the troy weight of an equal sphere of water,

$$1 : ·523598 :: 253·1808 \text{ grains} : 132·5648 \text{ grains,}$$

the weight of a sphere of water of 1 inch in diameter.

Now the volumes, and therefore the weights, of spheres of the same substance being as the cubes of their diameters, we have

$$1^3 : d^3 :: 132·5648 \text{ grains} : 132·5648d^3 \text{ grains,}$$

the weight of the small sphere of water of diameter d; therefore the weight w of the proposed sphere is

$$w = 132·5648d^3. S \text{ grains} \therefore d = ·19612 \sqrt{\frac{w}{S}}$$

the *number* of grains being put for w.

When a body is wholly immersed in a fluid, the weight lost is to the whole weight as the specific gravity (S) of the fluid to the specific gravity (S′) of the solid.

The weight lost is the weight of the displaced fluid (page 174); and, from the general relation $W = SVg$, the weight of bodies of the same volume are as their specific gravities; therefore, wt. of displaced fluid : wt. of solid :: S · S′

that is, wt. lost : whole wt. :: S : S′ . . . (1)

If the same body be immersed in another fluid whose specific gravity is S_1, then

wt. lost : whole wt. :: S_1 : S′ . . . (2)

The second and fourth terms being the same in the proportions (1) (2), it follows that the weights lost in the two fluids are as the specific gravities of those fluids; and hence may be ascertained the specific gravity of any fluid (obtainable in sufficient quantity for the experiment), when the specific gravity of any other fluid is known.

Also, the true weight of a body may be readily determined, from knowing what it weighs in each of two fluids of known specific gravities; for let W be the true weight, and w, w' the weights in two fluids whose specific gravities are S, S′: then the weights lost are $W - w$ and $W - w'$ respectively.

$$\therefore W - w : W - w' :: S : S'.$$
$$\therefore S'W - S'w = S W - Sw'.$$
$$\therefore (S' - S)W = S'w - Sw'.$$
$$\therefore W = \frac{S'w - Sw'}{S' - S}, \text{ true weight of the body.}$$

And thus, from having the true weight of the body, and the weight lost in one of the fluids, we may, by the proportion (1) above, find also the specific gravity of the body.

By the true or absolute weight of a body is to be understood the weight of it in a vacuum, that is, free from the presence of even the air: the absolute weight, therefore, is equal to the weight of the body in air, increased by the absolute weight of a volume of air equal to that of the body; but for bodies of small volume this minute increase is inappreciable.

Example 1.—A piece of uniform substance, whose true weight is 64 grains, is found to weigh only 48 grains when immersed in distilled water: required the specific gravity (S) of the substance.

The weight lost is $64 - 48 = 16$ grains,

$$\therefore 16 : 64 :: 1 \text{ (specific gravity of water)} : 4.$$

Hence the specific gravity of the body is $S = 4$.

2. A body weighs 130 grains in one fluid, and 68 in another; but the true weight of the body is 240 grains: required the relative specific gravities of the two fluids.

The weights lost are 110 grains and 172 grains, and (page 178) these are as the specific gravities of the fluids.

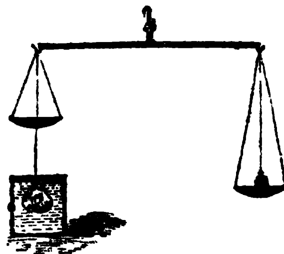
$$\therefore \frac{S}{S'} = \frac{110}{172}, \text{ or } S = \frac{110}{172} S'.$$

If one of the fluids be air in which the body is weighed, and the other water of specific gravity 1, then knowing the specific gravity S of the air, we have for the real weight W of the body, that is, for its weight in vacuo,

$$W = \frac{w - Sw'}{1 - S} = w + (w - w') S + (w - w') S^2 + \&c.$$
$$= w + (w - w') S \text{ very nearly,}$$

because S being a small fraction, S^2 is an equally small fraction of *it*.

The scales used in weighing bodies for the purpose of ascertaining specific gravities are called the *hydrostatic balance*. The body to be weighed is suspended by a fine thread from one of the scale-pans, and immersed, as in the margin; and when the weights in the other scale-pan bring the scale-beam into a horizontal position, the weight of the body in the fluid will be determined.



But in what has preceded, the solid is supposed to be heavier than an equal volume of the fluid in which it is immersed; when it is lighter than the fluid, the method of proceeding is this:—

Take a body sufficiently heavy to sink the lighter body with it, when both are united and immersed together in the fluid. Find the weight, in the fluid, of the heavier body by itself; then the weight, in the fluid, of the united mass: this latter weight

will obviously be *less* than the former; as the lighter body displaces more than its own weight of water, the difference will show how *much* more. Call this difference between the two weights w', and the true weight of the lighter body w, then the weight of the fluid displaced by the lighter body will be $w + w'$. Consequently, $w + w' : w :: S$ (specific gravity of fluid) : S' (specific gravity of the solid).

$$\therefore S' = S \frac{w}{w + w'}.$$

The weight w', by which the weight of the displaced fluid exceeds that of the body, expresses the *buoyancy* of the body, or the upward pressure upon it, in virtue of which it would begin to ascend if left to itself.

If the specific gravity of a body in the form of powder is to be found, the preceding method may still be employed: the powder may be imbedded in wax, or some such yielding material, sufficiently heavy to cause the compound to sink in the fluid; the weight in the fluid of the wax by itself being found, and then the weight in the fluid of the compound, the specific gravity of the powder, previously weighed in vacuo, will be given by the foregoing formula.

PROBLEM.—If in two fluids which do not mix a solid be immersed, and rest partly in one fluid and partly in the other, to find the ratio of the two parts when the specific gravities of the solid and the fluids are known.

Let V' be the volume immersed in the lower or heavier fluid, and V the volume immersed in the upper or lighter fluid; then from the condition $W = VSg$ we necessarily have

$$VS + V'S' = (V + V')S'',$$

where S', S are the specific gravities of the two fluids, and S'' the specific gravity of the solid.

$$\therefore V(S - S'') = V'(S'' - S') \quad \therefore \frac{V}{V'} = \frac{S'' - S'}{S - S''}, \text{ the ratio.}$$

Hence, adding 1 to each side,

$$\frac{V + V'}{V'} = \frac{S - S'}{S - S''}.$$

If the solid float on *one* fluid, then in these results $S = 0$, that is, the upper portion V of the body is in vacuo; therefore $\frac{V}{V'} = \frac{S' - S''}{S''}$. Also $\frac{V + V'}{V'} = \frac{S'}{S''}$.

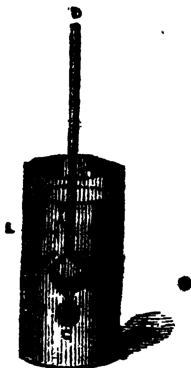
Consequently, if a body float on a fluid in vacuo, and then the air or any other lighter fluid be admitted, the body will rise, and leave a less portion of it immersed in the original fluid. Also, if a body float on a fluid, the part immersed V' is to the whole body $V + V'$ as the specific gravity S'' of the body to the specific gravity S' of the fluid.

Consequently if the body float upon a second fluid of specific gravity S_1, and if V_1 be the part of the body immersed, then we have the two proportions

$$\left.\begin{array}{rl} & V' : V + V' :: S'' : S' \\ \text{and} & V_1 : V + V' :: S'' : S_1 \end{array}\right\} \therefore V'S' = V_1 S_1$$
$$\therefore V' : V_1 :: S_1 : S'$$

That is, the parts of the body immersed, or the volumes of the fluids displaced, are inversely as the specific gravities of those fluids. Upon this principle the instrument called the hydrometer, for measuring the specific gravities of different liquids, is constructed.

The Hydrometer.—There are several modifications of this useful instrument: in its simplest form, the hydrometer consists of two hollow spheres, usually of glass—a larger, and smaller one below it, communicating with each other. Into the lower sphere mercury or small shot is introduced, in order that the spheres may sink in the liquid. From the upper sphere rises a cylindrical stem, the axis of which is in the straight line through the centres of gravity of the spheres; so that when the instrument rests in the fluid, the stem may be vertical. Let the instrument be placed in water, the stem will sink to some point A; let it then be placed in a liquid L that will cause the stem to sink to B; then, by the above-mentioned proposition, the specific gravity of L is to that of water (1), as the volume or magnitude of the body A C, to the volume or magnitude of B C.



$$\therefore \text{ specific gravity of } L = \frac{\text{volume of } AC}{\text{volume of } BC} = \frac{\text{volume of } (DC - DA)}{\text{volume of } (DC - DB)}$$

The whole volume of the instrument—that is, the volume of water it would displace if wholly immersed in it—is regarded as consisting of 4,000 parts, and the stem is divided so that each division is one of those parts. Suppose the stem to contain 50 of the parts, numbered from D downwards, and that it sinks to 30 in one liquid L_1, and to 20 in another liquid L_2: then

$$\frac{\text{Specific gravity of } L_1}{\text{Specific gravity of } L_2} = \frac{4000 - 20}{4000 - 30} = \frac{3980}{3970}$$

And in this way are the specific gravities of different liquids compared.

Nicholson's Hydrometer.—This differs from the common hydrometer chiefly in having a dish at each end, and in serving for measuring the specific gravities of solids as well as fluids. The stem in this instrument is a slender wire of hardened steel; and the adjustment is such that when a given weight—1000 grains for instance—is placed in the upper dish A, the instrument will sink, in distilled water at the temperature of 60° Fah., to the point P in the middle of the stem. It is used as follows:—



To Compare the Specific Gravities of Two Liquids L, L'.—Let W be the weight of the hydrometer, w the weight which must be placed in the dish A to sink the instrument to the point P in the liquid L, w_1 the weight which must be placed in A to sink the instrument to P in the liquid L': then

$$\text{weight of the volume of L displaced} = W + w$$
$$\text{weight of same volume of L' displaced} = W + w_1$$
$$\therefore \frac{\text{specific gravity of L}}{\text{specific gravity of L'}} = \frac{W + w}{W + w_1}.$$

To find the Specific Gravity of a Solid.—Let w be the weight which placed in the dish A causes the instrument to sink to P in the water. Place the solid to be examined, or a fragment of it, in A, and let w' be the weight to be added to sink the instrument to P. Then remove the solid to the lower dish B, and let w'' be the additional weight to be added to the upper dish to sink the instrument to P.

It is plain that the weight of the solid in air, in A $= w - w'$

and that its weight in water, in B $= w - w''$

Hence the weight of water displaced by the solid $= w'' - w'$

$$\therefore \frac{\text{specific gravity of the solid}}{\text{specific gravity of water}} = \frac{w - w'}{w'' - w'} = \text{specific gravity of the solid, the specific}$$

gravity of water being 1.

Or, without placing any additional weight in the upper dish, observe to what depth the solid alone sinks the instrument in water; then remove the solid to the lower dish, and some additional weight w will be necessary to sink the instrument to *the same* depth; so that, calling the weight of the solid W, we shall have

$$\text{specific gravity of the solid} = \frac{W}{w}.$$

The weight W of the solid under examination is readily ascertained by finding what weight supplying its place in the upper dish will sink the instrument to the same depth

It is evident that the instrument becomes more sensitive the slenderer the wire stem be made. If an additional small weight—a grain for instance—be placed in the dish A, an additional grain weight of the liquid must be displaced by the sinking of a suitable additional length of the stem; so that the finer this stem is the greater must the additional length be. The diameter of the stem is in general $\frac{1}{40}$th of an inch; and the instrument will rise or fall nearly one inch by the abstraction or addition of $\frac{1}{10}$th of a grain. For the weight of a cubic inch of water is 253·1808 grains troy; and therefore the volume of $\frac{1}{10}$th of a grain is $\frac{1}{2531 \cdot 808}$ of a cubic inch. Now the area of a transverse section of the wire stem is $\frac{1}{40^2} \times \cdot 7854 = \frac{7854}{16000000}$, and if the volume due to $\frac{1}{10}$th of a grain, namely $\frac{1}{2531 \cdot 808}$, be divided by this area, the quotient will be about $\frac{4}{5}$ths of an inch, for the length of wire that will rise or fall in water when the weight differs by $\frac{1}{10}$th of a grain. In a liquid lighter than water the length will of course be greater.

The hydrometer is the instrument used by officers of the Excise to ascertain the strength of spirituous liquors, or in what degree they are above or below proof. Proof-spirits consist of half alcohol, or pure spirit, and half water; when placed in this mixture, the exciseman's hydrometer sinks to the point marked *proof* upon the scale; if the liquor be below proof, or have more than half water, the instrument will not sink so far as proof; and if it be above proof, it will sink further, and the surface of the liquor will stand *above* proof.

But in all determinations of specific gravity account must be taken of the temperature of the substance at the time of the experiment; all bodies expand by heat, and therefore become specifically lighter. Spirits more especially are susceptible of this increase of volume by increase of temperature: a cubic inch of brandy, for instance, will weigh about ten grains more in winter than in summer, unless the change of temperature is obviated by artificial means. It is necessary, therefore, in measuring the specific gravity of a liquid to notice to what extent the temperature of it differs from the standard temperature of 60° Fahrenheit; and then, from knowing by previous experiment the amount of expansion or contraction of the liquid for different degrees of temperature, to reduce the specific gravity to what it would be at the temperature of 60°.

We shall now give a few examples in practical illustration of the preceding articles on specific gravity.

Examples: 1. If a piece of stone weigh 10 lb. in air, and only $6\frac{3}{4}$ lb. in water, required the specific gravity of the stone.—Ans. 3·077.

2. A piece of elm weighs 15 lb. in air, and a piece of copper weighing 18 lb. in air

and 16 lb. in water is affixed to it; the compound weighs 6 lb. in water: required the specific gravity of the elm.—Ans. 6.

3. A piece of cast-iron, of specific gravity 7·425, weighed 40 ounces in air, and 35·61 ounces in a fluid: required the specific gravity of the fluid.—Ans. 1.

4. Required the number of cubic inches in a block of stone of specific gravity 2·52, which weighs 1 cwt.—Ans. 1228¾ nearly.

5. A composition of 1 cwt. consists of tin of specific gravity 7·32, and of copper of specific gravity 9; the specific gravity of the composition is 8·784: required the quantities of tin and copper in the mixture.—Ans. 100 lb. of copper and 12 lb. of tin.

6. The dimensions of one of the marble stones in the walls of Balbeck are, length 63 feet, and breadth and thickness each 12 feet; the specific gravity of the stone is 2·7: required the weight of the block.—Ans. 683⅜ tons nearly.

7. A solid weighing 250 grains in air, weighs 147 grains in one fluid, and 120 in another: what is the ratio of the specific gravities of the fluids?—Ans. As 103 to 130.

8. A cubical iceberg is 100 feet above the level of the sea, the sides of it being vertical: given the specific gravity of sea-water 1·0263, and of the ice ·9214: required the number of cubic feet in the mass.—Ans. 936302452 cubic feet.

9. The weight of a common hydrometer is equal to that of a volume v of water; the part to which the stem is attached weighs a volume v' of water; the radius of the stem is r; and the specific gravity of a liquid, in which the instrument is immersed, is s: required the length l of stem immersed when in equilibrio.

$$\text{Ans. } l = \frac{v - v'}{\pi r^2 s}$$

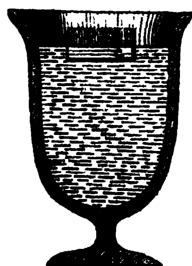
For a description of certain methods of taking specific gravities with extreme accuracy, the student may consult the ELEMENTARY CHEMISTRY, p. 5.

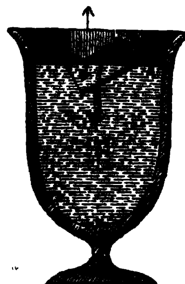
Equilibrium of a Floating Body.—In order that a body may float on a liquid and remain at rest, two conditions are necessary:

1. The weight of the body must be less than the weight of a volume of the fluid of the same bulk as the body; it must be equal to the weight of the volume displaced by that portion of the bulk which is immersed in it.

2. The centre of gravity of the body and the centre of gravity of the fluid displaced by the partial immersion, must both be situated in the same vertical straight line (p. 173).

These are the only conditions necessary to secure the equilibrium of the floating body. If either of them be wanting, the body (though it may not sink) will move, and can come to a state of rest only when these two conditions are secured.





Suppose, for instance, a body floating at rest on the water, as in the first of the annexed figures, the centres of gravity of the body and of the displaced fluid being in the same vertical line, to be disturbed from its position, as in the second figure. The fluid displaced then becomes changed in form and situation, though not in volume; its centre of gravity has shifted, and is no longer in the same vertical with the centre of gravity of the

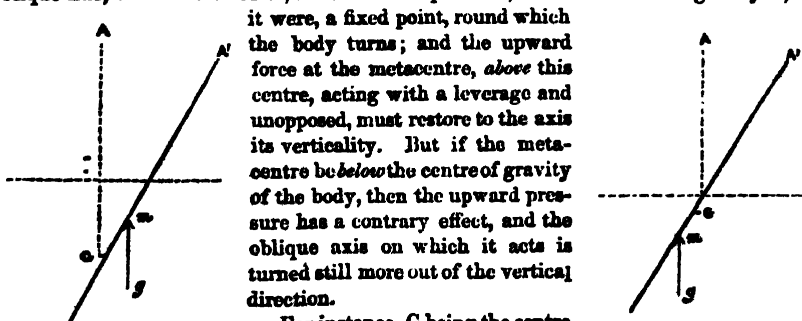
body. The body, therefore, is now acted upon by two forces tending to turn it round into the position from which it has been disturbed; namely, the force due to the weight of the body acting downwards through G, and the force due to the pressure of the displaced fluid acting upwards through G'.

When a body thus slightly disturbed tends, as in this case, to return to its original position of equilibrium, that position is said to be one of *stable equilibrium*, in reference to the disturbance in question. If the disturbance were to act vertically on the body through its centre of gravity, and therefore tending either to depress the body lower in the fluid or to raise it further out; in the former case the additional upward pressure, and in the latter the additional downward weight, would bring the body into its original state of rest, the equilibrium being always stable in reference to disturbance in a vertical direction.

Unstable equilibrium is that position of the floating body, from which, if disturbed, it yields to the disturbance; and instead of recovering its original condition of equilibrium, departs from it more and more, till it finds either some new position of equilibrium or upsets and sinks.

When a floating body is slightly disturbed from rest by being turned about its centre of gravity, the line through that centre, originally vertical, becomes slightly oblique; the downward pressure or weight is then in the direction of a new vertical line through the same centre; the upward pressure, too, in consequence of the change of figure and position of the displaced fluid causing a change in the centre of gravity of that fluid, is also in a new vertical line. The point where this new vertical meets the slightly oblique line through the centre of gravity of the body, is called the *metacentre*; and the vertical itself is called the *line of support*. We have already seen that it is necessary for the line of support to pass through the centre of gravity of the body when the body is at rest (p. 173).

The position of the metacentre, in reference to the centre of gravity of the body when slightly disturbed, determines whether or not the disturbance will be counteracted, and the body right itself by returning to its former position of equilibrium; that is to say, whether or not this position of equilibrium is a *stable* position. If the metacentre be at a point on the oblique line, slightly turned out of its vertical position by the disturbance, which is *above* the centre of gravity on that line, then it is plain the upward pressure, acting along the vertical (line of support) through the metacentre, must re-establish the oblique line, or *axis* referred to, in its vertical position; for the centre of gravity is, as it were, a fixed point, round which the body turns; and the upward force at the metacentre, *above* this centre, acting with a leverage and unopposed, must restore to the axis its verticality. But if the metacentre be *below* the centre of gravity of the body, then the upward pressure has a contrary effect, and the oblique axis on which it acts is turned still more out of the vertical direction.



For instance, G being the centre of gravity of the floating body, if G A' be the axis disturbed from its original vertical

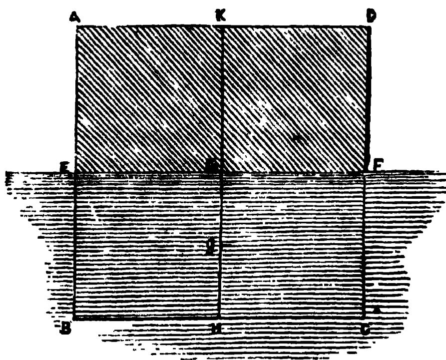
position G A, by any force which no longer acts, and if m be the metacentre, or ponit where the line of support gm, passing through the centre of gravity g of the displaced fluid, meets G A′, it is plain that the progress of the disturbance is arrested, and G A′ restored to its vertical position. If, on the other hand, the metacentre be situated, in reference to the centre of gravity G of the body, as in the second diagram, that is below G, then the upward pressure upon it assists the deviation from the vertical; and thus G A′ goes on inclining more and more.

It is thus obvious, that when the metacentre is *above* the centre of gravity of the disturbed body, the disturbance will be rectified, and the equilibrium which has been disturbed is *stable*.

But when the metacentre is *below* the centre of gravity, the disturbance becomes aggravated; and the equilibrium thus disturbed is *unstable*. It is possible that the line of support gm may pass through G, the centre of gravity of the body; in this case, the body remains at rest in its new position, for the conditions of equilibrium are then as rigorously fulfilled as at first: the equilibrium in such circumstances is said to be *indifferent;* it is also sometimes called *neutral* equilibrium. This kind of equilibrium evidently has place in a floating sphere and cylinder, and also in a body, of whatever shape, floating in a fluid of equal specific gravity as itself—that is, if it be allowed to call the equilibrium in this case *floating*, no part of the body being above the surface of the fluid.

From what has now been said, it is obvious how important it is, in the construction and loading of ships, to make the centre of gravity of the whole body so low that the metacentre may always be above that point, even for a considerable disturbance. The centre of gravity is sometimes elevated above its original position, in a gale of wind, by the shifting of the cargo. if the elevation be sufficient to bring it above the metacentre, the vessel must capsize.

Example.—A rectangular beam, the transverse vertical section of which is a square, and the specific gravity of which is one-half that of the water on which it floats, rests with one of its faces parallel to the surface of the water: it is required to determine whether the equilibrium is stable or unstable in reference to a slight transverse disturbance.

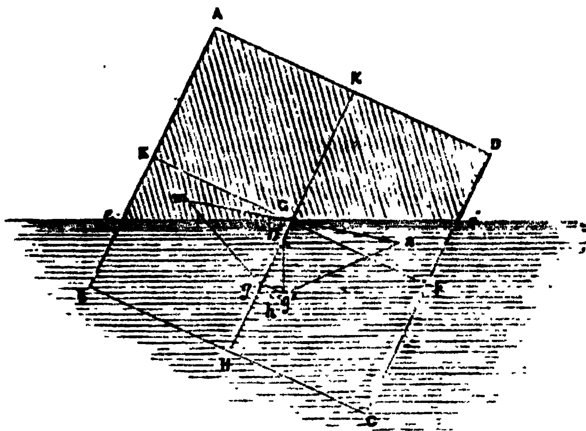


As the transverse vertical sections are all equal squares, it will be sufficient to consider merely the middle section A B C D.

The centre of gravity of this is at G the middle of the square, that is $GH = \frac{1}{2}$ AB; and if $gH = \frac{1}{4}$ AB, then g will be the centre of gravity of the displaced fluid; for the body floats half in and half out of the water, as by hypothesis it is only half the weight of its bulk of water. E F is the plane of flotation, and H K the vertical axis through the centre of gravity G.

Let now the body be turned round its centre of gravity, through a small angle $FGf = \theta$, causing the line of flotation to be ef, and the former vertical axis to beco me

the oblique line H K; we have to find g', the centre of gravity of the displaced fluid $efCB$ in this new position of the body, and thence the point O where the vertical $g'O$ cuts the axis H K: this point O is the metacentre.



Let m, n be the centres of gravity of the portions EGe, FGf, and let h be the centre of gravity of the portion $eGFCB$. Then by STATICS,

$hg : mg :: EGe : eGFCB$

and

$hg' : ng' :: FGf : eGFCB$

But $EGe = FGf$; consequently

$hg : mg :: hg' : ng'$

therefore gg' is parallel to mn

$$\therefore \frac{gg'}{mn} = \frac{hg}{gm} = \frac{EGe}{eGFCB}, \text{ by the first proportion, } = \frac{EGe}{EFCB - EGe}$$

Now, as the angle of disturbance θ is considered as very small, the portion EGe must be insignificant in comparison with E F C B: hence

$$gg' = mn\frac{EGe}{EFCB} \quad . \quad . \quad . \quad . \quad (1)$$

Moreover, on account of the smallness of θ, $\sin\theta$ and θ may be regarded as equal; and we shall then have for the three quantities mn, EGe, $EFCB$, the values $mn = 2Gm = \frac{4}{3}GE$, GE being the distance of the vertex G from the base, or from the *middle* of the base, the base being small in consequence of the minuteness of θ.

$$EGe = \tfrac{1}{2}GE^2 \times \theta, \; EFCB = 2GE^2$$

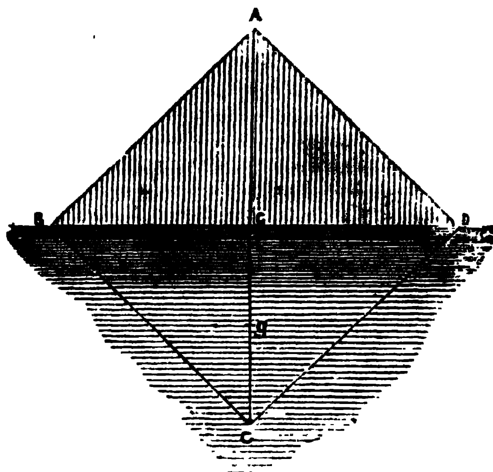
$$\therefore (1), \; gg' = \tfrac{4}{3}GE \times \tfrac{1}{4}\theta = \tfrac{1}{3}GE \times \theta = \tfrac{1}{3}GE \times \text{angle } gOg'$$

But $\quad Og \times \text{angle } gOg' = gg' \; \therefore \tfrac{1}{3}GE = Og, \text{ that is } Og = \tfrac{1}{6}GH$

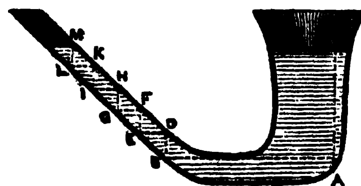
$$\therefore OH = (\tfrac{1}{6} + \tfrac{1}{2})\,GH = \tfrac{2}{3}GH \; \therefore GO = \tfrac{1}{3}GH$$

Hence, the equilibrium is unstable, and therefore the beam will roll partly over, though displaced through only a very small angle. When such a position is reached that the diagonal A C becomes vertical, the body will slightly oscillate till the resistance of the fluid destroys its motion, when it will firmly settle in a position of stable equilibrium with the diagonal A C vertical. For it is plain that when this position is arrived at, the centre of gravity G of the body, and the centre of gravity g of the displaced fluid, are again in the same vertical; but the inertia of the rolling mass will cause the axis C A to incline a little beyond the vertical position, the centre of gravity g of the displacement will thus move a little towards the right, as in the former case, and g being now nearer to G, the vertical from g' will meet the slightly inclined axis above G, so that the new position of equilibrium into which the body finally settles will be stable. (See next figure.)

In reference to the foregoing investigation, it may be proper here to remark, that the student is not to infer that the resulting determination of the metacentre is merely a close approximation to its true position from our having regarded θ as deprived of appreciable value. The metacentre has been defined to be a point where certain two lines intersect: these two lines approach nearer and nearer to coincidence as θ diminishes. Now if θ, from any finite value, continually diminish, the point of intersection referred to will move along the axis, and will cease to move and become stationary only when θ becomes zero; that is, when the two intersecting lines actually coincide. It is this extreme limit of the intersections that is, in strictness, the metacentre. It is rigidly and accurately determined, from any general investigation in which θ is assumed to be a small value, only in the extreme case of that hypothesis; that is, only when θ has diminished down to zero. It will be seen, by re-examining the operations above, that the final result strictly follows when θ becomes zero; or when O is the extreme limit of the intersections of the line of support with the axis H K. The metacentre, in reference to a displacement in a given direction, is thus a fixed point on the vertical through the centre of gravity; and has nothing to do with extent of disturbance. When the body is at rest, the vertical through the centre of gravity of the body, and that through the centre of gravity of the fluid displaced, become one and the same line: if the equilibrium be disturbed, these verticals separate, and the point about which, in this separation, they *begin* to turn is the metacentre.



Fluids Communicating through Bent Tubes.—In speaking of the Levelling Instrument at page 168, we have regarded it as a vessel of fluid; and from the proposition which suggested a reference to it, have inferred that the two surfaces of the bent tube were horizontal or level: but the following direct proof that the surfaces of the fluid in any bent receptacle are necessarily horizontal, is deserving of notice for its simplicity. We take it from the "Cours Elémentaire de Mécanique" of De Launay.



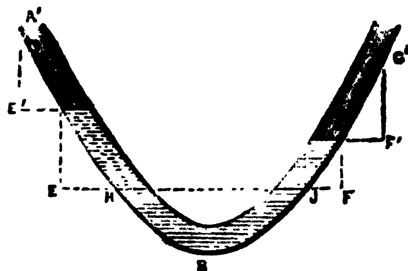
Let A B be two points taken in the interior of the fluid, and on the horizontal line wholly in the channel of communication A B, between the two ascending portions of the bent tube; these two portions being of any relative bulk whatever.

In order that the two points A and B may be at rest, the pressures at A and B must

be equal, and must be the same as the pressure upon every point of the line A B. Now the pressure on A is equal to the weight of the vertical column of particles C A: the pressure on B, on account of the obliquity of the arm B M, is not so easily found. It may be obtained as follows:—The pressure on B is equal to the pressure on D, augmented by the weight of the column of particles D B. The pressure on D is the same as that on E; but the pressure on E is equal to the pressure on F, augmented by the weight of the column of particles F E: consequently the pressure on B is equal to the pressure on F, augmented by the weight of the two columns of particles D B, F E. Proceeding in this way, and observing that the pressure on M is nothing, we see that the pressure on B is equal to the weight of the five columns of particles D B, F E, H G, K I, M L. And since the pressures on A and B are equal, it follows that the five vertical lines D B, F E, H G, K I, M L, are together equal to the single vertical line C A. Consequently every point in the surfaces at C and M is at the same vertical distance from the horizontal plane passing through the two points A, B: these surfaces are therefore horizontal.

In order that the foregoing phenomenon may always be exhibited, it is of course necessary that the fluid introduced into the tube be of uniform density, or that equal vertical columns of it press with equal weights. When two distinct fluids, which do not intermix, balance in a bent tube, or in any two vessels having a channel of free communication with each other, the proportion becomes modified, as follows:—

If two fluids which do not intermix are in equilibrio in a bent tube, the heights of their surfaces above the horizontal plane, where the one fluid rests upon the other, are inversely as their special gravities.



Let A B C be the bent tube, with a fluid A H of specific gravity S resting upon another fluid H B C of specific gravity S′, the horizontal plane of their separation being at H.

The pressure of the fluid A H on the horizontal plane H is $H \times AE \times Sg$; the pressure of the fluid C J, upward on the same plane, is $H \times CF \times S'g$; and the plane H sustains only the upward pressure communicated by C J, since, H J being horizontal, if C J were removed, there would be no upward pressure on H.

Hence, as the fluids are at rest, the pressures on H must be equal.

$$\therefore H \times AE \times Sg = H \times CF \times S'g \quad \therefore AE \times S = CF \times S', \text{ or } \frac{AE}{CF} = \frac{S'}{S}$$

that is, the heights of the surfaces A, C, above the horizontal plane of separation H J, are inversely as the specific gravities of the fluids A H, C J.

If $S' = S$, we then have the case of a single uniform fluid before considered, and the result gives $A E = C F$; that is, the surfaces are at the same height above a horizontal plane, or are level. Suppose above A and C, two other fluids of specific gravities S'', S''', to be introduced into the tube, and that the equilibrium remains undisturbed; then the new fluid in C′ C communicates to the horizontal plane A the only upward pressure it receives, and this by hypothesis is balanced by the downward pressure of the new fluid in A′ A. Hence, as before,

$$A'E' \times S'' = C'F' \times S''' \quad \therefore AE \cdot S + AE'' \cdot S'' = CF \cdot S' + C'F' \cdot S'''$$

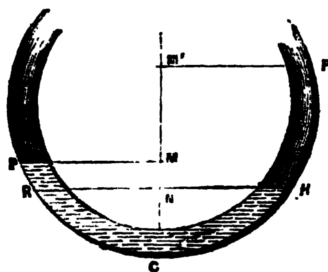
and so on for any number of superposed fluids in equilibrio. Hence, if any number of fluids be in equilibrio in a bent tube, the sum of the products of the vertical heights into the specific gravities of the fluids in one branch of the tube, will be equal to the sum of the corresponding products in the other branch. The vertical height of each fluid is estimated from the horizontal plane on which it rests; the height of the lowest fluid H B C is estimated from the lowest point B; the vertical distance of B from H is the height in one branch of the tube; and the vertical distance of B from C is the height in the other branch.

In illustrations such as the above, the tubes are always represented as circular; but, as the reasoning shows, the shape of the channel which connects the two upper surfaces of a fluid together does not enter into consideration. A pool of water connected by underground open passages, however tortuous, with an adjacent river, will, on the above principle, have its surface on the same level as that of the river. Also, water conveyed by pipes from a reservoir can never deliver a supply to any place on a higher level than the surface of the source, without the application of mechanical force or pressure.

The student must be apprised that in discussing the circumstances of the equilibrium of fluids in tubes, though the shape of the tube is of no moment, nor yet the sectional area of the branches at any height, provided only that these exceed a certain amount of smallness; yet if the section at any part be so small as to bring into operation what is called capillary attraction, the fluid surfaces in the two branches will not stand at the same level. If the section be circular, the diameter of it should exceed $\frac{1}{8}$th of an inch, in order that capillary attraction may not oppose that of gravity, and thus lessen downward pressure. We shall devote a short article to this curious subject presently.

PROBLEM.—Equal lengths of two fluids which do not intermix, and whose specific gravities are as m to 1, are poured into a uniform circular tube; required the position in which they will rest.

Let P C H be the heavier fluid, and P′H the lighter, the lengths of the arcs PH, P′H being equal; let a represent each of these lengths; and draw P M, P′M′ each perpendicular to the vertical diameter, as also H N R.



Then the lighter fluid P′H is upheld entirely by the pressure of the portion P R of the heavier fluid; hence the downward pressure of the lighter fluid on the horizontal plane H must be equal to the upward pressure of the portion P R of the heavier fluid, transmitted to that plane. The heights of these equilibrating portions of fluid are respectively M′ N, and M N; and as these are inversely as their specific gravities, from what is shown above, we have

$$M'N : MN :: m : 1 \therefore M'N = m . MN \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

Let the arc C P be represented by $x \therefore CH = a - x$, and $CP' = a - x + a = 2a - x$, then M′N and M N may be found in terms of a and x as follows, observing that

$$M'N = M'C - NC$$

$$\text{and } MN = MC - NC.$$

$$M'C = \text{ver sin } (2a - x), \; NC = \text{ver sin } (a - x), \; MC = \text{ver sin } x;$$

$$\therefore M'N = \text{ver sin } (2a - x) - \text{ver sin } (a - x),$$

$$MN = \text{ver sin } x - \text{ver sin } (a - x);$$

$\therefore$ (1) ver sin $(2a - x)$ — ver sin $(a - x) = m$ ver sin $x - m$ ver sin $(a - x)$ . . (2)

Or, since ver sin $=$ R — cos, where R is the radius of the circle, we may use cosines instead of versed sines; and reduce the geometrical to the ordinary trigonometrical cosines, that is, to the scale R $= 1$; we shall only have to regard a and x as standing, not for lengths of arc, but for the *degrees* in those lengths; we shall thus have from (2)

$$1 - \cos(2a - x) - 1 + \cos(a - x) = m - m\cos x - m + m\cos(a - x);$$

$$\text{that is, } \cos(a - x) - \cos(2a - x) = m\cos(a - x) - m\cos x;$$

$$\therefore \cos(2a - x) + (m - 1)\cos(a - x) = m\cos x;$$

that is,

$$\cos 2a\cos x + \sin 2a\sin x + (m - 1)(\cos a\cos x + \sin a\sin x) = m\cos x;$$

$$\therefore \cos 2a + \sin 2a\tan x + (m - 1)(\cos a + \sin a\tan x) = m;$$

$$\therefore \tan x = \frac{m - \cos 2a - (m - 1)\cos a}{\sin 2a + (m - 1)\sin a} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

From this expression the trigonometrical tangent of the arc x may be found from the tables; and thence the number of degrees in that arc, or the angle which C P subtends at the centre of the circle.

Let $a = \frac{\pi}{2}$, then $\cos a = 0$, $\cos 2a = -1$, $\sin a = 1$, $\sin 2a = 0$, and the expression in these circumstances becomes,

$$\tan x = \frac{m + 1}{m - 1} \quad . \ . \ . \ . \ . \ . \ . \quad (4).$$

If $m = 1$, that is, if the two fluids be the same, then (3) gives

$$\tan x = \frac{1 - \cos 2a}{\sin 2a} = \frac{1 - (1 - 2\sin^2 a)}{2\sin a\cos a} = \frac{\sin a}{\cos a} = \tan a,$$

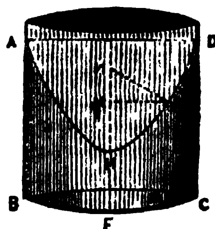
that is, the arcs a and x are equal, as we otherwise know they must be.

Equilibrium of a Rotating Fluid.—In all the preceding investigations the only force supposed to act on the fluid is the force of gravity; we shall give an illustration of a case in which the equilibrium is due to the united effects of gravity and centrifugal force.

PROBLEM.—A cylindrical vessel containing water or any other liquid revolves round its axis, perpendicular to the horizon, with a given angular velocity: to determine the form assumed by the internal surface of the fluid.

The rotation of the vessel necessarily puts the contained fluid in motion, and this motion continues till all the forces balance and the fluid attains a fixed form and position; it is in this state of equilibrium that we have to consider it.

Suppose A H D to be the form assumed by the surface of the fluid when the state of equilibrium is attained, and P any particle on that surface. From P draw P M perpendicular to the axis of the cylinder, and P N perpendicular to the curve at P. The forces which keep P in equilibrio are—1. Gravity, acting in the vertical direction N M; 2. The centrifugal force, acting in the horizontal direction M P; and 3. The pressure or reaction of the particle in the direction P N perpendicular to the surface; for it is plain that the vertical and horizontal forces acting on P are the only forces which cause the normal pressure of P on the surface H P D; so that this normal pressure is the resultant of the other two, and therefore, taken in the opposite direction P N, neutralizes the former and keeps P at rest.



Let α be the angular velocity of rotation, that is, the angle turned through by M P or F C in a second of time; then the circular arc turned through by P in a second will be αy, putting y for P M, that is to say, the velocity v, of the point P, is $v = \alpha y$.

Now the centrifugal force acting on P is

$$\frac{v^2}{y} = \frac{\alpha^2 y^2}{y} = \alpha^2 y. \quad \text{(Dynamics, page 145.)}$$

Also (Statics, Prop. IV., page 48),

$$\text{NM} : \text{MP} :: \text{gravity} : \text{centrifugal force} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{that is, NM} : y :: g : \alpha^2 y \therefore \text{NM} = \frac{g}{\alpha^2}.$$

The line N M, thus determined, is called the subnormal to the curve A H D at the point P; and we see that the curve is such that the subnormal at any point is a constant quantity. This property belongs exclusively to the *parabola*. Hence the surface of equilibrium is that generated by the rotation of a parabola about its axis; that is, it is a *paraboloid.*

In reference to the foregoing investigation, it may be useful to the student to attend to the following remarks:—

It will have been perceived that the problem proposed is of a mixed character—partly dynamical and partly statical. The forces first generate motion in P, which point continues to move till these forces become balanced, and P takes a position of rest. The force of gravity g, and the centrifugal force $\alpha^2 y$, both act dynamically, and are therefore to be expressed in *feet*. The symbol α, representing the angular velocity of rotation, is an abstract number:—it is not an *angle*. Here, as in all dynamical inquiries, angular velocity is estimated thus:—A circle, whose radius is represented by 1, is imagined to be described about the centre of motion (about M in the present case); the semicircumference of this circle is 3·1416; and the extent of arc of the same circle, turned through in a second, is α, the angular velocity of the rotation, which is therefore 3·1416 multiplied by some number, whole or fractional. If, therefore, y or M P be measured in inches, the velocity of P is αy inches; if in feet, it is αy feet. As g is measured in feet—viz. $g = 32{\cdot}2$ feet, it is necessary that M P or y should also be measured in feet; otherwise the proportion (1) would be incongruous. The final result, namely, $\text{NM} = \frac{g}{\alpha^2}$, gives the length of NM in feet: it is 32·2 feet divided by the abstract number α^2. Suppose the vessel performed 10 rotations in a second; then the angular velocity would be

$$\alpha = 3{\cdot}1416 \times 20 = 62{\cdot}832 \therefore \alpha^2 = 62{\cdot}832^2 = 3947{\cdot}86$$

$$\therefore \text{NM} = \frac{32{\cdot}2}{3947{\cdot}86} \text{ feet} = \frac{386{\cdot}4}{3947{\cdot}86} \text{ inches.}$$

We see that, with the same angular velocity of rotation, the curve is the same whatever be the magnitude of the vessel: it is further obvious that, as the cylindrical shape of the vessel is no item of consideration in the above solution, the conclusion remains the same whatever the shape may be, provided that the horizontal sections be circles, and the vertical axis of revolution pass through their centres.

Suppose the vessel to be itself in the form of a paraboloid: then the velocity of rotation, necessary to cause all the fluid to run over, may be easily determined. For the vessel will be emptied as soon as every point P has, through the intensity of the centrifugal force, reached the side of the vessel; that is, when the subnormal N M at

any point P is identical with the subnormal at the point where the horizontal line M P meets the side of the vessel.

Calling, then, the constant subnormal of the given parabola l, we have $l = \frac{g}{a^2}$

$\therefore a = \sqrt{\frac{g}{l}}$, the angular velocity per second at which the vessel must rotate in order that the empty paraboloid, formed by the rotation, may coincide with the concavity of the vessel itself.

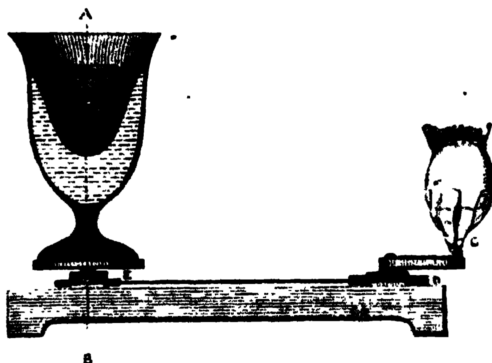
From this expression for the angular velocity, the time in which each complete revolution must be performed may be ascertained thus:—

$$a : 3\cdot1416 :: 1 \text{ sec.} : \frac{3\cdot1416}{a} \text{ sec.} = \text{time of a semi-rotation}$$

$$\therefore \text{time of a rotation} = 2\frac{3\cdot1416}{a} = 3\cdot1416 \times 2\sqrt{\frac{l}{g}} \text{ seconds}$$

the subnormal l being measured in feet, and g being 32·2 feet. It appears, therefore, that with this rate of rotation, the fluid will rise and run over the vessel, till only what covers and adheres to the sides is left.

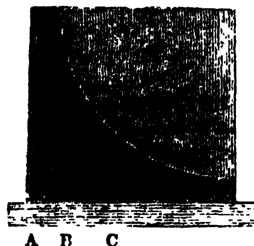
The annexed figure will illustrate the manner of communicating rapid rotatory motion to a vessel of water. D and E are two pulleys, round which passes an endless cord; by means of a winch C, the vessel may be made to rotate round its vertical axis A B as rapidly as we please.



Capillary Attraction and Repulsion.—The remarkable phenomena of capillary attraction and repulsion has already been described and illustrated at pages 19 and 24 of the present volume; and little more than mere description and illustration can, with propriety, be given in this work. Common observation shows that the surfaces of solids in general, unless they are unctuous surfaces, attract the particles of water: this is more especially noticeable in glass. If a plate of glass be dipped in water, and then gently withdrawn, a line of fluid particles will be seen suspended from the lower edge, and the whole surface will be *wetted*, showing that there exists an attraction between glass and water.

It is to this attraction that the rise of water in a capillary tube is attributed: the bore of such a tube being very minute, from $\frac{1}{20}$th to $\frac{1}{50}$th of an inch in diameter, the thread of water is so slender that the surface of glass, above its upper extremity, may exercise attractive force sufficient to raise the upper particle, when the contiguous particles must follow to fill up the vacuum; this lengthening of the thread increasing till gravity, or the weight of the little column, counterbalances the capillary attraction. It is observed that the height to which the water is raised in a capillary tube, is inversely as the diameter of the tube. And this fact is sufficient to teach us the form

of the curve which the upper surface of water assumes when acted upon by the two glass plates which embrace it, as mentioned at page 20.



For the distance between the plates at B, is to the distance between them at C, as the height of the liquid at C, to the height at B. Hence, putting $AB = x$, and $AC = x'$, and the corresponding heights y and y', and remembering, from Euclid Prop. 2, Book VI., that the distances between the plates at B and C are as AB to AC, we have

$$x : x' :: y' : y \therefore xy = x'y'$$

that is, the rectangle xy is a constant quantity, wherever B may be within the limits of capillarity. As this is the distinguishing property of the hyperbola, the axes of reference being the asymptotes, we infer that the curve presented by the upper surface of the fluid interposed between the plates, is an hyperbola. The curve ceases, of course, to be that of the hyperbola beyond the limits of capillarity: at a distance from the junction of the plates, at which the interval between them exceeds about $\frac{1}{20}$th of an inch, the curve will terminate: there will still be an elevation of the liquid up the side of each plate, but the line traced by its surface will become a straight line.

Where the distance between the plates is $\frac{1}{200}$th of an inch, the fluid, if water, is observed to rise 5 inches; and therefore where the distance between the plates is $\frac{1}{100}$th of an inch, the rise of the water is $2\frac{1}{2}$ inches. The elevation of the liquid is found to be quite independent of the thickness of the glass: if the bore of the tube, or the distance between the interior surfaces of the plates, be the same, the elevation is unaffected by the thickness, so that the quantity of matter in the glass does not contribute to the ascent of the fluid: and if the interior surfaces of the plates, or the bore of the tube, be coated with a film of oil, the capillary attraction is destroyed. On this account it has been inferred that the attraction is exerted entirely by the surface of the glass, and that the liquid must be in complete contact with that surface for the phenomenon to take place: the attractive energy, it would seem, has no penetrating power, it being stifled and rendered inoperative by the interposition of even the thinnest lamina of oil or grease.

It may be remarked, too, that the elevation of the fluid is not near so great between plates, as in a tube whose diameter is equal to the distance of the plates: the rise is something less than half as great in the former case as in the latter. But this might be expected, as in the tube the thread of fluid is completely surrounded by the glass surface; the same law, however, namely, that the height of the fluid is inversely proportional to the distance between the plates, is observed to have place.

The fluids which exhibit the phenomena here noticed are exclusively the watery fluids, all of which rise to different heights in the same tube; and some of the heavier rise higher than the lighter: spirits of wine, for instance, rises only about $\frac{2}{5}$ths as high as water, which of all liquids rises to the greatest height. Mercury, however, does not rise at all; on the contrary, the tube and the plates seem to exercise a repellant energy, and cause the mercury to descend: this is called *capillary repulsion.*

The diameter of a capillary tube is not found by actual measurement; the method recommended is this:—Put a known weight w of grains of mercury into the tube, and let the length of tube it occupies be l inches; then representing the diameter of a trans-

verse section of the tube by d, we have for the volume of the mercury $d^2l \times \cdot 7854$; and as a cubic inch of mercury, when pure, weighs 3443 grains, we have

$$1 : d^2l \times \cdot 7854 :: 3443 \text{ grains} : w \text{ grains};$$

$$\therefore d = \sqrt{\left(\frac{w}{l} \cdot \frac{1}{3443 \times \cdot 7854}\right)} = \cdot 09123 \sqrt{\frac{w}{l}} \text{ inches.}$$

Whatever the force called capillary attraction and repulsion may be, it is plain that it cannot be analogous to that so universally diffused throughout the material universe, and called the force of gravitation; because the intensity of *this* influence increases with the mass of the body exercising it: whereas the capillary influence is more like that of *electricity*, which, when in a state of equilibrium, seems to be confined to mere surfaces. It appears not improbable, therefore, that capillarity may be a peculiar manifestation of electric force exerted by the surface of the glass on the particles of liquid in its immediate neighbourhood. For an account of experiments to prove the "outsidedness" of electricity, the student is referred to the volume on ELEMENTARY CHEMISTRY.

Conclusion.—We here terminate the elementary treatise on Hydrostatics, which, like the preceding treatise on Dynamics, has been prepared exclusively for those whose knowledge of pure mathematics is limited within the range of the volume on the MATHEMATICAL SCIENCES in the present series of treatises. In the tract now brought to a conclusion, we have been able to enter a little more fully into the subject proposed than we could with propriety do into the matter fairly coming under the head of the former topic; because in statics generally, as well of fluids as of solid bodies, there is less demand upon the higher calculus than in Dynamics. But in both treatises our principal object has been to convey clear conceptions of fundamental principles, and more especially to impress an intelligible signification upon every symbol employed. We have always thought that if thus much be faithfully performed in any department of science, the elementary instructor may withdraw his aid, and may safely leave his pupil to his own efforts, and the ordinary books, in the further prosecution of his inquiries.

It is an unfortunate feature in many books on applied science, otherwise valuable, that the preliminary matters here noticed are too hastily disposed of. The learner is often left to grope his way to the precise meanings of the symbols he uses in an indirect and circuitous manner, without any real assistance from the writer who undertakes to guide him. And the consequence is that he is often compelled to regard the investigation of a problem in physical science merely as an algebraical exercise, in which symbols, and symbols alone, are alike the instruments and the subject-matter before his mind. Quantities concrete and abstract—weight and time—volume and space—seem to him so jumbled together, in one and the same incongruous expression, that he ignores the applicability of his formula to any material reality; and, in obedience to the dictates of common sense, is constrained to regard it as involving mere mathematical abstractions, having probably some mysterious connection with the outward objects of sense; but what connection, it is beyond his penetration to discover.

The student of the physico-mathematical sciences must vigilantly contend against the propensity, in dealing with his symbols, to leave out of sight the things signified: he had better abandon an investigation altogether, than prosecute it beyond the step at which it ceases to be intelligible in reference to the purpose for which it was undertaken.

HYDRODYNAMICS.

HYDRODYNAMICS treats of the motion of incompressible fluids, and, as the name implies, more especially of *water*. The consideration of compressible or elastic fluids, whether in a state of equilibrium or of motion, comes under the head of *Pneumatics*, a subject to which the conclusion of the present treatise will be devoted. But from our limited space we are precluded from discussing these remaining topics at any great length. This, however, is scarcely to be regretted, for the physical laws which regulate the motions of fluids are so imperfectly ascertained, that, except in those few particulars which constitute the mere elements of the subject, the investigations of hydrodynamics, founded on hypothetical data, are of little practical value; and sometimes even lead to results clearly contradicted by experiment. We shall, therefore, confine ourselves in this brief summary to those elementary portions of the mathematical theory which actual experiment confirms, or where the practical obstacles to such confirmation are clearly ascertained, and may be estimated and allowed for.

If water run through a pipe or other channel, always filling it, the velocity of the fluid at any part will be inversely as the area of the transverse section of the channel at that part. For let A, A′ represent the area of any two transverse sections, the water being supposed to run from A towards A′, then whatever quantity passes through A, in a given time the same quantity must pass through A in that time; since if a less quantity passed through, then the fluid between the sections, always remaining full, would be condensed; and if a greater quantity passed through, the fluid between the sections would be expanded, as the channel is always full. As each consequence is incompatible with an incompressible fluid, it follows that equal quantities of fluid must pass through A and A′ in the same time.

If, therefore, the velocity of the stream through A be v feet per second, and the velocity through A′, v' feet per second, we shall have

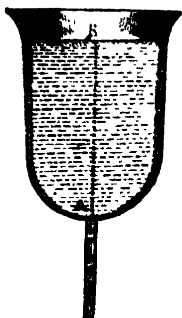
$$Av = A'v' \therefore A : A' :: v' : v;$$

that is, the velocities are inversely as the areas of the sections.

Spouting of Fluids.—The velocity with which a fluid issues from a small orifice in the bottom or side of a vessel, kept constantly full, is equal to the velocity that would be acquired by a heavy body falling freely through the height of the surface of the fluid above the hole.

Suppose the orifice A to be at the bottom of the vessel, then the thin lamina of fluid covering the orifice A is forced out by the action of gravity upon it, and by the superincumbent pressure of the column of fluid whose base is A and altitude A B. Conceive the column A B to consist of n such laminæ, thus supposed to be acted upon at A; that is, calling the thickness of the lamina 1, let A B $= n$; then before motion, the statical

forces or pressures on A are its own weight, and the weight or pressure of the column A B, and these are as 1 to n; consequently the dynamical effects of these forces, when motion takes place, must be also as 1 to n. Now the dynamical effect, that is the velocity generated in the lamina A, in the time of falling through its own thickness 1, regarding this time as the unit of time, is 2 (DYNAMICS, page 135): hence



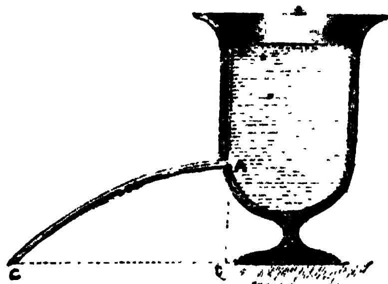
$$1 : n :: 2 : 2n;$$

therefore $2n$ is the velocity generated in the lamina A by the column of fluid in the assumed unit of time; and as this time is indefinitely small, it must be the velocity which the fluid has at the orifice itself, or that with which it actually spouts out; and as the vessel is kept constantly full, the initial circumstances continue invariable, so that the fluid is discharged with the same uniform velocity.

Now, if a heavy body fall freely through a height h by the action of gravity it acquires during the time of fall a velocity $v = 2h$, that is, a velocity that would carry it through $2h$ in the same time (DYNAMICS, page 134). Consequently the fluid issues from the orifice A with a constant velocity equal to that which a heavy body would acquire in falling through the height B A.

If this height be h feet, the accelerating force of gravity being $g = 32{\cdot}2$ feet, we have, for the constant velocity of the spouting fluid, $v = \sqrt{2gh}$, the velocity per second.

The fluid spouts out with the same velocity whether the small orifice be in the side or in the bottom of the vessel, provided only that in both cases the hole be at the same depth below the surface of the fluid, since the pressure of a fluid is the same in all directions at the same depth.



The stream being, as it were, thus projected with a uniform velocity v, the curve it will describe in issuing from the side of a vessel (as in the margin) will be a *parabola*, of which the directrix is the horizontal line through B the surface of the fluid (DYNAMICS, page 141). The hole through which the fluid spouts must be so small as to render the pressures at different parts of it insensibly different from that at the centre of the orifice.

Quantity of Fluid Discharged per Second.—As the velocity is constant, the quantity of fluid discharged per second is found by multiplying the area of the orifice by the length $\sqrt{2gh}$; thus, if a be the area of the orifice, the discharge per second is $a\sqrt{2gh}$ cubic feet. Let $h = 20$ inches, and $a = 1$ square inch, then

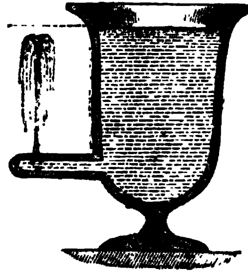
$$a\sqrt{2gh} = \sqrt{(2 \times 32{\cdot}2 \times 12 \times 20)} = \sqrt{15456} = 124{\cdot}3 \text{ cubic inches.}$$

Since $a\sqrt{2gh}$ is the discharge per second, in t seconds a quantity Q will be discharged, such that

$$Q = at\sqrt{2gh} \therefore t = \frac{Q}{a\sqrt{2gh}} \quad . \quad . \quad . \quad . \quad (1)$$

This, therefore, expresses the time in which a proposed quantity Q of the fluid will be delivered, Q being expressed in cubic feet.

The Vena Contracta.—The foregoing conclusions respecting the velocity with which a fluid spouts from a small hole in the vessel containing it, and the quantity of the fluid delivered in a given time, have been compared with many carefully conducted experiments. The most satisfactory way of ascertaining practically the velocity of the discharge, is by observing the altitude to which the fluid spouts when forced to take an upward direction, as in the accompanying figure. If the uniform velocity of the issuing fluid be that due to the fall of a heavy body from the surface to the orifice, the fluid ought to spout up as high as that surface; and such is found to be very nearly the case. We might expect that the theoretical result would not be rigidly fulfilled, but that the height reached would fall a little short of the surface of the fluid in the vessel; because friction and the resistance of the air would act as opposing forces. But there is another cause in operation which must now be noticed.



When a passage is made in a vessel for the exit of the fluid contained in it, the equilibrium of the entire mass is disturbed, and motion takes place among all the particles. The instant that the plug is removed, the velocity is due to the direct pressure upon it; but as lateral and oblique pressures all round the hole cause the neighbouring fluid to flow sideways towards the main stream, when this surrounding fluid reaches the aperture it does so with a certain velocity transverse to the direction of the main stream, which, in consequence, becomes slightly contracted a little beyond the opening. This contraction of the fluid vein is called by Newton the *vena contracta;* and it is through the section of the vena contracta that the fluid flows, as we have supposed it to flow through the orifice itself. The distance of this section from the orifice is nearly equal to the radius of the orifice, and its area about $\frac{5}{8}$ths of that of the orifice.

If, therefore, the orifice be small—as, for instance, a gimlet-hole in a cask—the confounding the orifice with the vena contracta can occasion but very little error in the height or distance to which the fluid spouts; but a considerable departure from the truth will result from calculating the quantity of fluid delivered in any time on the supposition that the sectional area of the spouting stream is A instead of $\frac{5}{8}$ A; for instance, in the example above the quantity of fluid discharged per second, instead of being 124·3 cubic inches, is only 77·7 cubic inches.

If the vessel be not kept full, and the surface be therefore allowed to descend, the velocity with which the descent commences will be to the velocity with which it passes through the vena contracta as the area of the section at the latter to the area of the surface (page 195).

Since the curve which the fluid spouting from a small hole in the side of the vessel assumes is that of a parabola—the same curve, in fact, that is described by a projectile impelled in the same direction and with the same velocity as the fluid at the vena contracta—everything connected with extent of range, time of describing it, greatest altitude, &c., may be determined as in the theory of projectiles: the following are a few illustrations:—

If the fluid spout from a point in the middle of the upright side of a full vessel, the

horizontal range will be the greatest; and from points at equal distances above and below the middle point the ranges will be equal.

Let A B be the upright side of the vessel perforated at the middle point C, and also at two points D, E, equidistant from C.

The velocity with which the fluid issues from one of these points, as D, is that which would be acquired in falling through B D. Since gravity acts during the whole fall of the fluid to P, a particle arrives at P in the time that a body would fall through D A.

Hence the particle at D has a horizontal velocity that would carry it forward through a space equal to 2 B D in the time that a body would fall from B to D; consequently we have only to find how often this time is contained in the time of falling from D to A and to multiply 2 B D by the resulting number, in order to get the horizontal range A P. Now the times of falling through any spaces are as the square roots of those spaces (DYNAMICS, page 134), therefore the required multiplying number is $\sqrt{\frac{DA}{BD}}$: consequently,

$$\text{Range AP} = 2BD \times \sqrt{\frac{DA}{BD}} = 2\sqrt{BD.DA}.$$

If, therefore, with centre C a semicircle be described upon A B, and the perpendicular D F be drawn, the range of A P will be twice the length of this perpendicular. (Euc. XIII. of VI.) [In the figure, P should be more to the right.]

In like manner, the range of the fluid spouting from E will be twice the length of the perpendicular E K; and as these perpendiculars are equal, being equidistant from C, the ranges from D and E are equal.

As the perpendicular C L from the centre C is the longest that can be drawn from A B to the arc of the semicircle, the range AR = 2CL, of the fluid spouting from C, is the greatest of all ranges.

These theoretical results are fully confirmed by experiment when the orifices in the vessel are small, so that the vena contracta may be sufficiently near the vessel to be regarded, without sensible error, as at the orifice itself.

As the perpendicular D F is the sine of the arc B F, we see from above that the horizontal range from any orifice is twice the sine of the arc of a circle whose diameter is the depth of the fluid, and whose versed sine is the depth of the orifice. By the depth of the fluid may be understood the depth of the horizontal plane on which the range is measured below the upper surface of the fluid; for whether the bottom of the vessel be in this plane, or be raised above it, makes no difference: the range from any orifice, measured on any horizontal plane, is the same, provided the distance between that orifice and the surface be the same, whether the fluid extend down to that plane or not.

PROBLEM.—A fluid issues from the side of a vessel through an oblique jet: to determine the horizontal range.

Let a be the angle of direction of the jet above the horizontal line, and h the height of the surface of the fluid above the orifice: then (DYNAMICS, page 143) the range r on the horizontal plane through the orifice will be $r = 2h \sin 2a$. This is greatest when $\sin 2a$ is greatest, that is, when $a = 45°$, the range then being $R = 2h$.

The velocity with which a fluid spouts from a small orifice, as the surface of the fluid descends, varies as the square root of the height of the surface above the hole.

For the velocity at any instant is always equal to that acquired by falling from the surface to the orifice, and this velocity varies as the square root of the space fallen through: hence the velocity varies as the square root of the height of the surface above the hole. The velocity, therefore, is retarded as the surface descends, but not uniformly retarded unless the horizontal sections of the vessel are all equal in area; for the more extensive the section—or the surface of the fluid—at any instant of the descent, the more slowly will it increase its distance from the top of the vessel, and the less, therefore, will the velocity of the issuing fluid be retarded.

But if the horizontal sections of the vessel be all equal, then the velocity of the issuing fluid, as likewise the velocity of the descending surface, will be uniformly retarded.

For let V be the velocity of the descending surface at any instant, and v that at the orifice: let also the areas of the surface and orifice be A and a. then (page 195)

$$V : v :: a : A \therefore V = \frac{a}{A} v.$$

Now, as shown above, v varies as the square root of the space s between the descending surface and the orifice. we may put, therefore, $c\sqrt{s}$ for v, and write

$$V = \frac{ac}{A}\sqrt{s} = \sqrt{\frac{a^2c^2}{A^2}s} = \sqrt{2fs}$$

by putting $2f$ for $(\frac{ac}{A})^2$. As this agrees with the general expressions for the velocity in uniformly accelerating or retarding forces (DYNAMICS, page 134), we infer that the velocity V is uniformly retarded; and since v is V multiplied by an invariable number, we conclude that v, the velocity of the spouting fluid, is also uniformly retarded. Consequently, the volumes or quantities of fluid discharged in equal times decrease as the numbers 1, 3, 5, 7, &c., taken in reverse order. (DYNAMICS, page 134.)

If a vessel of uniform horizontal section be kept constantly full, then twice the quantity which the vessel holds will run out from a hole in the bottom, in the same time that the vessel would empty itself.

For the surface of the descending fluid is uniformly retarded as the fluid runs out, and when it arrives at the bottom the velocity is destroyed: the space which the surface *would* describe in the same time, with the first velocity of it uniformly continued, would be twice the actual space described; and the quantity discharged in any time when the vessel is kept constantly full, is the same as what *would* be discharged if the surface descended uniformly with the first velocity: hence the quantity discharged in the one case is double that discharged in the other.

PROBLEM.—To find the time in which a vessel of uniform horizontal section will empty itself through a small orifice in the bottom.

Let A be the area of any horizontal section of the fluid, h the height of the vessel, and a the area of the orifice. Then the quantity of fluid that would be discharged in the time t of emptying, if the first velocity were to continue uniform, would be twice

the contents of the vessel; that is, the quantity would be $2\mathrm{A}h$, so that if v be the velocity with which the fluid would then uniformly issue, we should have

$$atv = 2\mathrm{A}h \quad \therefore t = \frac{2\mathrm{A}h}{av}$$

$$\text{But page 196, } v = \sqrt{2gh} \therefore t = \frac{\mathrm{A}}{a} \times 2\frac{h}{\sqrt{2gh}} = \frac{\mathrm{A}}{a}\sqrt{\frac{2h}{g}}$$

which therefore expresses the number of seconds occupied in emptying the vessel through the orifice.

It follows from this expression for t, that the times of emptying cylinders or prisms through equal orifices in their bases, vary as $\mathrm{A}\sqrt{h}$. If the bases of the vessels be equal in area, then the times vary as $\sqrt{h}$. If the altitudes are equal the times vary as the bases. The time of emptying any altitude $h - h'$ of the vessel is found by subtracting from the time of emptying the whole height h, the time of emptying the height h': thus, since

$$\left.\begin{aligned} t &= \frac{\mathrm{A}}{a}\sqrt{\frac{2h}{g}} \\ \text{and } t' &= \frac{\mathrm{A}}{a}\sqrt{\frac{2h'}{g}} \end{aligned}\right\} \therefore t - t' = \frac{\mathrm{A}}{a}(\sqrt{h} - \sqrt{h'})\sqrt{\frac{2}{g}} \quad . \quad . \quad . \quad . \quad (1)$$

the number of seconds elapsed during the descent of the surface of the fluid from the height h to the height h'.

The time in which the surface would descend from the height h to the height h', or the time in which the fluid occupying this interval would run out, if the vessel were kept constantly full, by equation (1) page 196, is

$$\frac{\mathrm{Q}}{a\sqrt{2gh}} = \frac{\mathrm{A}(h - h')}{a\sqrt{2gh}} \quad . \quad . \quad . \quad . \quad (2).$$

Hence the time of discharging the proposed quantity Q, when there is no supply from without, is to the time of discharging the same quantity when the vessel is kept constantly full, as

$$\frac{\mathrm{A}}{a}(\sqrt{h} - \sqrt{h'})\sqrt{\frac{2}{g}} : \frac{\mathrm{A}}{a} \cdot \frac{h - h'}{\sqrt{2gh}}, \text{ or as } 2(\sqrt{h} - \sqrt{h'}) : \frac{h - h'}{\sqrt{h}}$$

$$\text{or as } 2 : \frac{\sqrt{h} - \sqrt{h'}}{\sqrt{h}}, \text{ or } 2 \,.\, 1 - \sqrt{\frac{h'}{h}}.$$

If the quantity Q be the whole contents of the vessel—that is, if h' be zero—then the times of discharging the vesselful of fluid in the two cases are as 2 to 1, as already inferred at page 199.

In order to test the accuracy of the theory by experiment, (1) is the most suitable formula for the purpose, since the time occupied by the surface in descending from one level to another can be correctly observed; but the instant when the last portion of fluid escapes cannot be accurately noted, for when the vessel is near exhaustion, the fluid ceases to fill the orifice, and is discharged in drops from the edges of the hole. The time occupied by the surface descending through a certain space is found to agree very closely with the theoretical expression for it.

The Clepsydra.—The clepsydra, or water-clock, is a contrivance for measuring time by the descent of the surface of water in a vessel, the water flowing through an orifice at the bottom. The most convenient form for the vessel is that in which the surface descends through equal vertical spaces in equal times. This form may be determined as follows:—

Let the altitude of the surface at any instant be x, and the radius of the surface, that is, the corresponding ordinate of the generating curve, y. Then the velocity at the orifice at that instant is $\sqrt{2gx}$ (page 196), and the area of the descending surface πy^2, where $\pi = 3{\cdot}1416$. The velocity V of the descending surface is found by the theorem at page 195, which gives

$$\pi y^2 : a :: \sqrt{2gx} : \mathrm{V} = \frac{a}{\pi y^2} \sqrt{2gx}$$

where a is the area of the orifice.

Since the surface is to descend uniformly, V must be a constant quantity, which can be suitably assumed in reference to the whole time of emptying: putting c for thi constant, we have

$$c = \frac{a}{\pi y^2} \sqrt{2gx} \quad \therefore y^4 = \frac{2a^2 g}{\pi^2 c^2} x.$$

This, therefore, is the equation of the curve, the rotation of which about its vertical axis x will generate the surface of the vessel. This generating curve is a parabola of the fourth order.

But the surface need not be a surface of revolution, or one having all its horizontal sections circles. They may all be rectangles. Let the sides of the rectangular section, at the altitude x from the base, be y and p, then the area of that section is py, and we have

$$c = \frac{a}{py} \sqrt{2gx} \quad \therefore y^2 = \frac{2a^2 g}{p^2 c^2} x,$$

so that if p be the same for every section—that is, if the rectangular sections have all the same breadth—the curve bounding the vessel towards the lengths of these rectangles will be the common parabola.

The Resistance of Fluids.—All bodies moving in a fluid are impeded in their progress by the resistance of the fluid, which must itself be moved, in order that motion may take place in the immersed body. The resistance to motion in the fluid is of course mainly due to its inertia; friction and the tenacity of the fluid add somewhat to the resistance, but the retarding influence of these is too inconsiderable to render the consideration of them of much practical consequence.

It may safely be assumed that the resistance on a plane surface, moving with a given velocity through stagnant water, is the same as when, the plane being at rest instead of the water, a stream moving with the same velocity acts against *it*.

If a stream act perpendicularly against a plane surface, or if the surface act against the quiescent fluid by moving perpendicular to its surface through the fluid, the resistance will be as the area of the plane, the density of the fluid, and the square of the velocity conjointly.

For the velocity v of the stream being regarded as uniform throughout the whole depth of the plane, the resistance is the same at every point of the plane, and therefore, other circumstances being the same, it is proportional to the area A of the plane.

But the quantity of fluid matter striking against the plane in a given time is proportional to the density and velocity of the fluid conjointly; and this quantity of matter strikes with a velocity v, and therefore with a momentum equal to

quantity of matter $\times v$.

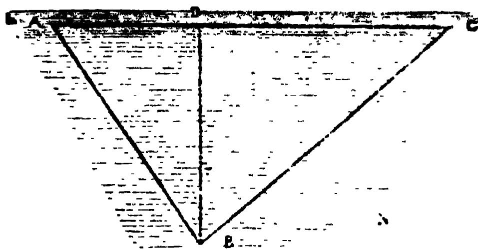
Hence, D being the density of the fluid, the resistance of the area A varies as $\mathrm{AD}v^2$.

If a body were to fall by the force of gravity till it acquires the velocity v of the

stream, the space fallen through would be $s = \frac{1}{2}\frac{v^2}{g}$ (DYNAMICS, page 134); this, therefore, would vary as the square of the velocity, so does the resistance ADv^2; hence the resistance varies as the weight of a column of the fluid whose base is the area of the plane, and altitude the space through which a heavy body must fall to acquire the velocity of the stream or of the moving plane.

The force with which a stream acts perpendicularly upon a plane opposed to it obliquely, varies as the square of the sine of the inclination of the plane to the stream.

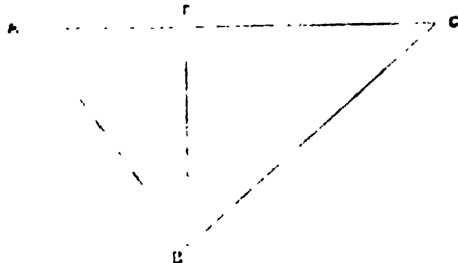
Let A B and L N be sections of the plane and stream, in the direction of the latter; draw A C in that direction, meeting B C perpendicular to A B in C; draw also B D perpendicular to A C.



Now the quantity of fluid acting against A B is the same as that which acts perpendicularly against D B; it varies, therefore, as the length of B D; that is, as the sine of the angle A. If the velocity with which this quantity moves in the direction of the stream be represented by A C, then B C will represent the velocity with which it moves perpendicular to A B; and just as B D varies as the sine of the angle A, so does B C vary as the sine of the angle A; hence the force with which the stream acts perpendicularly upon the plane varies as $\sin^2 A$.

The force impelling the plane in the direction of the stream varies as the cube of the sine of the inclination of the plane to that direction.

Let B C, perpendicular to A B, represent the force on A B perpendicular to it; this may be resolved into two forces B D, D C, the latter D C in the direction of the stream, and the former B D perpendicular to that direction. Now D C, the force in the direction of the stream, varies as BC sin DBC, that is as BC sin A; but, as shown above, BC itself varies as $\sin^2 A$, therefore the force in the direction of the stream varies as $\sin^3 A$.



This, of course, is on the assumption that the other component of the force of the stream in its own direction—namely, the component represented by B A, in the direction of the plane itself—is of no effect.

The force B D, perpendicular to the stream, is as BC cos DBC = BC cos A; hence the force with which the stream impels an oblique plane upwards, in a direction perpendicular to the stream, varies as $\sin^2 A \cos A$.

It must be observed that in the foregoing propositions the plane is regarded as fixed while receiving the force of the stream, or else the fluid is regarded as quiescent and the

plane as moving through it. If both be regarded as moving in the same or in opposite directions, then the velocity spoken of above must be considered as the relative velocity; that is, it is the difference of the velocities if the plane and stream move in the same direction, and the sum of the velocities if they move in the opposite direction. Thus, if V be the velocity of the stream, and v the velocity of a float-board of an undershot water-wheel, then the perpendicular pressure on the float-board will vary as $(V - v)^2$.

The student has already been apprised, at the commencement of the present brief tract on the motion of incompressible fluids, that the results of the mathematical theory often require considerable modification to render them accordant with actual experiment. This arises from the difficulty of assigning correct values to all the circumstances attending that motion. We have seen, in treating of fluids spouting from an orifice, that observation has discovered that the orifice is not the narrowest channel through which the stream uniformly flows, and consequently that the velocity there is less than at the vena contracta, which ought therefore to be regarded as the true orifice.

It was found by Newton that the velocity at the vena contracta is to the velocity of the orifice as $\sqrt{2}$ to 1; and since, in falling bodies, the spaces passed through are as the squares of the velocities acquired, it follows that the space to be fallen through to give a body the velocity with which the fluid spouts from the *orifice*, is only half the space necessary to give the velocity at the vena contracta; that is, the velocity at the orifice would be acquired by a body falling through only half the altitude of the surface above the orifice.

In like manner, in treating of fluid resistances, the commonest observation shows that a stream impinging upon a plane surface cannot have the full effect which theory assigns. The velocity of the stream is to a certain degree impeded by what may be termed the back-water; moreover, the velocity is not in general the same at all depths. Yet the results of theory may usually be applied with safety in the *comparison* of different similar cases: for instance, the comparative results as to quantity of fluid discharged, time of emptying, &c., from equal orifices in two different vessels, disregarding the vena contracta, would accord very nearly with experiment. And, in like manner, fluid-pressures may be relatively estimated and computed from theory: for instance, though theory may assign an erroneous value for the effect of a stream upon the rudder of a ship, yet the position of *greatest* effect as deduced from theory may very well accord with actual observation; so the effect of a stream of water upon the floats of a water-wheel may be incorrectly assigned by theory, and yet the degree in which the velocity of the wheel should fall short of that of the stream, in order that the *greatest* effect may be produced, is found to be verified pretty closely by practical experience.

To determine particular values for the unknown, or arbitrary quantities, which enter into a general expression, so that for those values the expression may be a maximum or a minimum, requires the aid of the differential calculus. But in the two cases mentioned above, the principles to be employed are so elementary, and the knowledge required of that science so trifling, that although in general we are interdicted from using the calculus, we shall give the two problems adverted to as a conclusion to the present portion of our subject. We may premise, however, that in order to determine x so that any expression of the form

$$A + Bx + Cx^2 + Dx^3 + \&c. \quad . \quad . \quad . \quad . \quad (1)$$

may be a maximum, all we have to do is to disregard the term independent of x—that is, the term A—to write down all the other terms, first converting each *exponent* of x into a factor, and writing each exponent a unit smaller; thus

$$B + 2Cx + 3Dx^2 + \&c.,$$

and then to equate this expression to 0: that is, to solve the equation $B + 2Cx + 3Dx^2 + \&c. = 0$. The value of x, necessary to render (1) the greatest possible—if it *can* be made the greatest possible for any value of x—will be found by solving this equation.

PROBLEM.—To find the angle at which the rudder of a ship must be inclined to the stream so that the effect, in the direction perpendicular to the stream, may be the greatest possible.

It has been seen above that the effect varies as

$$\sin^2 A \cos A, \text{ or as } (1 - \cos^2 A) \cos A = \cos A - \cos^3 A = x - x^3$$

where x is put for the cosine of the required inclination.

$$\text{Since } x - x^3 = \text{a maximum}$$

$$\therefore 1 - 3x^2 = 0 \quad \therefore x = \sqrt{\tfrac{1}{3}}$$

hence the angle whose cosine is $\sqrt{\frac{1}{3}}$ will be the inclination necessary.

PROBLEM.—The velocity V of the stream being given, to determine the velocity v of an undershot-wheel, so that the greatest possible effect may be produced.

The pressure upon the same area of float-board will vary as $(V - v)^2$. As this pressure moves with the velocity v, the effect will be greatest when

$$(V - v)^2 v = V^2 v - 2Vv^2 + v^3 = \text{a maximum};$$

in order to which, the unknown quantity v must satisfy the condition

$$V^2 - 4Vv + 3v^2 = 0$$

The solution of this quadratic gives

$$v = V, \text{ and } v = \tfrac{1}{3}V$$

The first value of v renders the proposed expression a minimum: that is, the least quantity of work is performed by the wheel when it moves with the same velocity as the stream—which is obvious: the other value of v is that which renders the expression a maximum; and shows that the greatest amount of work is performed when the wheel moves with a velocity equal to one-third the velocity of the stream. But for the practical effects of water-power, acting through the medium of water-wheels, and other hydraulic machines, the student is referred to the treatise on PRACTICAL MECHANICS in the present volume.

PNEUMATICS.

The fluids treated of in the foregoing articles are all regarded as inelastic or incompressible fluids: it remains for us to devote a few pages to the consideration of elastic or compressible fluids. The most important of these is the atmosphere with which we are surrounded; and it is an especial reference to this, that the examination of the mechanical properties of elastic fluids is said to belong to the science of *Pneumatics*—from a Greek term signifying, the air we breathe.

Transmission of Pressure: Compression.—The mechanical properties of elastic and inelastic fluids are in many important particulars the same: the fundamental principle in Hydrostatics, for instance,—namely, that a pressure applied to the surface of a fluid, is transmitted undiminished in all directions throughout the entire volume of the fluid,—holds equally whether the fluid be elastic or inelastic; and in both cases may be put to the test of experiment in the same way. If the piston A at page 162, play in a metal cylinder projecting into the interior of the vessel, and the piston be forced inwards along the tube by a pressure or weight of 1 lb. say, it will advance under this pressure along the tube and then stop · and it will be found that an additional pressure of 1 lb. must be applied to the piston B to prevent its being forced out.

In the case of a liquid the only effect of the pressure on A is the transmission of it to every portion of the surface of equal area. In the case of the air, there is another and a distinct effect observably produced—namely the compression of the fluid into smaller volume; and it is found by experiment that the volume diminishes just as the pressure increases: in other words, that the space into which any quantity of air is forced to compress itself, is inversely as the pressure applied. How this truth was ascertained we shall explain presently: it is necessary first to establish the fact that air has weight. It is possible to conceive that a substance like air may have the property of transmitting pressure, yet that it may not itself be a pressing or weighty substance.

Experimental Proof of Atmospheric Pressure.—But that the air excreises pressure, may be proved by many easy experiments: the boys' sucker is familiar to everybody. This toy consists of a piece of leather with a string passing through a perforation in the middle: upon being wetted so as to exclude the air, when it is pressed close to a smooth surface, it is found that considerable force must be applied to the string to detach the sucker; and if the surface of it be large, a stone of considerable weight may be thus lifted. What is it that thus presses the sucker and stone so firmly together? We are compelled to answer the *air*, which surrounds them both as if they were one body. Before the sucker was applied the air surrounded *it*: the pressure, if any, was alike on its upper and under surface: but when affixed to the stone the pressure on the under surface of it was excluded, while that on the upper surface remained the same; and as an equal pressure is exerted on the corresponding area

of the under surface of the stone, the two are, as it were, thus pinched or pressed together.

Again: take a glass tube, a foot or two long, open at both ends, and therefore full of air; plunge one end into a vessel of water: then we know from the first principles of Hydrostatics, that the level of the water will be the same both outside and within the tube. Let, now, the air in the tube be drawn out by the mouth, or pumped out by a machine fitted for the operation:—the water will be seen to rise within, and at length to fill the tube; and if the thumb be quickly applied to the top, so as to prevent the re-admission of the air, the water thus filling the tube will remain suspended, just as it would do if an additional column of water of the same height as the unimmersed part of the tube pressed upon the surface of the water in the vessel. In the absence of this additional column of water, what is it that sustains the water in the tube? We are again compelled to answer, the pressure of the air which supplies its place.

We have here supposed the tube to be only a foot or two long for convenience, but careful experiment has proved that with a tube of sufficient extent, exhausted of air, the water would not cease to rise till it had attained the elevation of about 32 feet; which limit attained, it would remain stationary. On this large scale the experiment was actually performed in 1647 by the celebrated Pascal: he procured glass tubes closed at one end, of forty feet long, and found that when filled with water in a deep river, and then raised vertically with the open end downwards, the fluid ceased to fall when water in the tube stood at about 32 English feet 2½ inches from the surface of the river. This experiment clearly proved that the pressure of the whole atmosphere on any surface was equal to the pressure of a column of water 32¼ feet high on the same surface. And thus was satisfactorily shown not only that air has weight, but what amount of weight was sustained, by a given area of the surface of the earth pressed upon by the atmospheric column resting upon it: the weight sustained would be equal to that of a column of water on the same area about 32¼ feet high.

The same conclusion is obtained with a more manageable length of tube by using mercury instead of water. And in this way the experiment had been tried previously by Torricelli, and it is hence called the Torricellian experiment:—it led to the invention of the barometer. When a tube about three feet in length, and closed at one end, is filled with mercury, and then inverted in a basin of that fluid, the column of mercury held suspended in the tube is found to be about 29 inches above the surface; and as mercury is about 13½ times the weight of the same volume of water, we arrive at the same result,—namely, that the pressure of the atmosphere on any area, is the same as the pressure of a column of water on that area of about the height

$$29 \times 13\frac{1}{2} \text{ inches} = \text{about } 32\frac{1}{2} \text{ feet.}$$

An exact correspondence between the results of such experiments made at wide intervals of time must not be expected, because the weight of the atmosphere fluctuates, like the other changes, in its condition. The average height of the mercurial column is about 30 inches, the ordinary range being between 28 inches and 31 inches; and as 30 cubic inches of mercury weigh about 15 lb., every square inch of the surface of the earth, at the level of the sea, sustains, on the average, 15 lb. of atmospheric pressure.

Weight of a Volume of Air.—Besides the foregoing methods of proving that the whole atmosphere exerts a pressure upon the globe of the earth equal to that which would be exerted by a sea of mercury 30 inches deep, covering its entire surface, a definite portion of the air about us may be taken, and, like any other material substance, be actually weighed in a balance.

Let a vessel containing a cubic foot, that is 1000 ounces, of water be provided with a stop-cock screwed upon its neck. This vessel may be exhausted of its air by aid of the air-pump, a machine which will be hereafter described. Thus emptied, and the stop-cock closed so as to prevent the readmission of air, let the vessel be accurately weighed in a very sensible balance. (See STATICS, page 111.)

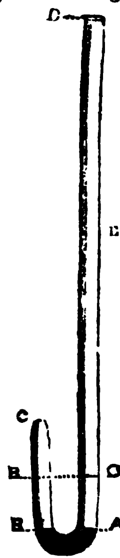
When the exact counterpoise is thus ascertained, let the stop-cock be opened and the air admitted into the vessel; the vessel thus filled, with the stop-cock again closed to cut off all external pressure, will be seen at once to preponderate, and we shall find it necessary to put about 523 grains of additional weight in the other scale-pan to restore the equilibrium; thus showing that a cubic foot of air at the surface of the earth weighs about 523 grains.

We cannot, of course, speak accurately as to a few grains more or less, but it will be always found that at least an ounce more weight will be required to counterpoise the vessel when full of air than was required before the air was admitted.

It is in this way ascertained that when the barometer stands at 30 inches and the temperature of the air is 52° of Fahrenheit's thermometer, the weight of water is to the weight of an equal volume of the air around us as 840 to 1.

Law of Marriotte.—Having thus established the truth of the proposition that air has weight, we may now proceed to explain the way in which the law of Marriotte, namely, that the volumes into which air may be compressed are inversely as the pressures applied, is arrived at.

Let DAC be a bent tube of equal bore throughout, or at least equally wide throughout, the shorter vertical leg. Conceive this tube to be at first open at both ends C and D, and let a little mercury be poured in, so as to fill the bend of the tube; the level of the mercury at A and B will be at the same height A B in each tube, however dissimilar the two portions of the tube may be (HYDROSTATICS, page 187). Now let the end C be closed, and let such a quantity of mercury be poured in at D as will cause the surface, originally at C, to rise to B′, half-way between B and C. Upon measuring the additional column of mercury thus introduced into the tube—that is, having previously marked the level O of B′—if we now mark the level E when the surface B has risen to B′, we shall find that the length of the mercurial column O E is exactly equal to that of the mercurial column (as shown by the barometer) which represents the weight of the atmosphere. The inference is this: when the surface A sustained the pressure of only the column of air above it, the air in the shorter leg occupied the space B C; the air in B C was thus compressed into the volume it then had, simply by the weight of the atmosphere applied upwards to the surface B.



But when the weight of *two* atmospheric columns pressed on the surface B, that is, the weight of the barometric column of mercury in addition to the atmosphere, then the volume of air in B C occupied only half the space, namely, B′ C; that is, the original volume was compressed by the double pressure into *half* its former bulk. Again, if another column of mercury be poured into the tube at D, till the air in the other leg occupies only one-third of its original volume, we shall find that the height of the entire column of mercury, measuring the difference of the levels in two legs, is twice

that of the barometric column; so that the air in B C, when pressed upwards by the weight of *three* atmospheres—the atmosphere itself pressing on the upper surface of the mercury and the double barometric column—is compressed into *one-third* of its original volume. Results in harmony with these are always found to follow whatever fractional portion of the whole atmospheric pressure be applied, and thus the law is experimentally established, that the volumes into which air is compressed by pressures are inversely as the intensities of those pressures. And this is only saying that the densities are directly proportional to the pressures.

As it is the elastic force of the condensed air that thus balances the additional pressures, we further infer that the elastic force is proportional to the density or to the force of compression.

A form somewhat more mathematical may be given to the foregoing account as follows :—

The mercury, at first poured into the tube at D, standing at the same level A B in both legs, and the end C of the tube being then closed, the space B C is occupied by the air in its existing state of atmospheric pressure at the time of the experiment, which pressure is indicated by the column of mercury in the barometric tube at that time. Fresh mercury is now poured in at D: its surface reaches the level E in one leg, and some lower level B′ O in the other, showing that the pressure of the air originally occupying C B, by now being forced to contract itself into C B′, exercises an increased pressure; so that now it not only balances the pressure of the atmosphere, in its original state at D, but also the column of mercury O E.

Let H be the height of the mercury in the barometer; then the pressure on the original volume of air B C is that of a column of mercury of base B and height H, while the pressure on the condensed volume, B′ C, is that of a column of mercury of the same base and of the height H + E O. Now, wherever B′ may be, it is found by actual measurement that

$$H + EO : H :: BC : B'C;$$

and the pressures and volumes being as these linear dimensions, it follows that

Pressure on B′ C : Pressure on B C :: Volume B C : Volume B′ C.

And the same proportion is found to hold whatever be the original density of the air experimented upon; and it equally has place for all elastic fluids.

The foregoing law, although generally called the law of Marriotte, is equally entitled to be called Boyle's law; as it was announced, independently, by the latter philosopher, at nearly the same time. It was discovered by Boyle in the year 1662.

Since the force of compression on the air near the surface of the earth is that due to the weight of the superincumbent atmosphere, it follows that the air must be rarer the higher we ascend; rarer, for instance, at the top of a mountain than at its base, a truth that has been turned to practical account in the way of measuring the height of a mountain by observing, with the barometer, the difference of atmospheric pressure at top and bottom.

With the general theoretical principles and the practical construction of this useful instrument, we here presume the reader to be already acquainted; for although some account of the theory of the barometer might reasonably be expected in a treatise on Pneumatics, yet, as the subject has been sufficiently discussed in the METEOROLOGY, in the present series, and as the space now at our disposal is too limited to permit of our indulging in repetition, we must refer for the requisite information, upon what concerns the barometer and thermometer, to the interesting treatise just mentioned.

Density of the Atmosphere at Different Heights.—Since the density of air varies directly as the pressure sustained by it, the higher regions of the atmosphere having a less superincumbent load than the lower, must be proportionally less dense. This weight or pressure, as already noticed, will be affected by temperature: and moreover, the same particle, or volume of air, will weigh less, the higher it is situated above the surface of the earth, on account of the diminution in the force of gravity. But, leaving these comparatively unimportant modifying influences out of consideration —unimportant at least for moderate altitudes—and regarding both the temperature and the force of gravity as uniform, we may prove that:—The density of the atmosphere at different heights above the surface of the earth, varies thus,—namely, when the heights increase in arithmetical progression, the densities decrease in geometrical progression.

Conceive a uniform slender column of the atmosphere to be divided by horizontal sections into n equal parts; the number n being so great, that each thin stratum of air may be regarded as of uniform density: then commencing at the bottom or first stratum a_1, the several strata may be denoted by

$$a_1, a_2, a_3, a_4, \quad . \quad . \quad . \quad . \quad a_n \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and their densities by

$$d_1, d_2, d_3, d_4, \quad . \quad . \quad . \quad . \quad d_n \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It has been sufficiently explained (HYDROSTATICS, page 166) that, in speaking of the density of any substance, we always have reference to some assumed unit of density, and that the weight of the unit of volume (the unit of weight) of the standard substance, multiplied by the numerical expression (D) for the density of any other substance, gives the weight of a unit of volume of that other substance. The abstract numbers (2) therefore will equally express the number of units of weight of the volumes (1), provided we take the magnitude of a_1 or a_2, &c., for that of the unit of volume: and this we are of course at liberty to do, for in all investigations our unit of measure may be chosen at our convenience.

It may be well, however, in order to prevent confusion, to write the numbers (2), when regarded as so many units of weight, thus, namely:—

$$w_1, w_2, w_3, w_4, \ldots \ldots w_n.$$

Then, as the density of any stratum of air varies with the weight or pressure it sustains, we have

$$\left.\begin{array}{l} d_1 : d_2 :: w_2 + w_3 + w_4 + \ldots + w_n : w_3 + w_4 + \ldots + w_n \\ d_2 : d_3 :: w_3 + w_4 + w_5 + \ldots + w_n : w_4 + w_5 + \ldots + w_n \\ d_3 : d_4 :: w_4 + w_5 + w_6 + \ldots + w_n : w_5 + w_6 + \ldots + w_n \\ \quad \&c. \qquad \qquad \&c. \qquad \qquad \&c. \end{array}\right\} (3)$$

But

$$\left.\begin{array}{l} d_1 : d_2 :: w_1 : w_2 \\ d_2 : d_3 :: w_2 : w_3 \\ d_3 : d_4 :: w_3 : w_4 \end{array}\right\} (4)$$

Hence, putting w for d, in the foregoing proportions, alternating and compounding the ratios (ALGEBRA, page 220), we have

$$\begin{array}{l} w_1 : w_1 + w_2 + w_3 + w_4 + \ldots + w_n :: w_2 : w_2 + w_3 + w_4 + \ldots + w_n \\ w_2 : w_2 + w_3 + w_4 + w_5 + \ldots + w_n :: w_3 : w_3 + w_4 + w_5 + \ldots + w_n \\ w_3 : w_3 + w_4 + w_5 + w_6 + \ldots + w_n :: w_4 : w_4 + w_5 + w_6 + \ldots + w_n \end{array}$$

Consequently, alternating, and having regard to the proportions (3) and (4), we have

$$d_1 : d_2 :: d_2 : d_3 :: d_3 : d_4, \ \&c.$$

Therefore the densities of the strata a_1, a_2, a_3, a_4, &c., of which the heights are in

arithmetical progression, are themselves in geometrical progression; and consequently the pressures sustained at different altitudes are also in geometrical progression.

Determination of Altitudes by the Barometer.—Let A be the altitude in feet of any spot above the surface of the sea, then measuring downwards from that spot, the distances 0, 1, 2, 3, &c. feet, the pressures upon each foot will form a geometrical increasing progression, by what is proved above. The relation, therefore, between the numbers expressing this progression, and the numbers 0, 1, 2, 3, &c., is the same as that between any numbers in geometrical progression, and their logarithms. The numbers 0, 1, 2, 3, expressing the distances downwards from the highest point, must therefore be the common logarithms of the numbers expressing the corresponding pressures, multiplied by some constant factor or modulus, which constant factor we may call K.

Hence a expressing any lower altitude in feet, and P, p the pressures at the altitudes A, a, we shall have

$$A - a = K \log P - \log p = K \log \frac{P}{p}$$

If the lower station be at the level of the sea, then $a = 0$; and since the pressures P, p are indicated by the heights of the mercurial column in the barometer at the two stations, we have for A, the number of feet in the altitude of the upper station above the lower,

$$A = K \log \frac{P}{p} = K \log \frac{\text{height of barometer at upper station.}}{\text{height of barometer at lower station.}}$$

But before this formula can be turned to practical account, we must be able to assign the numerical value of the constant multiplier K.

Let H be the height of the barometer at the level of the sea, and H′ the height at some known altitude A′; then

$$A' = K \log \frac{H}{H'} = K \log \frac{P}{p}.$$

Let A′ = 1 foot, then

$$K = 1 \div \log \frac{H}{H'}, \text{ or } K = 1 \div \log \frac{P}{p}.$$

Now when the barometer stands at 30°, and the temperature of the air is 55° Fah., the weight of a volume of air is to that of an equal volume of mercury as 1·22 is to 13568: hence the pressure of the 30 inches of mercury is equal to the pressure of a column of $\frac{13568 \times 30}{1 \cdot 22}$ inches of air of the same uniform density as that in which the barometer is placed—that is, of the air at the level of the sea. The height of this equiponderant column of homogeneous air is therefore

$$\frac{13568 \times 30}{1 \cdot 22 \times 12} \text{ feet} = 27803 \text{ feet};$$

and, therefore, the height of the column measured from a foot above the level of the sea, is 27802 feet.

$$\therefore K = 1 \div \log \frac{27803}{27802} = 1 \div \log \left(1 + \frac{1}{27802}\right).$$

$$\text{But } \log \left(1 + \frac{1}{27802}\right) = \cdot 43429448 \left\{ \frac{1}{27802} - \frac{1}{2} \frac{1}{27802^2} + \text{\&c.} \right\}$$

(Mathematical Sciences, page 279), therefore, neglecting the powers of so small a fraction,

$$K = 1 \div \frac{43429448}{27802} = \frac{27802}{\cdot 43429448} = 64020 \text{ feet};$$

or dividing by 6, K = 10670 fathoms. Consequently the formula for the computation of any altitude A above the level of the sea is

$$A = 10670 \log \frac{\text{height of bar. at alt. A}}{\text{height of bar. at the sea-level}} \text{ fathoms.}$$

The constant factor 10670 has been determined on the supposition that the temperature of the air is 55° Fah. The more convenient multiplier 10000 may be employed instead by a suitable alteration of the temperature; that is, by assuming the air to be in a different state. Experiment has shown that the altitude of a place, as deduced from the foregoing formula, will vary by $\frac{1}{435}$th of its whole value for every degree by which the mean of the temperatures at the two stations differs from 55°; the variation to be added when the mean exceeds 55°, and subtracted when the contrary is the case.

Now, the difference between 10670 and 10000 is 670, and the difference between 10670 and the value it would have for one degree less of temperature is $\frac{10670}{435} = 24{\cdot}5$: hence, to find for how many degrees less of temperature the difference is 670, we have the proportion

$$24{\cdot}5 : 670 :: 1° : 27°.$$

Consequently 55° — 27° = 28° is the temperature of the air for which the altitude A in fathoms is

$$A = 10000 \log \frac{\text{height of bar. at alt. A}}{\text{height of bar. at the sea-level}}$$

And this value must be increased or diminished by the $\frac{1}{435}$th part of itself for every degree shown by the mean temperature of the two stations above or below 28°.

In taking the logarithms, we may employ only five figures for each, including the index 1. Regarding these as whole numbers, we take their difference; then if E be the excess of the mean temperature of the two stations above 28°, we must multiply this difference by $\frac{E}{435}$, and add the result to the said difference to obtain the altitude in fathoms. This fraction of the difference must be subtracted if E be negative. The following is an example:—

Required the height of a mountain when the barometer at the bottom stands at 29·68 inches, and at the top at 25·28 inches, the mean temperature, that is half the sum of the temperatures at the two stations, being 47°.

The excess of the mean temperature above 28° is 47° — 28° = 19°

$$10000 \times \begin{cases} \log 29{\cdot}68 \quad . \; . \; . \; . \quad 14725 \\ \log 25{\cdot}28 \quad . \; . \; . \; . \quad 14028 \end{cases} \qquad \frac{E}{435} = \frac{19}{435}$$

$$697 \times \frac{19}{435} = \quad \begin{array}{r} 697 \\ 30 \\ \hline \end{array}$$

∴ the height is . . . . . . 727 fathoms = 4362 feet.

Where minute accuracy is required, certain particulars, here disregarded, must be taken into consideration; as, for instance, the latitude of the place of observation, and the dilatation of the mercurial column, which lengthens to the extent of $\frac{1}{10000}$th of the whole for every additional degree of temperature: but the determination of altitudes by

the foregoing formula will, in general, differ from the truth only by a fathom or two, and will therefore be sufficiently accurate for most practical purposes (see METEOROLOGY, page 18).

The higher we ascend into the regions of the atmosphere, the colder does it become. There are two reasons for this: in the first place, the direct rays of the sun pass through the air without being arrested, as it were, as in the case of a solid body, which absorbs a great portion of the heat poured upon it; and, in the next place, experiment proves that air by rarefaction loses part of its original heat, so that when air of a certain temperature is relieved of part of the pressure upon it, and allowed to expand (as the air in the receiver of an air-pump), it is found that the temperature becomes less and less as the pressure diminishes; so that if the original temperature is to be maintained, fresh heat must be communicated. As there is no external source for the supply of such heat in the more elevated regions of the atmosphere, the superior coldness of those regions becomes sufficiently accounted for.

Weight of the whole Atmosphere.—We have already seen that the weight of the atmosphere surrounding the earth is equal to the weight of a surrounding coating of mercury, on the average 30 inches thick: the amount of this weight may be found by multiplying the volume of the mercurial shell by the specific gravity of the mercury, and the product by $g = 1000$ ounces (HYDROSTATICS, page 166). Let $R =$ radius of the earth in feet, $r =$ height of the mercury in feet, and s its specific gravity then subtracting the volume of a sphere of radius R from that of a sphere of radius $R + r$, in order to get the volume of the spherical shell of mercury, we shall have for the weight W of that shell,

$$W = \left\{\frac{4\pi(R+r)^3}{3} - \frac{4\pi R^3}{3}\right\} sg$$

$$= \frac{4\pi}{3}\left\{3R^2r + 3Rr^2 + r^3\right\} sg$$

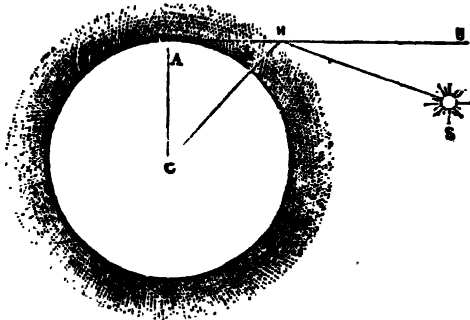
$$= 4\pi r\left\{R^2 + Rr + \tfrac{1}{3}r^2\right\} s \times 1000 \text{ oz.}$$

This weight the celebrated Cotes calculated to be equal to that of a globe of lead of 60 miles in diameter, or upwards of 77,670,000,000,000,000 tons.

The weight of the atmosphere can be much more accurately estimated than its height, since the former can be submitted to experimental examination, while the latter is beyond our reach. But there are many cogent reasons which preclude the notion that the atmosphere is illimitable. Thus, we have seen that it has weight; that is, that it is a material substance, like all other material substances, operated upon by the attraction of gravitation. There must be an elevation, therefore, at which its elasticity, or its tendency to expand further upwards, is just balanced by its gravitating tendency downwards, and which must therefore mark the limit of its altitude. Again, if its extent were boundless, the moon and all the other planets would each, by its attraction, appropriate a share of it; and, as in the case of our earth, the density of it would increase towards the surface of each planet; and, more especially as regards the moon, its presence would become manifest by astronomical observation. But astronomers find the moon to be quite destitute of an atmosphere: it has no clouds, no rain; and the planet Jupiter seems to be in the same predicament. That the moon has no atmosphere, and consequently can have no breathing inhabitants, is a fact interesting in itself; and it involves this other fact, equally interesting, that our atmosphere has its limits. Accurately to assign these limits is beyond the power of science; but the phenomenon of the

refraction and reflection of light leads to the conclusion that it extends to the height of about forty or fifty miles. The average between these, namely, forty-five miles, is usually regarded by philosophers to be the height of our atmosphere. But that this conclusion may rest upon something better than mere conjecture, we shall give the reasonings by which it has been arrived at.

Atmospherical refraction is caused by the bending of the rays of light from a luminous body upon entering our atmosphere: till the outer boundary of the atmosphere is reached, nothing diverts the direction of a luminous ray. This refraction of the rays of the sun adds intensity to the twilight, or the light we enjoy after sunset. The twilight is found to continue till the sun is 18° below the horizon. Let, therefore, A H be the horizon of an observer at A, who will continue to have twilight till the ray S H from the descending sun makes with B H the angle S H B = 18°, and therefore the angle A H S is 162°.



The last glimmer of twilight is due, no doubt, mainly to the reflection of the ray S H from the particles of the atmosphere at H, and the radius C H being perpendicular to the reflecting surface at H, the angles A H C, S H C must be very nearly equal, and therefore each equal to about 81°. Now, taking the radius C A of the earth at 4000 miles, we have by Trigonometry,

$$CH = CA \operatorname{cosec} H = \frac{CA}{\sin H} = \frac{4000}{\sin 81^\circ} = 4050 \text{ nearly.}$$

So that, according to this calculation, the height h H of the atmosphere is 4050 — 4000 = 50 miles.

The height of an homogeneous atmosphere that would press, as the actual atmosphere is found to do, may be readily ascertained. Thus, if we take the specific gravity of air to that of water as 1 to 850, when the barometer stands at 30 inches, and the specific gravity of water to that of mercury as 1 to 14, we shall have the specific gravity of air to mercury as 1 to 11,900.

$$\therefore 1 : 11{,}900 :: 30 \text{ inches} : 357{,}000 \text{ inches} = 5{\cdot}63 \text{ miles.}$$

This, therefore, would be the height of an atmosphere pressing with the same weight as ours does, and of which the density is uniformly the same as the air at the surface of the earth. If the specific gravity of the air at the surface to that of mercury, when the barometer stands at 30 inches, be taken as 1 to 12,000, the height of the homogeneous atmosphere will be 12,000 × 30 inches = 10,000 yards = $5\frac{1}{8}$ miles nearly. Hence the height of the equiponderant homogeneous atmosphere would be about $5\frac{1}{8}$ miles.

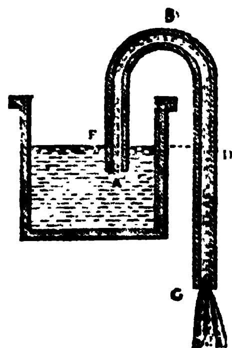
The Syphon.—The weight of the atmosphere, like all the other forces spontaneously offered to us by nature, has by the ingenuity of man been made subservient to his wants and conveniences in a great variety of ways. Next to the force of gravitation, of which, indeed, the weight of the air is only a particular manifestation, atmospheric pressure is perhaps the most important of terrestrial phenomena; and it is not easy to estimate the amount of extra toil and privation to which mankind would be subjected,

if the air we breathe, like the light we see, had no appreciable weight. It is our business now to give a short account of some of the contrivances by which atmospheric pressure has been turned to advantage in the practical affairs of life, and to explain the principles upon which these contrivances accomplish the purposes intended by them. The least complicated of these is the Syphon.

This instrument is simply a bent tube A B C, employed for the purpose of exhausting a vessel of the liquid it may contain, or of transferring it to another vessel without the practical inconvenience—often the practical impossibility—of actually pouring the liquid from the one vessel into the other.

There are two ways of bringing the instrument into operation: the end of the shorter leg A may first be inserted in the liquid, and then the air in the tube withdrawn by the mouth through a small pipe communicating with the tube near the extremity C of the longer leg. In this case there must be a stop-cock between the pipe and C to





cut off communication with the atmosphere pressing through C. Such a syphon is called the distiller's syphon: it is exhibited in operation in the first figure above. In the second way the tube is inverted and filled with the fluid, and the ends A, C closed; the shorter end is then immersed and both ends are opened.

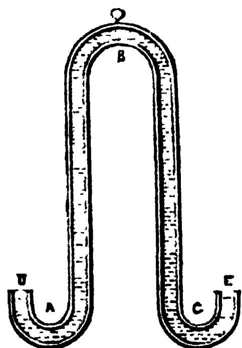
Suppose, as in the first way, that the tube, with the end A in the liquid and the end C stopped, has been exhausted of air: the pressure of the atmosphere on the exposed surface of the fluid in the vessel acts upwards at A, and forces the fluid into the vacant space with an energy sufficient, if it were water and the tube were straight, to carry it to the height of thirty-two or thirty-three feet; and to a greater height if it were a fluid lighter than water. As, however, the tube bends at a moderate height B, the ascending column is forced to accommodate itself to the course of the tube, and descends into the leg B D C. Arrived at B, the highest point, its descent down B C is expedited by the direct influence of gravity upon it; and C being opened, the liquid flows out.

Now it must be observed that when the column has attained the height E B it is not forced forward by the whole of the atmospheric pressure at E, but only by that pressure diminished by the weight of the column E B; so that when the column had extended itself to D, if the atmospheric air were admitted, the upward pressure on D, like that on E, would be equal to the whole atmospheric pressure diminished by the weight of the column D B; the two pressures, therefore, balance. But as soon as D is passed and the column in B C lengthens, the equilibrium is destroyed, the downward pressure prevails,

and the liquid falls through C upon finding a passage there: and the longer the part D C of the syphon below the surface D of the liquid, the greater will be the velocity with which it will issue, since the greater will be the preponderating pressure downwards.

In the second way of preparing the syphon, that is by first filling it with the fluid, it is brought, in another manner, into the same circumstances as when it is filled, as above, through the pressure of the air on the surface of the liquid. The liquid will evidently cease to flow as soon as the surface of it descends to a level with the rising surface in the vessel to which it is transferred; at this stage, however, the syphon will remain full, as there will be a complete equilibrium.

In the syphon just explained it is necessary that the legs be of unequal length; but there is another kind of syphon called the Wurtemburg syphon, in which the two legs are of the same length. In this instrument (see the first figure below), the extremities of the equal legs are turned upwards so that the two open ends D, E may be on the horizontal level when the syphon is held upright. The instrument is kept constantly filled with water, which remains suspended in the two legs D A B, E C B, because the equal atmospheric pressures at D and E place them in a state of equilibrium; but if the leg terminating in D be immersed in water, the end D will sustain the additional pressure of the water reaching from D to the surface, and consequently E being free from such additional pressure, the fluid is forced out at that end. When the





surface of the water in the vessel has descended to D the stream from E stops, but the syphon remains full, and, thus filled, is taken out and hung up by a loop at B till again wanted.

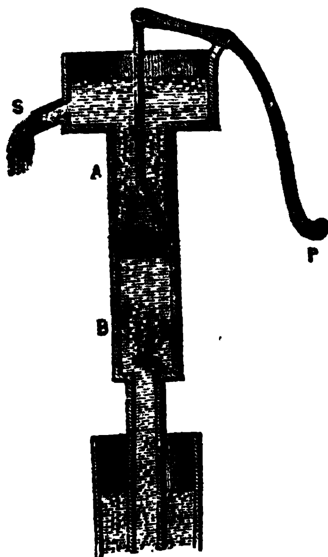
Intermitting Springs.—The curious phenomena of intermitting springs are referable to the foregoing principles. These springs, issuing from a fissure in a mountain side, flow for a certain period and then stop; after a while the water flows again, and so on. These effects are produced through the operation of one of Nature's syphons. The preceding figure exhibits a section of the mountain and the stream. A cavity exists in the former, as here represented; this by degrees is filled by gradual infiltrations, or by slender fissures communicating with the upper surface.

Suppose, now, that there is a syphon-like communication between the reservoir of water, supplied by these channels from above, and the mountain-side; as this syphon is not artificially exhausted of air, it will not deliver the water in the reservoir till the

surface of that water rises as high as the bend of the syphon B, after which it will begin to flow, pouring out a continued stream till the reservoir is emptied, or at least till the level reaches the immersed end of the syphon; the stream will then stop and will commence to flow again only when by a fresh accumulation of water from above the level of B is again reached.

It is of course a condition necessary to the production of these results that the surface of the reservoir be subjected to the pressure of the atmosphere; but through the fissures which supply the cavity with water, air must have previously found its way; and, indeed, from the principle of the equal transmission of fluid-pressure, the weight of the atmosphere acting on the upper surface of the water in a single downward column, as on the water in the slender perforation to the right in the foregoing figure, acts equally on every part of the surface of the water in the reservoir.

The Household Pump.—The common suction-pump is another important contrivance which owes its usefulness entirely to the pressure of the atmosphere, although its dependence upon this agency was little suspected for many ages after its invention. As already noticed (page 206), Torricelli, a pupil of Galileo, early in the seventeenth century, was the first who fully recognized the influence of atmospheric pressure. He accounted for the ascent of water in the pump, and of mercury in an exhausted tube; and the invention of the barometer naturally followed.



The marginal figure represents a section of the suction-pump; A B is the tube, or working pump-barrel, communicating with the water in the well. In this barrel an air-tight piston C, moved by the lever or pump-handle P, freely plays. At the lower extremity of the working barrel there is a valve V opening upwards. this valve separates the barrel from, and completely covers, what is called the suction or feeding-pipe, which is usually of smaller bore. In the piston C there is also a valve similarly opening upwards. The piston C is called the *sucker*.

Now, imagine the piston C to be at first at the bottom V of the barrel, and then to be raised by the action of the pump-handle. As the piston is air-tight, the pressure of the air in the barrel upon the ascending valve C keeps it closed, so that that air, having no escape below, is forced up and pumped out at the spout S. The vacuum thus produced in the barrel is immediately filled up by the ascent of the water through the feeding-pipe, for the water in the well sustains the pressure of the atmosphere on its surface, all except that portion of the surface which the feeding-pipe covers; and from *this* portion, as just explained, the air has been withdrawn. The barrel is thus filled with the water forced up by the pressure of the atmosphere on the exposed surface of that in the well; but no water escapes through the spout S, since all that has been raised is below the closed valve C, but it is retained suspended in the barrel, though the valve V still

remains open, the upper surface of the water, immediately below the piston, being, of course, not more than 32 or 33 feet above the surface of the water in the well (p. 206).

Upon now lowering the piston, the valve C opens by the resistance of the water confined in the barrel, for the downward pressure communicated to the confined water, and exercised through the lever P, closes the valve V, and prevents the water from being, by this extra pressure, forced back again into the well. Hence, upon again raising the piston, the upward resistance of the water above it closes the valve C, and the fluid is forced out of the spout S, while another vacuum is forming below the piston, and fresh water rising in the barrel; and thus every time the piston is raised the lower valve opens and the upper one closes, while, on the contrary, every time the piston descends the lower valve closes and the upper one opens, and in a well-made pump the barrel is kept constantly full.

It will of course be understood, in all that is said above, that the pressure of the atmosphere on the surface of the water in the well, or reservoir, need not be sufficiently great to cause the water in the pump to ascend as high as the spout, when, by the action of the handle, the air is withdrawn from the tube: if the length of the suction-pipe be 28 or 29 feet, or rather, if the valve at the top of it be at this height above the water in the well, the exhaustion of the air will be followed by the ascent of the water, through the suction-pipe, to some distance up the barrel of the pump. The subsequent raising of the water, thus introduced into the barrel, to the spout, is the sole result of the mechanical force applied to the pump-handle.

Quantity of Water Discharged, and Force Applied to Raise it.—The quantity of water discharged at each stroke of the handle, supposing the barrel to be constantly full, that is, supposing the spout not too high, is a column of water whose base is the horizontal section of the piston, and altitude the height to which the piston is raised, called the length of the stroke: thus, if r be the radius of the section in feet, and l the length of the stroke in feet, we shall have

$$\text{Quantity discharged} = 3{\cdot}1416r^2l \text{ cubic feet;}$$

or, since a cubic foot of water weighs 1000 ounces, or about $62\frac{1}{2}$ pounds avoirdupois, and since an imperial gallon contains 10 lb. of water, we have

$$\text{Quantity discharged} = 3{\cdot}1416r^2l \times 6{\cdot}25 \text{ gallons.}$$

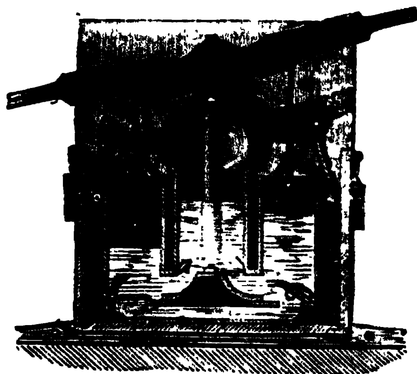
But the force necessary to raise this quantity, will be that required to raise the entire column of water extending from the surface of the reservoir to the surface of the water in the pump, the base of the column being the section of the piston. For let a be the number of linear feet of water above the piston, and b the number of feet below it, to the surface of the reservoir; let also p be the length of the column of water equal to the atmospheric pressure: then since, in raising the piston, the downward pressure of the atmosphere has to be overcome, the height of the equivalent column of water lifted is $a + p$; but the upward pressure against this, is equal to the weight of a column of water of height $p - b$: hence

$$(a + p) - (p - b) = a + b$$

Consequently, the force necessary to lift the piston is that necessary to lift a column of water having the same section as the piston, and the height $(a + b)$, of the water in the pump, from the surface of that in the reservoir; and, in fact, additional force must be applied to pump out the water, on account of friction, and the weight of the piston and rod.

In the returning stroke of the pump-handle, the piston descends by its own weight, which is sufficient to overcome the friction, and the slight resistance of the water.

Forcing Pump and Fire Engine.—As the pressure of the atmosphere varies, pumps are not constructed to raise water to a height above 28 or 29 feet; but the water thus raised may, by an additional contrivance, be forced upwards as much higher than this as we please. A pump for this purpose is called a *forcing pump*. In this pump there is no valve C in the sucker or piston; so that, after the exhaustion of the pump-barrel of air, and the consequent filling of it with water, no downward pressure on the piston could cause its descent, since the water is incompressible. A pipe, therefore, is inserted in the side of the barrel near the bottom, with a valve at the insertion opening outwards; the downward pressure on the solid piston forces open this valve, and drives the water into the tube, which may be carried upwards to any height, and be made to deliver the water there, provided only sufficient downward pressure act on the piston.



The fire-engine is essentially the combination of two forcing-pumps, the pistons of which are worked by a lever whose fulcrum is at its middle, and the power is applied at each end alternately; the water from the fire-plug is forced into the central receptacle, called the air-vessel, the air in which is thus condensed the more, the more water is forced in; the elastic force of this compressed air drives the water up the leathern hose, from which it issues through the delivery-pipe with a velocity proportionate to the pressure applied, or the condensation produced in the confined air. If the pistons be worked with sufficient energy to supply the air-vessel with water as rapidly as it is thus delivered, the stream will be invariable; if more water be forced into the vessel than can escape out of the delivery-pipe, the air-vessel will be in danger of bursting by the increased pressure of the condensed air.

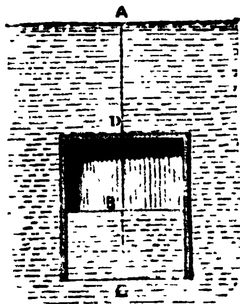
The intention of the air-vessel will be readily perceived by the student: the force applied to the two ends of the lever, which works the double pump, is necessarily an intermitting force; and without some contrivance to render the effect continuous, the water would issue from the delivery-pipe in jerks, by the separate impulsions given to it; but the continuous pressure of the condensed air in the air-vessel causes a continuous flow of water along the hose, up to the point of issue. If the supply of water to the air-vessel vary, the velocity of the issuing stream will, of course, vary likewise; but there will be, nevertheless, no interruption in the continuousness of the discharge.

In fact, the air-vessel performs an office a good deal like that performed by the fly-wheel in ordinary machinery: it gives continuity to intermittent action, tends to equalize the effects of it, and prevents that strain on the structure which irregularities of action are apt to produce.

The metal pipe, from which the stream of water issues, is considerably smaller in bore than the leathern hose along which the fluid flows to it. A double purpose is accomplished by this disparity. In the first place, the friction in the hose is diminished, as a comparatively smaller portion of water comes in contact with the

interior surface; and in the next place, the contraction of the stream, at the outlet, gives to it a proportional increase of velocity, so that the water is driven out of the narrow pipe with much greater force (page 195). It may be observed here, that the common syringe, or boys' squirt, is a miniature forcing-pump, or embryo fire-engine; the great contraction of the tube near the outlet is the cause of the velocity with which the water issues, being so much greater than that with which the piston moves. In the syringe there is certainly no air-vessel, as in the fire-engine, but the pressure on the water is continuous, so long as the tube contains any.

The Diving Bell.—The diver's bell is a heavy iron chest open at the bottom; it is called a *bell*, because in its original construction a bell-form figure was given to it. The interior is furnished with seats, sufficiently high above the mouth to enable the persons sitting on them to keep their heads free from the water, which rises in the bell, in virtue of the upward pressure, as it descends into the sea, and to breathe the air which is thus forced up and condensed above the rising water. By means of a flexible tube, communicating with the upper atmosphere through the top of the bell, fresh air can be pumped in, and the air unfit for respiration let out.



It is easy to find the space into which the air, originally in the bell, will be compressed when the chest is sunk to any depth below the surface.

Let A B be the distance between the surface of the water above and the surface of the water in the bell. Let also h be the height of a column of the water, the pressure of which is equal to that of the atmosphere: then the original air in the bell sustained a pressure equal to that of a column of water of the same horizontal section and height h; but when condensed by being forced up to B, it sustains a pressure equal to the same column h, and the additional column of water that is above B; that is, the pressure now sustained is equal to that of a column of water of height $h + \text{AB}$.

$$\therefore \frac{\text{vol. of condensed air}}{\text{vol. of original air}} = \frac{\text{DB}}{\text{DC}} = \frac{h}{h + \text{AB}} \text{ (page 207).}$$

Putting, therefore, x for DB, the depth of the compressed air between the roof of the bell and the surface of the water within it, we have

$$\frac{x}{\text{DC}} = \frac{h}{h + \text{AD} + x} \quad \therefore x^2 + (h + \text{AD})x = h\text{DC}.$$

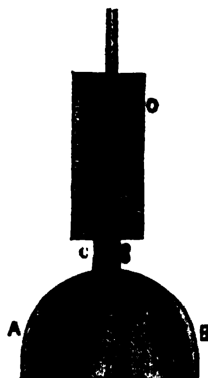
Hence, by solving this quadratic equation, which has but one positive root, the depth DB of the compressed air may readily be ascertained when the height DC of the bell, and the depth AD, to which its upper surface is sunk, are given.

The Condenser.—The condenser consists of a strong vessel AB, called the *receiver*, into which atmospheric air is forced and condensed by means of the following apparatus:—

A cylindrical barrel, opening into the receiver and having a valve opening downwards at C, is furnished with a piston D, having a valve also opening downwards. As the piston descends, the valve D closes by the resistance of the air in C D, and the pressure opens the valve C, the compressed air passing into the receiver A B.

This piston having thus forced all the air originally in the barrel into the receiver, is raised up; and the superior pressure of the condensed air in A B immediately closes

the valve C, while the downward pressure of the external atmosphere opens the valve D; so that the barrel becomes again filled with common air, and the operation is repeated.



In some condensers the piston D is made solid, and a small orifice is made at O near the top of the barrel, through which, upon raising the piston above it, the barrel is supplied with fresh air from without.

The density of the compressed air in the receiver, after n descents of the piston, may be ascertained thus:—

Let R and B be the respective capacities of the receiver and the barrel, and let D be the density of the atmospheric air: then, R being the volume of this air in the receiver at first, and $R + B$ the volume of it in the receiver after the first descent of the piston, the entire mass of air then in A B will be $D . R + D . B$; as the additional mass $D . B$ is forced in at the second descent, A B will then contain the mass $D . R + 2D . B$; and a fresh mass $D . B$ being thus forced in at every descent, we have for the whole quantity of air, after n descents,

$$\text{Quantity of air} = D\,(R + n\,B) \quad \therefore \text{ its density} = D\,(1 + n\,\frac{B}{R})$$

which increases in arithmetical progression.

The Wine Taster.—We may here notice, as among the minor contrivances by which the pressure of the air has been made available for practical purposes, the convenient little instrument called the wine-taster, which is much used in wine and ale cellars to draw out through the bung-hole of a cask, a specimen of its contents. The marginal representation will give a sufficiently clear idea of the form of this contrivance, which is hollow, and has a small perforation at each end. When dipped into any liquid, the upper orifice being kept open to the atmosphere, the fluid rises through the lower orifice, till the level is the same inside and outside; the thumb is then pressed on the upper orifice, and the vessel withdrawn. The air previously within—the common atmospheric air, reaching from the surface of the liquid to the upper hole—expands and fills the enlarged space which the descent of the liquid leaves as the vessel is raised. The pressure within is thus diminished, while the external pressure upwards, on the lower orifice, is that of the unrarefied atmosphere; so that a portion of the liquid remains suspended in the tube, under which if a glass be held, and the thumb removed, the sample will run out by its own weight.



The Air-Pump.—The office of the air-pump is the opposite of that of the condenser, the purpose of it being to exhaust a receiver of the air contained in it. There are several forms of this machine; that best known is represented in the following page (Fig. 1), and is called Hawksbee's air-pump. The receiver, containing the air to be withdrawn, communicates by means of a pipe with two barrels, generally of polished brass, in which two closely-fitting pistons move by rack-work, as in the sectional outline exhibited in the following page (Fig. 2), where it will be observed that the descent of one piston compels the ascent of the other. The four valves, marked a, b, e, f, all open upwards. As one of the pistons descends its valve opens, and the air in the barrel passes through, and rests above the piston when it has arrived at the bottom. During this operation the other piston ascends, with its valve closed,

emptying the barrel in which it is fitted of its air, the vacuum being supplied from the air in the receiver with which the barrel is in communication. The first piston is now raised and *its* barrel emptied in like manner, fresh air rushing in from the air in the receiver communicating with it. And these alternate ascents and descents of the pistons render the air in the receiver so rarefied that at length its pressure is inadequate to open the valves and pass into the barrels. It is plain that each turn of the toothed wheel, driving one piston to the bottom of the barrel and raising the other to the top, withdraws one barrel full of air; so that the air in the receiver, previously to this turn of the wheel, now occupies both the receiver and one of the barrels.

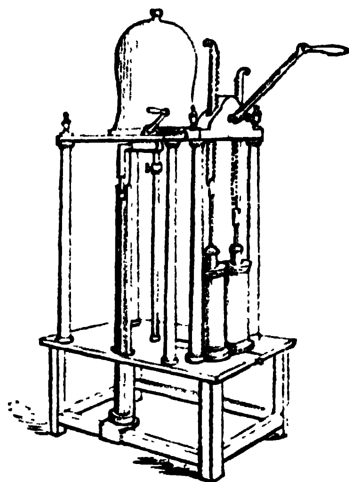


Fig. 1.

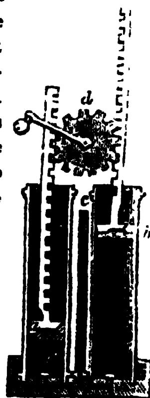


Fig. 2.

Let R be the capacity of the receiver and B that of each barrel; and the density of the atmospheric air being D, let D_1, D_2, D_3, &c., be the densities of the air in the receiver after 1, 2, 3, &c., strokes or turns of the wheel: then since at every turn the volume of air R is dilated into R + B, we have

$$D_1 (R + B) = D . R \quad \therefore D_1 = D \frac{R}{R + B}$$

$$D_2 (R + B) = D_1 . R \quad \therefore D_2 = D \left(\frac{R}{R + B}\right)^2$$

$$D_3 (R + B) = D_2 . R \quad \therefore D_3 = D \left(\frac{R}{R + B}\right)^3$$

And generally $D_n = D \left(\frac{R}{R + B}\right)^n$, the density after *n* turns. Hence the density decreases in geometrical progression. As the mass, or quantity of air, is equal to its volume multiplied by its density (page 150), therefore after *n* strokes the quantity of air in the receiver is

$$D_n R = \left(\frac{R}{R + B}\right)^n R = D \frac{R^{n+1}}{(R + B)^n}.$$

The quantity of air in the receiver decreases, therefore, in geometrical progression; but as a decreasing geometrical progression may be continued indefinitely, it follows that the quantity can never be actually exhausted in any number of strokes of the air-pump.

As the pressure of the atmosphere on the receiver becomes very great after the pump has long worked, it is necessary that it possess considerable strength; and the bell-form, as exhibited in the first figure, is given to it to secure this condition, the

glass of which it is formed being also very thick. The valves are usually of strong oil-silk, in little brass frames, traversed by a grating.

Problem.—To find the number of strokes necessary to reduce the air in the receiver to a given density.

Let d be the given density: then D being the original density,

$$d = D\left(\frac{R}{R+B}\right)^n$$

$$\therefore \log d = \log D + n\{\log R - \log(R+B)\}$$

$$\therefore n = \frac{\log d - \log D}{\log R - \log(R+B)}$$

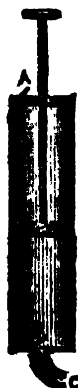
And if, as is usual, the original density, or that of the atmosphere in its ordinary state, be represented by 1, we shall have

$$n = \frac{\log d}{\log R - \log(R+B)}$$

The density d is here, of course, a proper fraction, and therefore $\log d$ is negative, but the denominator being evidently negative also, the expression is positive; and here, as inferred above, if $d = 0$, that is, if the receiver be absolutely exhausted of air, $\log d$, and therefore n, is infinite.

Although two barrels are connected with the receiver in Hawksbee's air-pump, but only one barrel is exhausted at each stroke; yet the working of the two pistons simultaneously, at each turn of the wheel, considerably diminishes the labour of the operation; for the atmospheric pressures on the upper surfaces of the two pistons being the same, the force required to work the pump is only that necessary to overcome the difference of the pressures on the under surfaces of the pistons, and the friction of the pistons themselves. If only one barrel were employed, the ascent of its piston would be opposed by the difference, constantly augmenting, between the pressures on its upper and under surfaces, in addition to the friction. If, however, the ascending single piston could be relieved from the pressure of the atmosphere, a single exhausting barrel would answer every purpose, and the machine would be simplified.

Smeaton's Air Pump.—Such a simplification is given to the pump constructed by Smeaton, and called after his name. A B is the barrel or cylinder communicating with the receiver by means of the pipe B C. In this construction the barrel is closed at top, but furnished with a valve A opening upwards; the piston also has a valve D opening upwards, and a third valve B, likewise opening upwards, covers the pipe B C.



As the piston ascends from the bottom of the barrel, forcing up the air above it, and leaving a vacuum below, D is the only valve that closes; the atmospheric air originally in the barrel is forced out at A, and the exhausted cylinder is instantly supplied with air from the receiver through the pipe B C. Upon the descent of the piston, the valve A closes, the pressure from without exceeding that from within the cylinder, so that in its next, and in every succeeding ascent, the piston is relieved from the pressure of the atmosphere upon it.

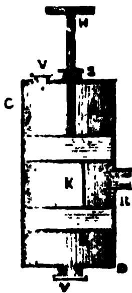
If, as before, R, B represent the capacities of the receiver and barrel respectively, D the density of the atmospheric air, and D_n the density of the air in the receiver after n strokes of the piston, we shall have, as in the former case,

$$D_n = D \left(\frac{R}{R+B}\right)^n,$$ the density after n strokes.

In this pump the exhaustion may be carried on to a much greater extent than in the common pump before described, because the valve D, being relieved from the downward pressure of the atmosphere, will open for a very slight pressure upwards, and consequently will allow of the passage of air in a more rarefied state.

But as the foregoing expression for D_n is the same as that furnished by Hawksbee's machine, and consequently the expression for n, the number of strokes producing a given degree of exhaustion, must be the same in both constructions, provided only that the barrels are all equal, it may seem that there is no difference in the exhaustive powers of the two. It must be remembered, however, that in the former investigation the valves are supposed to open at even the nth stroke, however high a number n may be; but as in the former construction the valve of the descending piston sustains the pressure of the atmosphere, which in the present contrivance is removed, a greater elasticity in the air below the valve is necessary to open it; so that, in Smeaton's pump, a greater degree of rarefaction will be ultimately attained.

Tate's Air Pump.—This pump is of very recent contrivance, and as yet but little known; it was first described by the inventor in the "Philosophical Magazine" for April, 1856. Its chief peculiarity is, that while it has, like Smeaton's pump, only a single cylinder or barrel, it has a double piston. This double piston performs the work of the two pistons in the ordinary double-barrelled air-pump, and that with only half the motion.



In the annexed diagram, C D is the cylinder or barrel; A and B are solid pistons, rigidly connected by a rod K, and moved by the piston-rod A H, passing through a stuffing-box S. V and v are valves opening outwards; and R an open pipe, at the middle of the cylinder leading to the receiver from which the air is to be exhausted.

The distance between the extreme faces of the pistons is about $\frac{3}{8}$ths of an inch less than one-half the length of the cylinder; this $\frac{3}{8}$ths of an inch being the space requisite for clearing the exhausting-pipe R. The pistons are each about $1\frac{1}{2}$ inch in thickness, and the rod K connecting them may be of any section consistent with strength. The effective length of the stroke is equal to the space between one side of the pipe R and the corresponding end-face of the cylinder, or it is very nearly equal to one-half the length of the cylinder.

In an upward stroke, the air above the piston A is propelled through the valve V into the atmosphere, while a vacuum is being formed beneath the piston B. When the piston A strikes against the top of the cylinder, the air from the receiver rushes through the pipe R, and diffuses itself through the lower half of the cylinder. In a downward stroke, the air beneath the piston B is propelled through the valve v into the atmosphere, while a vacuum is being formed above the piston A, and so on. It will be observed that the double piston performs a double duty at every single stroke; for while a vacuum is being formed in one half of the cylinder by one piston, the other piston is propelling the air from the opposite half into the atmosphere.

In the paper referred to, the inventor enters into an investigation of the exhausting power of this air-pump, and gives the results of some experiments which our limits preclude us from transcribing; and in a second paper in the "Philosophical Magazine" for May, 1856, he has offered some modifications of the above design, more, however,

for the sake of variety than for the purpose of superseding the original conception. We take the liberty of suggesting, however, that we think it would be a slight improvement if the exhausting-pipe were furnished with a valve, opening into the cylinder at R, since at present a portion of the air between the pistons flows into the receiver at every alternate stroke: this valve would be virtually equivalent to making the interval between A and B solid. Those interested in this new air-pump may see it at Messrs. Murray and Heath's, philosophical instrument makers, Piccadilly.

All air-pumps are furnished with a barometer-gauge, a vertical glass tube not less than 31 inches long, the lower end being immersed in a cistern of mercury, and the upper, by means of a horizontal tube, in communication with the receiver. If h be the height of the barometer, measuring the pressure before the pump is worked, the density of the air in the receiver then being D, and if h' be the height of the gauge afterwards, when the density is D_n, we shall have

$$D_n = D\frac{h - h'}{h}$$

for h represents the pressure equally of the external atmosphere and of the air in the receiver at first, and $h - h'$ represents the pressure of the latter after n strokes, and the densities are as the pressures. From what is proved at page 220, it follows that at every stroke of the pump, $h - h'$, the defect in the height of the mercury in the gauge, from the standard height h of the barometer, must decrease in geometrical progression.

J. R. YOUNG.

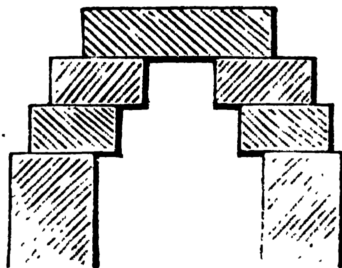


PRACTICAL MECHANICS.

Introductory Remarks.—Practical Mechanics may be defined to be the art of applying the theoretical principles of mechanical philosophy to materials, so that they are fashioned, arranged, and combined in the various forms required by man. It is difficult in any case to draw an exact line of separation between the theoretical and the practical: for all true theory is founded on practice, and all skilful practice is the application of true theory. In the case of Mechanics, the separating line between the theory and the practice is eminently difficult to be traced; for the development of principles depends at every step upon practical experiment, and the improvements in mechanical arts are generally the results of extended research into the principles. It is true, indeed, that a great range of mechanical theory can be investigated by abstract reasoning upon a few simple principles, just as mathematical science is built upon a few definitions and truths; but it is remarkable that little progress was ever made in mechanical science until men began to translate the actual results of experiment. In Mathematics, which is certainly the science of all others requiring the least amount of experimental proof, the ancients made considerable progress, because the minds of men in former times were quite as well fitted for the investigation of abstract truths as those of modern mathematicians; but in the sciences which demand experimental proof—such as Chemistry, Optics, Astronomy, and Mechanics—little progress was ever made until philosophers began to observe facts, and thence reasoned to their causes. For many ages of the world's history, it was the custom of men pretending to science to

start in their own minds some hypothesis or conjecture about some law of nature; to build upon this slippery foundation a vast scheme of nature; and then to wonder that natural phenomena did not suit their arrangement. In some such way as this the astronomers of old, with a few illustrious exceptions, took it for granted that an enormous, in their notions a boundless, mass, such as the earth, could not move; and therefore that the sun, moon, planets, and stars, revolved in daily cycle round it.

Architecture of Antiquity.—In respect of mechanical science, the ancients appear to have been very deficient. They had, indeed, tolerably true notions respecting the construction of buildings in such a manner as to ensure permanence and stability; but for these qualities they seem to have depended almost entirely on rude strength and massiveness, instead of skilful arrangement. Among the gigantic remains of Egyptian architecture, there is not found a single arch; the only approach to it is the vault of a passage hollowed like an arch, but made of two or three immense blocks of stone cut to the vaulted form. Again, among the ruins of Central America, remarkable for their enormous extent and elaborate decoration, the passages of such buildings as remain tolerably entire are found to be covered by blocks of stone, arranged as in Fig. 1. The beautiful and stable arrangement of materials in the form of an arch does not seem to have occurred to the architects of antiquity, until the comparatively recent era of the Roman Empire. It is difficult to ascertain whether the ancients were acquainted with any of the forms of roofs now in use, on account of the perishable nature of the materials: but it is probable that they knew no such method of covering large spaces, and for covering apartments of more limited extent, they had no resource but the use of large masses of stone.


Fig. 1.

Machinery of Antiquity.—In respect of machinery, the ancients appear to have been extremely deficient, for we have records of no apparatus except certain implements of war and of the chase, and a few rude contrivances for irrigating and cultivating the ground. But, indeed, even in modern times, the mechanical arts had made little advance until the genius of Watt had given to man a power applicable alike to manipulation of the most delicate character, and to labour demanding the most gigantic strength. Could a man who lived in England but a hundred years ago now revisit his country, he would in mechanical art find a change far greater, an advance more astonishing, than the whole progress made from the creation of the world up to the time at which he lived.

Modern Machinery.—That a man seeing all the wonders of modern art, as they have lately been seen at the great Exhibitions, should endeavour to attain a knowledge of their nature and construction, may at first sight appear presumptuous, because of the vast extent and variety of the objects which he would have to master; and, indeed, unless one were prepared to devote his lifetime to mechanical art, he could scarcely hope to know accurately the details of modern machinery, much less to attain the skill of a master in all its varied departments. But, with a fair knowledge of some simple first principles, and an introductory glance at a few of the leading details that are common to all mechanical arrangements, we believe it would not be difficult to attain a good general knowledge of mechanical art, and a readiness at comprehending any mechanical

arrangements that may be brought under his notice. Few will be inclined to dispute the advantages of possessing even such a moderate knowledge of Mechanics as this implies, for in modern times but little advance can be made in any useful pursuit without the aid of mechanical knowledge. Practical Mechanics may indeed be called the handmaid of the other arts. The sculptor, the architect, the musician, the chemist, the astronomer, the surgeon, the merchant, the manufacturer, the builder, the military man, the civilian, the traveller, and the emigrant, all require her services—all reap the benefit of her labours.

Mechanical Inventors.—It is generally supposed that inventors in the arts, as well as discoverers in the sciences, owe their success to some fortunate accident. We think it may, in most cases, be shown that such is not the fact. Almost every famous discovery, or useful invention, has been the result of long, often painful and laborious, thought and research: and success has often followed repeated failures, accuracy has succeeded repeated faults. That there is a natural genius for invention peculiar to some minds, cannot be doubted; but that genius alone, without labour and study, has ever led to brilliant results, may as firmly be denied. Mechanics, as an art, in a peculiar manner demands concentrated thought; there should be no waiting for some inspiration to bridge over a mechanical difficulty. Let a man gird himself to his task, and determine thoroughly to think out the subject he may have on hand, and we venture to predict that he will find few difficulties insurmountable.

Connection of Mechanics and Chemistry.—The two sciences that deal with matter in its various affections and modifications are Chemical and Mechanical Philosophy. The former treats of the influences affecting its minute particles, and the combinations of these particles; the latter has reference to masses of matter, and the forces and motions of masses or aggregations of particles. These two sciences frequently encroach upon the other's domain, for there are many similar phenomena developed by the action of natural laws on masses as well as on particles, and it is often difficult to draw a line of separation between the subjects of the two sciences. Questions relating to heat, light, electricity, and magnetism, form part of both sciences alike, and can scarcely be said to belong to the one more than to the other. There are questions, however, which are purely chemical, and others as distinctly mechanical.

The practical mechanic, examining a structure or a machine, sees, feels, and measures the parts of which it consists, traces the laws that govern its equilibrium or its movements, and determines the principles which have been adopted in its construction.

Statical and Dynamical Mechanics.—The objects of Practical Mechanics may be divided into Statical and Dynamical. Statical mechanics has reference to the formation and arrangement of materials intended to remain in a state of rest; its principal objects are permanence and stability. It has to consider the strength, elasticity, flexure, weight, and durability of the materials with which it deals. It has to employ the proper substances for its purposes, put them in their right places, make them of suitable forms, and unite them firmly together, or so arrange them that the tendency to change their relative position when affected by external forces shall be the least possible. The civil and the naval architect or the builder, whose business it is to make large fabrics by putting together numerous small pieces, is indebted for the durability and strength of his structure to the application of statical mechanics. So also the miner or the civil and military engineer, who has to form extensive excavations, erect embankments, bridges, tide works, or fortifications, requires an intimate knowledge of statical mechanics.

Dynamical mechanics, on the other hand, has reference to the forms and combinations of materials with a view to motion. Its principal objects are the generation, communication, and application of power and force, in order to change the forms or arrangements of materials. Its products are tools, implements, machines, engines, or apparatus; it teaches us how to choose the most suitable materials, produce the strongest and most serviceable forms, arrange their respective motions so as to secure regularity and diminish resistance, with a view to economy of labour and durability of structure. The farmer, the brickmaker and stoneworker, the carpenter and smith, the spinner and weaver, the worker in pottery and metals, are all employed in changing the forms and arrangements of the materials submitted to them; and they have all to use the apparatus provided by the machinist, and the machinist practises the art of dynamical mechanics in devising and executing the implements suitable for their and his own use.

In statical and dynamical mechanics, the chief study is economy of material, labour, and cost. In any structure, whether for stability or movement, by an unsparing use of materials it may generally be possible to secure the necessary strength. But it frequently happens that the usefulness of a construction would be seriously diminished by a rude accession of mere strength; and in all cases, elegance of construction is manifested by the careful and ingenious arrangement of materials, and not by their absolute mass. Accordingly, we find that as mechanical art advances, lightness and neatness of structure take the place of magnitude and weight; and improvements in existing constructions or arrangements are made to depend for their merits on the saving of material and labour which they effect. Perhaps the most stable structures that mankind have ever formed are the pyramids; and as works of enormous magnitude, of great age and mysterious purpose, they excite admiration and wonder. But when one considers their form and the arrangement of their parts, their durability is not to be wondered at. A hill of mere sand, not blown upon by strong winds or washed by heavy rains, would retain for ever its pyramidal form, and would be an object of as great beauty and utility as the pyramid itself. The sovereigns of Egypt had probably at their command an enormous amount of human power, and knew no better object on which to employ it than the erection of some immense pile to gratify their pride while living and contain their bodies when dead. We, living thousands of years after them, do not see the pain and labour of their slaves, and scarcely know how much misery might have been spared mankind had their labour been applied to better uses. In the mere paving of the metropolis we have more material and more labour applied to a useful purpose than was required for the largest of the pyramids. In our harbours and docks, our canals and railways, we have works far more stupendous; and in our steam-vessels and manufactories we have power at work exceeding the capabilities of the whole ancient world. But with all this it is the object of the engineer and machinist to study the greatest possible economy of material and of labour.

Knowledge of Strength of Materials Necessary.—Whatever be the elements which constitute beauty, in works intended for ornament, we see beauty in things made for use, just in proportion to their fitness for their purpose. Parts out of place, deficiency or redundance of material, elaboration not called for, and deviation from just proportion, become glaring defects in any structure or apparatus formed for useful purposes; and no amount of decoration or finish can reconcile us to a disproportioned or unskilfully designed fabric. It is therefore most important that the practical mechanic should form an intimate acquaintance with the strength of the various materials with

which he deals; their powers to resist strains in various directions, and the arrangements suitable for making use of these qualities to the greatest advantage. In the theoretical portion of this subject there is much that is abstract and mathematical, but nothing so difficult that the fair exercise of judgment may not lead to very sound conclusions respecting it. All the main facts, however, respecting the strength of materials, are the result of experiment and observation; and the true use of theory in a subject like this, is to analyze and classify these results. If it is found, for instance, that several beams of timber of known dimensions are capable of sustaining certain strains applied to them in certain directions, it is not difficult to form general rules by which the strength of other beams differing in dimensions, or their strength to resist strains in certain other directions, may be computed. The careful mechanical designer either makes experiments for himself, or accepts the observations of others, and devises his structures in such a manner as to take advantage of these results. Practice in design, and frequent observation of successful works, do much to form the eye of a designer, so that the mere appearance of a thing, whether drawn or executed, satisfies his sense of just proportion, or the reverse. It is indeed a wonderful, as it is a most valuable property of the mind, that it can readily recognize and prefer the useful and suitable; and as the just and the true recommend themselves to the mental and moral sense, so does the fitting in design recommend itself to the eye. Some individuals are gifted by nature with a keener sense and readier power of discriminating these qualities, and can therefore criticise more truly or design more skilfully than others. But where there is a love for mechanical pursuits, or a desire to study mechanical works, we believe much of this power may be readily acquired.

Communicating Power.—In the dynamical branch of Mechanics there are subjects of great interest and extensive application which demand the utmost attention; we allude to the means of deriving and communicating power. If we ask ourselves what power means, we can scarcely define it by any simple term; we may say that it is the capability of doing work or producing change. We start with the idea, which is a true one, that all matter is inert, incapable of receiving motion, of being brought to rest, or of undergoing any kind of change whatever of its own accord; some extraneous force or power has to be impressed upon it, and the quantity of change effected is the work done. When we lift a weight from the ground, the hand exerts a certain

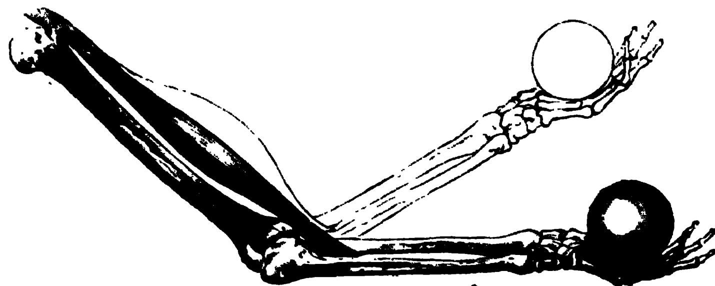


Fig. 2.

amount of power, and produces a certain amount of work; an act of mental volition communicates through the nerves, in some way unknown to us, an influence to the muscles of our arm, which causes them to contract. The muscles are attached at both

ends to the bones, which are rigid levers jointed together: one end of such a bone or lever being fixed, as at the elbow-joint, and the muscle being contracted; the other end of the lever, to which is attached the hand holding the weight, is caused to move through a certain space, and thus to lift the weight a certain height (Fig. 2).

This is a case of power employed in producing motion; but, after the weight is lifted, power must still be exercised in retaining it in its position, for were the muscles for one instant relaxed, the hand and weight would drop downwards. If such be the case, we may at first sight imagine that our notion respecting the inertness of matter is incorrect, otherwise the hand and weight would of themselves remain where our muscular power had placed them; but, looking a little more closely into the facts, we find that while the matter with which we deal is really inert, there is a constant power acting upon it in opposition to our muscular power—the power of gravitating attraction, by which all bodies near the earth's surface are drawn down to it.

This power has been found, by extended experiment and observation, to act equally on all bodies and all parts of every body; and the measure of the force with which it acts on any body is its weight. In estimating power, therefore, weight is the measure of one important element; and if we know the weight which any given power can lift from the earth, we have correct data for calculating its amount: in fact, we know the quantity of matter on which the power has effected a change. But there is another element of power quite as important as the quantity of matter acted on,—we mean the quantity of change produced upon it. If we find a certain amount of exertion necessary to lift a weight one foot from the ground, we shall find greater exertion necessary to lift it two feet. The change in both cases effected by the power applied is a change of position; but the amount of change—that is, the distance through which the weight is moved—is in the one case double that in the other. Experiment and observation, as well as reasoning, prove distinctly that the force required to lift a given weight is exactly proportional to the distance through which it is lifted. We thus acquire the means of computing another element of power; and can combine this element, expressive of the amount of change effected, with the former, which expresses the quantity of matter upon which the change is produced.

In comparing numerically one power with another, we are therefore warranted in multiplying the weight lifted by each, by the distance through which it is lifted, and comparing the results. Thus, if one man lift 20 lbs. 1 foot high, and another lift 30 lbs. 1 foot high, we say that the powers exerted by the two men are as 20 to 30, or as 2 to 3; or if one lift 20 lbs. 2 feet high, and the other lift 20 lbs. 3 feet high, the powers they exert are as 20 multiplied by 2 (that is, 40), to 20 multiplied by 3 (that is, 60); or as 2 to 3. Again, let the one lift 20 lbs. 2 feet high, and the other 30 lbs. 3 feet high, their powers are as 20 multiplied by 2 (that is, 40), to 30 multiplied by 3 (that is, 90); or, more simply, as 4 to 9.

But, recurring to our definition of power as being the capability of effecting change, we must see that there is yet another element of calculation necessary for estimating different capabilities; as yet we have reckoned only the amount of work done, and have paid no regard to the time required for doing it. A few coral insects, labouring successively for centuries, may form an island, which it would require thousands of workmen to raise in a short period; but yet the power of each workman far exceeds that of the insect, and the two can only be compared by supposing them to be exerted during equal times. Now, it is clear that a certain work being done, the shorter the period required for doing it, the greater is the power exerted; and conversely. In com-

paring powers numerically, we must therefore divide the work done by each by the time occupied in performing it, in order to be able to estimate their comparative amounts. Suppose one man lifts 20 lbs. 12 feet high in 3 minutes, and another lifts 30 lbs. 10 feet high in 2 minutes, the work done by each is 20 lbs. × 12 feet, or 240, and 30 lbs. × 10 feet, or 300, respectively; but 240 divided by 3 minutes are 80, and 300 divided by 2 minutes are 150: the powers of the two are therefore in the proportion of these numbers, 80 to 150; or, more simply, 8 to 15.

Standard of Power.—We have now found means of measuring all the elements of power, and thus determining its amount. For the sake of convenience, we fix upon some standard power with which we can compare others, just as we fix on a standard weight, such as a pound, or a standard length, as a foot; and knowing the value of several powers as compared with this standard, we can estimate their values as compared with one another. The standard power usually adopted in practical mechanics is a horse-power, which has been defined to be 33,000 lbs. lifted 1 foot high in 1 minute. The number of pounds lifted, 33,000, was determined from the average of numerous experiments made with horses at work. Whether it be an accurate expression of the power of an average horse or not, is of little consequence, provided it be generally accepted and understood as a standard measure of power. Were there no such standard, each mechanic might make an estimate of his own: one might compute the work of a very strong horse, the other of a very weak one. Thus, steam-engines or other apparatus might be supplied of all different strengths and sizes, and yet purporting to be of equal powers; or apparatus of like strengths and dimensions might be stated to be of very different powers. But, this standard once fixed, it is the duty of a mechanic to estimate, by experiment or calculation, the weight which the engine he makes can lift a given height in a given time, or the weight which moving through a given height in a given time, can work efficiently some apparatus which he may have fabricated; and he can then state the power which his engine furnishes, or which his apparatus requires, in terms intelligible to all the world. It is clear from what has preceded, that a horse-power does not mean precisely 33,000 lbs. lifted one foot in one minute, but a power equivalent to that; as, for instance, 330 lbs. lifted 100 feet in one minute, 3,300 lbs. lifted one foot in one-tenth of a minute; or, in fact, any weight on multiplying which by the distance moved and dividing the product by the time occupied in the motion, the result shall be 33,000.

These considerations may be somewhat simplified by combining the distance and time into one term, which we call velocity or speed. Velocity is directly proportional to the distance passed over, and inversely proportional to the time occupied in the transit. A railway train that passes over 50 miles in an hour, has double the velocity of one that travels 25 miles an hour, because 50 is double 25. Again, a train that travels 25 miles in an hour has double the velocity of one that passes over the same distance in two hours. To compare the velocities numerically, we divide the distances by the times of each.

In estimating powers, since our standard is given in feet for distance, and minutes for time, we divide the distance in feet by the time in minutes, and thus get the velocities. The weight in pounds lifted, multiplied by the velocity thus reckoned, gives a product which, being divided by 33,000, shows the number of horses'-power. Suppose an engineer were required to furnish a steam-engine capable of pumping 165,000 gallons of water every hour, to a height of 120 feet, he would reckon thus:—An hour contains 60 minutes; and 120 divided by 60, gives 2 feet per minute as the velocity with which the required volume of water must be lifted. A gallon of water weighs 10 lbs.;

therefore, 165,000 gallons weigh 1,650,000 lbs.; this weight moved at the velocity of 2 feet per minute is equivalent to 1,650,000 × 2, that is 3,300,000 lbs. lifted 1 foot in 1 minute. Dividing this by 33,000, the quotient is 100 horse-power as the actual force required to do the work in the time given. He would, therefore, proceed to make an engine which, after providing for all mechanical losses in the operation, should be capable of producing this effect.

The principal sources of power are, the muscular forces of men and animals, the natural motions of air and water, the weight and elasticity of materials, and the changes effected in bodies by the action of heat and electrical and chemical action. It is the business of the mechanic to utilize these forces, to regulate and control them, to change or modify their directions, velocities, or intensities, so that they may be made to do certain work in the best, most economical, and expeditious manner. In addition, therefore, to his knowledge of the materials on which, or through which, these forces have to act, he must have an intimate acquaintance with the nature and laws of the forces themselves, as discovered by experiment, or investigated abstractly. It is a fortunate circumstance for man, considering the brief period allotted for individual research, that the laws of nature are of the most simple character, and that he is gifted with faculties that enable him to communicate the results of his investigations through great distances and over lengthened periods. Every successive discovery in natural science throws additional light over all that has preceded, opens up new fields for research, and suggests new modes of practical action. In the arts likewise, an ingenious invention simplifies much that formerly was rude and cumbrous, facilitates operations formerly deemed impracticable, and furnishes the means of practising new and unheard-of branches of art. Let one but compare the state of mechanical art as it was before the time of Watt, and as it is now; and the more deeply he investigates the question, the more will he be surprised at the enormous change effected by a few simple but most ingenious modifications of a machine for utilizing the power developed by subjecting water to the action of heat.

The steam-engine has indeed exerted so vast an influence on the arts, that we shall think it necessary to devote considerable space in what follows to its description in detail. We are the more strongly urged to this, because we believe that the student who can master the details of the construction and application of this apparatus, will be prepared to find his way through any branch of Practical Mechanics.

Application of Power.—The mere supply of forces for our various purposes would be of little benefit to us unless we had ingenuity sufficient to find modes of applying them. All work, as we have already hinted, means the effecting of changes of some kind or other; and all changes of matter imply motions either of masses or of parts. A most interesting and extensive branch of Practical Mechanics consists, therefore, in the discussion of the various modes of communicating and modifying motion. The two elements of which power consists—viz. the weight or mass acting, and the velocity with which it acts—are convertible. Having a certain power to work with, we can give up a certain amount of its weight, and thereby gain in velocity, or sacrifice velocity in order to gain in weight. We cannot create any addition to the power either in weight or in velocity, nor can we annihilate any portion of it. We can, doubtless, apply the power with better effect to the work, and thereby save a loss; or we can misapply a portion of the power, and thereby render it ineffective for the purpose intended. A power applied to put in motion any part of a train of machinery, will be found acting at any other part of the same train.

If we could make our workmanship absolutely perfect, could have materials abso-

lutely rigid and frictionless, and could remove our machinery from the influence of all extraneous resistance, we should find in every part of it an equal development of power. Some parts may be moving more slowly than others, but then they act with greater pressure or weight; some parts may act with less pressure, but then they are moving with greater velocity: in short, in every part of such a perfect machine we should find the pressure multiplied by the velocity—that is, the power or momentum exactly alike. Even with our comparatively imperfect workmanship, we are quite safe in estimating according to this rule, without the necessity for making much allowance for external resistance. In the works of a clock, the wheel fixed to the barrel which carries the weight revolves so slowly, that its motion is quite imperceptible; and through the teeth of this wheel is conveyed the force which puts the whole clock-train in motion, a weight of many pounds. Again, the escapement-wheel, which sustains the motion of the pendulum, revolves with comparative rapidity, but with so little apparent power, that an opposing pressure of a few ounces might completely stop it; yet, on stopping thus the escapement-wheel, we stop also the barrel-wheel, and the few ounces applied to the one effectually oppose the many pounds acting on the other: in fact, the weight which acts on the barrel, multiplied by the velocity with which it moves, is equal to the weight necessary to stop the escapement, multiplied by the velocity with which it moves.

Friction.—We have alluded to resistance in machinery, the principal of which is friction, or the retardation caused by the rubbing together of imperfectly-smoothed surfaces. This is a subject about which little is known, or probably ever can be known. Some of its general laws have been successfully investigated, and a few general principles have been carried into practice; but its effects vary so much with every change of material—of speed, pressure, workmanship, and even temperature and other circumstances—that, after all, experience is the only real guide in all matters where it is to be considered. Except in a few mechanical arrangements where friction is employed as a useful resistance, as in the case of the friction-break of a locomotive or of a crane, it is generally the mechanic's object to diminish it as much as possible. The overcoming of friction is, in fact, the wasting of so much power; and as all machines are devised with a view to economy in the application of power, it becomes most important to reduce the waste to its minimum. In all devices for communicating and modifying motion, the question of friction becomes nearly as important as the question of strength; and many arrangements which are ingenious, and would be profitable, did friction not interfere, become comparatively useless, in consequence of its influence.

If one who had never made mechanics his study were introduced into a large manufactory, and had pointed out to him the steam-engines, the cranks, levers, wheels, pinions, straps, and other contrivances for communicating power throughout the building, and setting in motion each machine,—he would at first be bewildered, and inclined to believe that the art of communicating motion was intricate and complicated; but were he really to analyze the process carefully, and trace exactly the progress of the power or motion from the prime mover to the last machine, he would find the whole effected by the combination of a few simple mechanical elements; and wonder that through means so simple, results apparently so complicated could be attained.

Students of astronomy cannot help being struck by the circumstance, that the various motions of planets, satellites, and comets, and probably also of the stars and nebulæ, of the farthest and greatest, as well as of the nearest and smallest, of the heavenly bodies, are all the result of two simple forces, acting in various directions and

with different intensities. The movements of the vast machine of the universe present, to a superficial observer, inextricable confusion and inexplicable irregularity; but to a mind like that of Newton all was skill, order, and beauty. The very disturbances to which the different parts of the grand machine are subjected are elements of stability, the irregularities are sources of permanence. In a system of machinery devised by human ingenuity and executed by human hands, there can, indeed, be no approach to this simplicity, order, and harmony; everything must be imperfect in proportion as man is imperfect, when compared with the Divine Mechanic of the Universe.

In every human work there must be elements of decay, sources of irregularity, and causes of instability; and these can only be reduced by diminishing the extent and complication of the work itself. Simplicity is, therefore, the great aim of the mechanic, especially in arrangements for communicating motion. Every wheel, every lever, every pulley that can be saved, is saved—not so much to avoid the cost of its introduction, as with a view to simplify the machinery, and thereby diminish the amount of wear, and increase the permanence of the whole. To the eye of a practised mechanic, therefore, the beauty of a piece of machinery seems to depend more on its simplicity than on any other principle. The fewer the parts required in any apparatus to render it effective, the more ingenious is its contrivance, and the less is it liable to derangement, irregularity, and decay.

Governing Power.—Next in order to contrivances for communicating power or motion, may be studied those for regulating and governing it, so as to secure uniformity of action. All the forces we employ, with the exception of gravity, are subject to continual variations of intensity; and even the most uniformly regulated forces, when transmitted through trains of machinery necessarily imperfect, are subjected to considerable variations. Further, the forces we employ are chiefly used for effecting changes on materials; and as the qualities and conditions of the materials vary, so the quantities of force required to do the work upon them differ. It becomes, therefore, most important that arrangements should be devised for compensating all these variations of force; and accordingly great ingenuity has been developed in contrivances for that purpose. We may quote a very beautiful specimen of mechanic skill applied for a purpose of this kind, in order to illustrate the great use of such arrangements. Wind-power is used to a great extent for putting machinery in motion; and yet, as we all know, nothing is more variable than the force of wind, both in direction and intensity. In former times wind-mills were made so that the miller, watching the direction of the wind, could turn round the sails of his mill to face it, and furl or unfurl those sails as he found the breeze too strong or too light to give the velocity of movement he might require. But in modern wind-mills the wind itself is made to regulate the machinery by simple but ingenious arrangements. In the first place, if the wind change in direction, it necessarily blows upon a small subsidiary set of sails placed so that they can be acted on by a wind that does not directly blow upon the main arms; by a little simple machinery, this side-action of the wind is made to turn round the head of the mill until it brings the great sails into the proper position to receive its direct impulse—just as the wind acting on a weathercock brings the arrow-point round to face it, by its pressure on the broad feather at the other end. Again, should the force of the wind increase, this very increased force is made to act upon a regulating apparatus so as to furl the different divisions of the sails, or to turn the surfaces of which they are made up edgeways to its impulse, and thereby to diminish the surface on which it acts proportionally to its increase of intensity. The consequence of these contrivances is, that from whatever quarter the wind

may blow, the sails of the mill are always directly opposed to it so as to receive its full action; and whatever be its force, whether a gentle gale or a stiff breeze, the power communicated to the machinery does not greatly vary. Perhaps the most ingenious devices for regulating power and velocity are those employed in apparatus for the measurement of time—clocks, watches, and the like, which form one most interesting branch of mechanical art, Horology.

Nature of Machines.—Having obtained the power necessary for our purposes, and having found the means of communicating it to our machines, and of regulating its intensity so as to suit the work to be done, we have next to inquire into the nature of the machines themselves, or the contrivances through which the power is made to act on the inert material subjected to it.

We can scarcely venture to offer a very distinct classification of machines; nor indeed, in a work of limited compass like the present, could we pretend to discuss in detail all the different classes of machinery now in use. The subject is so extensive and demands so minute a knowledge of what has been done by thousands of ingenious mechanics, that even the professed machinist cannot pretend to an intimate acquaintance with every branch of it. But the qualities of mind and the experimental training which render a man skilful and adept in one or two of these branches, make it easy for him to take up intelligently any other branch that may be brought under his notice. The same general mechanical principles pervade all kinds of machinery, the modes of applying them being varied according to the nature of the material subjected to their operations, or to the kind of work to be done. With the exception of certain special apparatus, we believe almost all the machinery used in modern times may be classed under some of the following heads:—

I. *Machinery for raising Weights and giving Pressure.*—Among these we may notice especially the simple mechanical powers, the lever, wheel and axle, pulley, inclined plane, wedge, and screw. Next, in order of complication, may be mentioned such machines as cranes, crabs, capstans and windlasses, slips, tackle and travellers, jacks, screw and lever presses, printing presses, and the like.

II. *Machinery for effecting Transit and Communication.*—This branch has of late years attained immense importance by the extended use of railways and steam vessels; and under the head of transit, it will embrace the consideration of vehicles for land transport, and vessels for water carriage, and the various modes of putting them in motion. For the communication of intelligence, we now practise an art unknown in former times, and one deserving of detailed consideration from its marvellous nature and the important influences it is likely to exercise upon human civilization. We allude to the electric-telegraph, to which perhaps greater ingenuity has been devoted within a few years than has been displayed in any other branch of mechanical art during preceding ages.

III. *Machinery for Moving Fluids.*—This branch naturally includes hydraulic apparatus, such as pumps, fire-engines, hydraulic presses and lifts; and also machinery for moving air, such as bellows, blowing-cylinders, fanners, and the like.

IV. *Machinery for changing the forms of Solid Materials.*—This is perhaps the most extensive branch of mechanical art, as it includes all apparatus for cutting, piercing, moulding, bending, crushing, and such like operations. It may be subdivided, according to the materials on which the operations are to be effected, as follows:—

1. Machinery for preparing animal and vegetable products; as oil, tallow, leather, flour, sugar, vegetable oils and extracts, caoutchouc, and gutta-percha.

2. Machinery for preparing mineral products; as stones, bricks, cements, ores, pottery-ware, glass, pigments, and the like.

3. Machinery for sawing, planing, moulding, bending, and carving timber.

4. Machinery for working metals; as in the operations of moulding and casting, forging, rolling, and wire-drawing; bending, shearing, punching, and rivetting; drilling, turning, and boring; planing, shaping, and the like.

V. *Machinery used in the manufacture of Textile Fabrics.*—This is likewise a most extensive and interesting branch of mechanical art, and one of the highest importance to us as a nation, as well as to the world at large. It includes directly the apparatus and processes employed in preparing the crude materials, such as wool, flax, cotton, and silk; the various operations of dressing, carding, spinning, weaving, dyeing, bleaching, calico-printing, and the like. Under the same head may also be discussed the machinery for making ropes and cordage, and for the manufacture of paper.

VI. *Machinery for Measuring and Calculating.*—The apparatus included in this class are mostly of that exact character required for philosophical experiment and observation, such as indicators, dynamometers, gauges, balances, and mathematical and optical instruments; but there is one extensive and interesting branch, horology, devoted to apparatus for the measurement and division of time, which will deserve especial consideration, as well from its usefulness as from the great ingenuity displayed in it. In this class we may include some other apparatus, which, though not distinctly falling within the scope of its title, yet present in some respects considerable similarity to some of those included; we mean automata and musical instruments.

These six classes, we believe, include the greater part of the machinery used in modern times, with the exception of such as rather fall within the range of other treatises than of one devoted to practical mechanics,—as, for instance, agricultural implements, and implements of war. In what follows we will not pretend to give detailed descriptions of many of the different machines included in those classes, we shall endeavour rather to select a few of such as are most generally used, and involve in their construction the principles which are applied, in a modified form, to others,—reserving the subjects themselves for future exposition.

While improved education and extended acquaintance with principles have placed those who direct mechanical labour in a better position as to knowledge of their art, the division of labour and the extensive use of mechanical contrivances in the place of manual labour have certainly lowered the position of the workmen as to general knowledge of their trade. Formerly, the millwright knew all about the machinery he made, and could turn his hand to all the operations required in its construction; now, there are mechanics who can only turn, others who can only file, and only very few who to skill of hand unite a knowledge of the machinery of which they execute portions. We are convinced that this state of things is injurious both to workman and to master; for a man can never labour with hearty good-will at work which he does not understand, and in which he therefore takes no intelligent interest; he can only act as the machine at which he works, and he is thus morally and intellectually degraded. We would have every workman understand the character and tendency of his operations, and take an interest in them. He would thus be relieved from much of that monotonous drudgery which his ignorance forces him to undergo; he would be prepared for emergencies; he would see modes of economising labour, and of improving the work on which he might be engaged; and he would be provided with a source of rational amusement in his leisure hours.

But not to the mechanical workman alone do we think that a general knowledge of practical mechanics should be supplied; we think that it should form part of the education of every one, whatever be his position and his prospects. We have seen of late deplorable instances of want of contrivance in military affairs, and of sacrifices of blood and treasure in consequence. We are eminently a commercial, manufacturing, agricultural, and colonizing nation; our commerce is conducted by means of railways, steam and sailing vessels, and requires warehouses, docks, and quays; our manufactures are all the product of mechanical contrivances; our agriculture is now rising to its just position as a mechanical and chemical art; and our colonies have been successful from the energy displayed by our emigrants in giving scope to their natural genius for mechanical adaptation of means to an end. Of late we have assumed the position of a warlike nation, and we have sadly asked ourselves why there has not been employed in war any of that mechanical skill and ingenuity which characterize our peaceful arts.

Notwithstanding all this, practical mechanics have never yet been cultivated as a branch of general education; and consequently every man, whatever be the walk of life in which he chooses to tread, has to begin his real education after he leaves school. We think this evil should be remedied,—that every man should have instilled into his mind in his early years a thorough knowledge of common things, so that when he advances in life he may enter more readily on any of the professions or trades practised at home, and be the better prepared for the emergencies of commerce, colonization, or warlike expeditions abroad. Among the population of the land there lies dormant a vast amount of talent and ingenuity, which at present is so much loss of capital to our country and to the world. Let some opening be made for its cultivation, and we doubt not that a few years would bring about a more astonishing development of our resources, extension of our commerce, and improvement in our arts, than all the marvellous advance of the last thirty years. While we fully admit the high importance of other branches of mental and moral training, we would also strongly urge the advantages of insight into the mechanical arts, because through their extended cultivation we expect to secure a great increase of our material prosperity and comforts, a great diminution of labour, improvement of health and strength, and much diffusion of intelligence among all classes.

MECHANICAL DRAWING.

Knowledge of Drawing Essential.—In the actual practice of mechanical art, drawings are invaluable; they show the true forms, dimensions, and arrangements of machinery to those accustomed to their use with greater clearness than a model, or even the full-sized work itself. The draughtsman devises the arrangement of his machinery; sketches it on paper; calculates the strength and proportion of the parts; draws them out full-size or to some suitable scale; studies their combinations on paper; improves this part, strengthens that part; modifies the forms so as to save complication, material, or cost of workmanship; traces the action of the whole; provides safeguards against accident or undue wear and strain; and having at length fully embodied his ideas on his plans, sections, or elevations, places them before the workman for execution. The workman has, in general, no need to study any of those details which fall within the province of the draughtsman; he has only to put his rule to the drawing, and measuring every dimension there indicated, shape the material with which he deals in exact accordance with it. Without drawings it would be impossible to make any advance in

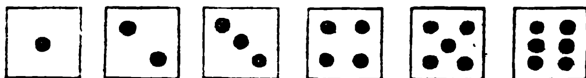
mechanical art. The paper, pencil, ink, and colours, are cheap materials; and the time and labour occupied in making a drawing are as nothing compared with the work in fashioning the solid material. To scheme on paper is, therefore, most advantageous in every point of view, as by that process only can be secured that economy of material and labour, harmony of action and justness of proportion, which constitute the beauty of a mechanical device. To scheme in the solid materials is a most expensive as well as unsatisfactory process, and one that seldom leads to successful results. We know of no circumstance that can warrant its adoption, except in cases where there is no practical experience to guide the mechanic as to the power required, or the mode of operation suited to the work he has in hand. Even in such cases it is generally possible, by the exercise of a little judgment and ingenuity, to arrange some simple and inexpensive experiment which may give to a practical man a tolerable notion of the kind and scale of machinery that will be required.

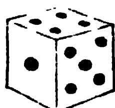
A system of mechanical notation was proposed some years ago by the ingenious Mr. Babbage. In devising his calculating machinery, which consisted of a great number of parts, many of them merely repetitions, he found it difficult to imagine all their simultaneous movements without an excessive and painful exercise of that mental power which has to deal with such matters. He therefore attempted, and with considerable success, to trace by written symbols the flow of motion through a train of machinery. We are not aware that his system of notation has been adopted by practical men. Indeed, most of the complex machinery with which the engineer has to deal is of a kind similar in many respects to works previously executed, embodying improvements that may have been from time to time effected in them; his mind is, therefore, in a manner prepared for the conception of the various motions and connections by his practice in watching those in machines in action, and he has comparatively little difficulty in fully imagining the intended action of the work on which he may be engaged. It will, therefore, be our first duty to offer a few practical suggestions on drawing as applied to machines.

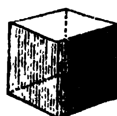
Plane Surfaces.—When we have to draw a triangle, square, circle, or any other superficial figure, upon paper, we find no difficulty in giving a full and accurate delineation of it in respect of form and dimensions. The paper on which we draw is itself a plane surface; and so long as we are not limited in length and breadth, we can delineate any form whatever upon it. In the same manner we can describe any of the flat faces of any object presented to us, such as a circle, or pyramid, or prism; and give an accurate representation of any two of its dimensions, whether we call these by the names of length and breadth, or height and width. But when we have to delineate a solid body which has three dimensions, length, breadth, and thickness, we must either cut our paper to pieces, shape it so as to correspond with the different faces of the body, and put them together in similar order, so as to form a model of it; or we must have recourse to some device that shall enable us to comprehend for ourselves, and to communicate to others, an accurate notion of the solid body we propose to delineate on a flat surface. When the painter draws a portrait, or executes a landscape, he has to imagine that between his eye and the object which he draws there is interposed a surface or sheet of some kind on which he sees the object drawn as in his picture. He delineates it on the canvas, therefore, exactly as it would appear upon this interposed screen. In a camera-obscura the rays of light proceeding from every point of objects presented to it, are concentrated or gathered together by a lens into a particular place or focus behind it. A piece of frosted glass placed in this focus offers a surface for these rays to illuminate, with their respective lights, shades, and colours; and an observer looking on the frosted

glass, sees a perfect miniature picture of the objects. A plate of prepared metal or glass placed in the focus, receives exactly the same picture; and certain substances spread over its surface, rendered sensitive to chemical properties of rays of light which the eye cannot appreciate, undergo changes in their constitution, which can be rendered permanently visible.

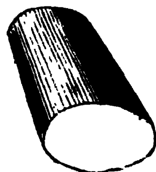
Drawing Cubes.—Pictures are thus produced by hand and by photography, which present upon flat surfaces delineations of solid bodies as they appear to the eye. But if we were to apply a compass or a rule to the measurement of the bodies represented in these pictures, and also measure the objects themselves, we should find the angles, dimensions, and proportions of the picture totally different from those of the real bodies. Thus a cube, such as a die, which has six equal square faces, marked by dots as in Fig. 3, when represented on a picture, might present such an appearance as in Fig. 4, where only three sides are visible, none of them squares, and no two alike: not a single angle or dimension equal to those in the die itself. And yet the picture may be a perfectly true delineation of the die as it appears to the eye of an observer. Now, if a workman were furnished with such a picture, and required to make a die according to it, he could have no conception of its form and dimensions. For instance, he might cut a piece of plate to the shape of an irregular six-sided figure, with three lines engraved on it meeting at a point near its middle, and certain dots marked at the distances indicated on the picture. He would then have made a solid body which viewed in one direction would certainly present the appearance of the picture. Even if the picture were shaded and marked by dotted lines, indicating the invisible edges of the die, as in Fig. 5, he could not, without further information as to its meaning, construct a solid body such as would answer to it. He would have to be told that it was a solid having six equal flat sides, which, viewed in a certain direction, and at a certain distance from the eye, under light falling upon it at a certain angle, presented the appearance of the picture. He would then either model all conceivable kinds of six equal-sided bodies, and try each in the given aspect, or he would work out geometrically some other properties of the body, such as might enable him to construct it. If, on the other hand, instead of having such a perspective picture presented to him, he had a drawing like Fig. 3 placed before him, and were told to make a piece of ivory of such a shape that each of its six sides should be exactly of the form drawn, and either of its dimensions, or some given multiple or fraction of these dimensions—as for instance, for every inch on the drawing, three inches in the die itself,—he would at once comprehend and execute the work required of him.

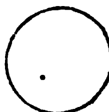

Fig. 3.

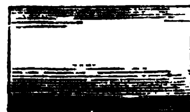

Fig. 4.


Fig. 5.

Drawing Cylinders.—In like manner, if a cylinder or roller were pictured as it appears to the eye, with its light and shadow as in Fig. 6, the workman might guess at its form, but could ascertain nothing


Fig. 6.




Fig. 7.

as to its dimensions. But if he were told to make a thing of which the drawing is, when delineated, one-quarter of its full size (Fig. 7), he could at once form his material to its proper diameter and length, by measuring the drawing, and allowing an inch in his work for every quarter of an inch on the drawing.

The object of mechanical drawing is, therefore, not to present a delineation of any object as it appears to the eye, but to furnish the exact figures and dimensions of its parts in such an intelligible manner, that solid materials may be fashioned into shapes, whose parts shall have similar forms and proportions. The principle on which mechanical drawing is founded, is that of projection on plane surfaces, which we shall now endeavour to explain.

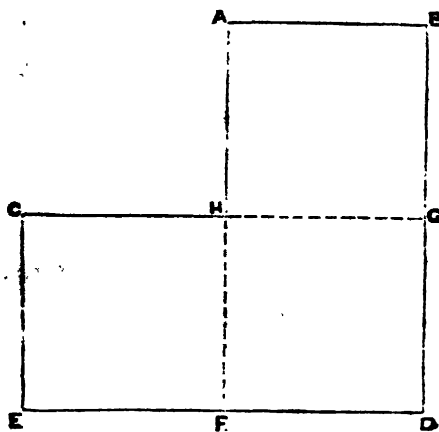


Fig. 8.

Suppose a sheet of paper were cut to the form A B D E C H (Fig. 8), and folded across where marked by the dotted lines H G and H F, the two parts A B G H and C H F E being turned up so as to stand at right angles or square to H G D F, and to one another as in Fig. 9, forming, as it were, three sides of a square box; then each of these three portions of the paper becomes what is called a "plane of projection" for receiving the representation of one of the sides of a solid body placed somewhere within the imaginary box of which they form the sides. Let us suppose, for instance, that a die were suspended within the box (Fig. 10), and that from every point in each of the surfaces of the die exposed to the three sides of the box, lines were supposed to be drawn perpendicular or square to those sides as indicated by the dotted lines;—then the lines joining the intersections of the dotted lines with the upright and horizontal surfaces would enclose

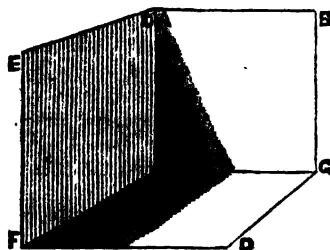
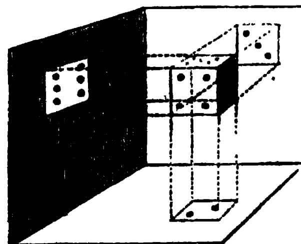


Fig. 9.


Fig. 10.

figures accurately representing in form and dimensions the sides of the die presented to them. The paper being then unfolded, would contain a mechanical drawing of the die (Fig. 11), exhibiting accurately the forms and dimensions of three of its sides. In order to get a drawing of the other three sides, the sides of the imaginary box

might have similar figures projected or thrown upon them. It is evident that whatever be the situation of the die as to distance from either of the three planes of projection, there will be no difference in the drawing; the form delineated will be the same, though it may appear on a different part of the paper.

Names of Projections.—The names usually given to the three drawings of an object are these:—The projection on the horizontal plane or bottom of the box, is called the *plan*; and the projections on the vertical planes or upright sides, are called *elevations*. One of the elevations may be called the side elevation, or side view; and the other the end elevation, or end view; while the plan, when projected upwards or on the top of the box, is sometimes called the bird's-eye view, having nearly the form which an object would present to the eye of a bird soaring above it. For drawing the exteriors of all solid bodies, bounded by straight lines and flat surfaces, these projections are generally sufficient. But it is often necessary to make such drawing as shall give a correct notion of the interior construction of bodies.

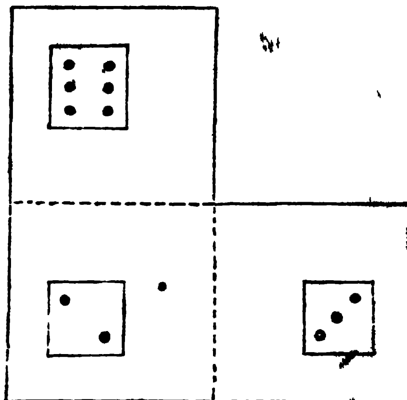


Fig. 11.

Suppose, for instance, we wished to have a cubical box constructed of wood, having a certain thickness, and fitted with a vertical partition of a certain height, and at a certain distance from the ends. In order to indicate this by a drawing, we must show its internal construction. To effect this, we may still use the three planes of projection; but as these planes are imaginary surfaces, we may easily conceive some of them to pass directly through the substance of the box: in other words, we must conceive the box to be sawn or cut across in any direction, and the form

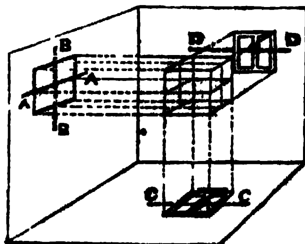


Fig. 12.

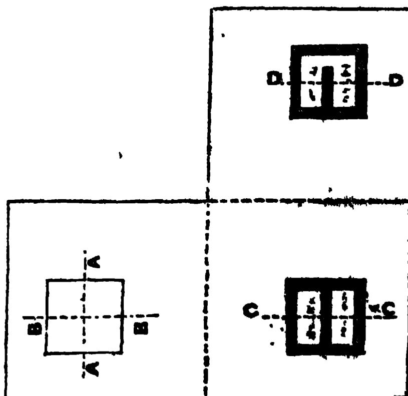


Fig. 13.

presented by the parts so cut projected on the planes (Fig. 12). Thus, if a horizontal and a vertical cut were made as indicated by the dotted lines, we should have a plan and a side elevation of the cut

surface, in which the thickness of the wood, and the position and height of the partition, would be clearly shown. These views would be called sections, or the drawings of cuttings. The horizontal projection would be a sectional plan, and one of the vertical projections would be a longitudinal or transverse section, while the other would still remain as an end or side elevation. The dotted lines A A and B B on the end elevation and section correspond with the horizontal plane of section; and the lines B B and C C on the end elevation and plan correspond with the vertical plane of section. In a drawing, then, of a box, such as in Fig. 13, which would be seen on unfolding the paper of projection, enough of the form, thickness, position, and height of the partition would be represented to enable a workman to construct the box.

Now, if the part of the paper containing the end elevation were cut away from the rest, and turned round so as to bring the letters on it into a more convenient position for reading them, as in Fig. 14, the accuracy of the drawing would in no respect be altered, provided always that it were clearly understood what elevations, plans, or sections, the different parts of the drawing are intended to represent. There is another mode of considering this question. The rays of light by which the eye is enabled to perceive the forms of an object, proceed from every point of the object in straight lines to the eye. Suppose, then, one face of a die were presented directly to the eye, as in Fig. 15, the rays of reflected light, proceeding from every point in the surface of the die to the eye, would all converge or draw together towards the small opening which the pupil presents; and entering there, would produce the image which enables the spectator to see the object.

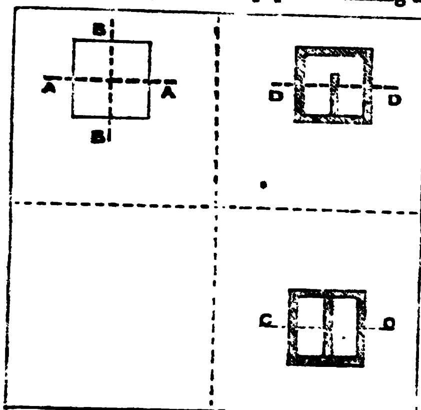


Fig. 14.

This would be the case however far off, or however near, the eye were to the die; but the greater the distance, the less the convergence of the rays, or the more nearly are they parallel to each other, as may be clearly seen by the diagram Fig. 16, where an arrow is supposed to be the object, and the rays of light proceeding from its extremities are indicated by the dotted lines to an eye at the various positions, *a*, *b*, and *c*.

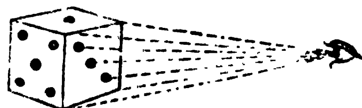


Fig. 15.

While those to the eye at *a* converge very rapidly, those to *b* still converge, but more slowly, and those to *c* more slowly still. But if we could conceive the eye removed to an incalculably great distance from the object, and suppose that its powers of vision were still sufficient to receive it, the convergence of the rays

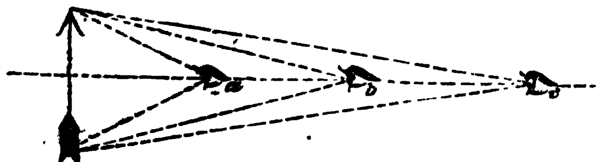


Fig. 16.

would be immeasurably slow: or, in other words, the rays would be so nearly parallel that we could not appreciate any convergence at all. Retaining, then, this conception of extreme distance, we may say without error that the rays are quite parallel. We have in nature actual cases of this kind; for the fixed stars are so very distant that no difference can be traced in the direction of rays of light coming from them to the earth at different parts of her orbit, although the distance across that orbit is nearly two hundred millions of miles.

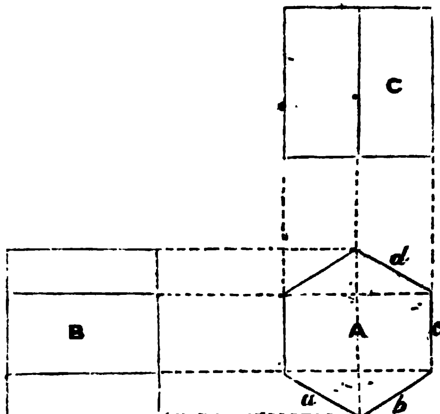


Fig. 17.

Now, we have already described that the different projections of an object are formed by supposing parallel lines to be drawn from the different points of it to certain planes; and if we suppose these parallel lines were rays of light, such as we have described, proceeding to an immeasurably distant eye, we attain a mode of understanding what these projections mean. According to this notion, the plan or horizontal projection is the view which an object would present when we look directly down on it, or directly up to it, from a great distance; the side and end elevations are the direct views of the side and end respectively; and sections are the direct views of the object when cut or sliced across in any direction. Bearing this in mind, we may then be prepared to make mechanical drawings of most objects with facility.

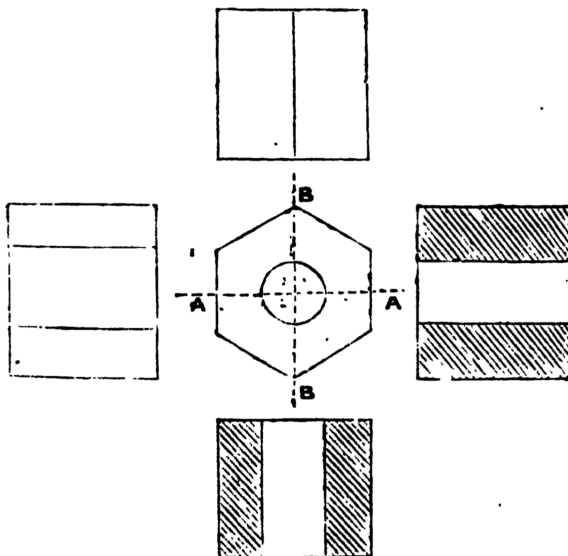


Fig. 18.

Suppose, for example, we had to draw an object such as mathematicians would call a hexagonal prism—that is to say, a solid figure having six equal oblong sides and two six-sided ends. Supposing it to stand on one of these ends, and that we looked directly

down upon it, we should see its plan, which is a hexagon (A, Fig. 17). Again, looking directly on the angle formed by the two sides *a* and *b*, we should see the elevation C, showing the two retreating sides *a* and *b*; and looking direct on the side *c*, we should see the elevation B, showing the full front view of *c*, and views also of the two retreating sides *b* and *d*. The dotted lines represent portions of the supposed parallel rays of light proceeding from the angles of the object, and prolonged to form lines, projections, or drawings of the two views, the position of the top and bottom lines of these projections being determined by the height of the prism which we suppose to be given.

Let us now suppose that the same prism is to be made with a circular hole passing through its centre from end to end, we may show the same views as before, and in addition to them two sections, as in Fig. 18.

The lines A A and B B, through which the sections or cuttings are supposed to be made, are marked on the plan, and the sections are formed exactly like the elevations by tracing the different points of the object where the section lines cut it, as indicated by the dotted lines. We have hatched or drawn diagonal lines across the solid parts which are supposed to have been cut asunder, in order to distinguish them by a sort of shading from the open circular part where no solid material is supposed to have been cut through.

We will now take a somewhat more complicated drawing as an illustration—for instance, of a toothed wheel, in front elevation (Fig. 19), sectional plan on the line A A, and elevation as seen edgeways, which we shall call side elevation.

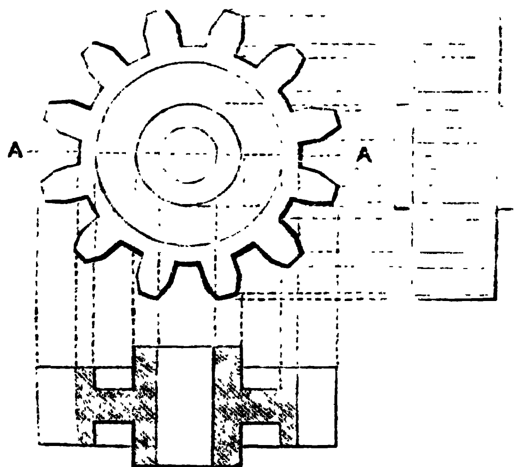


Fig. 19.

In this case, as in the former, the dotted lines are drawn parallel to each other from the different visible points or edges, and furnish us with the positions of these in the side elevation and plan; the width of the teeth and thickness of the material being supposed to be known independently.

Projection of Curves.—By the same method, drawings of curved lines may be made, some consideration being given in each case of the mode most suitable. Let us take for an example a drawing of a screw. In the first place, we must understand what a screw is. Suppose a cylinder or roller were revolving round its axis, and that while it was so revolving a pencil or sharp point were held against it. If the point were at rest while the roller revolved, a simple circle would be described on the surface of the latter. But suppose that while the roller revolved the point were made to traverse lengthways along it, then a screw-line would be drawn on the surface (Fig. 20).

The pencil having begun to mark the roller at *a*, and having advanced to *b*, while the roller has made one complete turn, the screw line *a c b* would be marked on its

surface. As we could not see both sides of the roller at once, we can only draw the part *a c* of the screw as visible, the other part *c b* being marked by a dotted line to show that it exists, and would be seen were the roller transparent. The whole line *a c b* would be called a thread or complete turn of the screw; and the distance *a b* would be called the pitch, or distance between two adjacent threads. It is clear that were the roller to continue to turn and the pencil to advance, the thread would continue to be marked as far as the surface of the roller might extend. The screw marked in the figure would be technically called a "drunken thread," or be said to be of irregular pitch, as there was no relation between the velocity with which the roller revolved and the pencil advanced respectively.

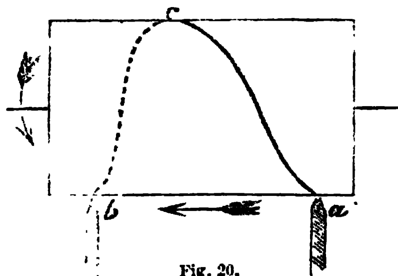


Fig. 20.

But if while the roller revolved uniformly or with regular and equal velocity, the pencil also advanced uniformly, a thread would be traced perfectly regular and equal in all its parts, or a true screw, such as is used in mechanics, would be formed. We will now draw such a screw. Let A (Fig. 21) be the end-view or section of the roller, which is merely a circle, and let B be a side elevation; *a* being the position of the pencil when it began to mark the thread, *b* its position after one complete turn, and *c* its position after two turns; it is our object to find the shape of the line, or at least its drawing or projection on the elevation B, connecting the points *a* and *b*, and *b* and *c*, respectively. Now recollecting that a regular screw implies that while the roller makes any part of a turn, the pencil must make an advance through the same part of its pitch, we will divide the circumference of A into four equal parts by the points *d*, *e*, *f*, *g*, and also the two pitches *a b* and *b c* into four equal parts, marked by the points *h*, *k*, *l*, *m*, *n*, *p*. Then we know that while the roller has turned one quarter round, so as to bring the part of its surface *e* to the pencil, the pencil itself has moved one quarter of its pitch along, so as to be at the point *h*. If then we trace a line of projection from *e* along the elevation, and a line square to it from *h*, the point e_1 where these lines cross must be a point in the projection of the thread. In the same manner, tracing lines from *f* and *g* and from *k* and *l* crossing them, we get other points, f_1 and g_1, in the thread; and repeating the process for the next thread, we get the points e_2, f_2, and g_2, similarly situated in it. But though we might between the points d_1, e_1, f_1, &c., draw any sort of lines straight or crooked, we should not be warranted in assuming them to be the proper representations of the screw. We can, however, again subdivide the circumference of A and the pitch

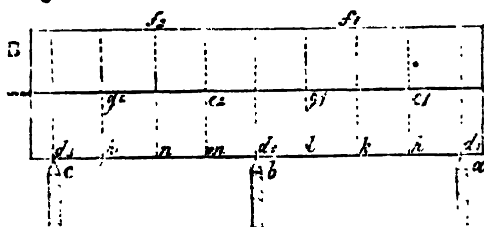


Fig. 21.

on B, and thus get a number of other points, as indicated in part on Fig. 22, which must be in the projection of half the screw. A curved line traced through these points will then be a correct representation of half the thread; and as the thread is perfectly regular, the same form of curve will be repeated at the equal intervals of pitch, from d_1 to f_1, and d_2 to f_2. The intermediate parts of the thread from f_1 to d_2, and from f_2 to d_3, being traced on the side of the roller where g is, cannot be visible at the same time as the other portions; but if they could be seen, as in the case of a transparent roller, they would be symmetrical with the visible parts, but inverted, as indicated by the dotted lines in Fig. 23.

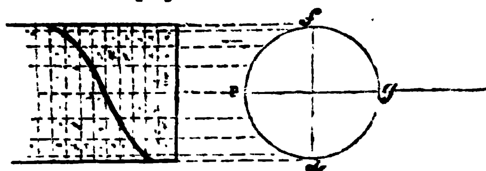


Fig. 22.

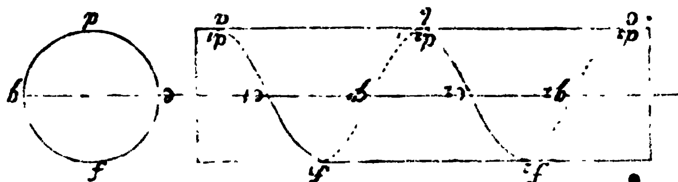


Fig. 23.

Next let us suppose that two pencils were held against the roller a little distance apart, so as to trace two screw-threads parallel with each other; the side elevation of this, as in Fig. 24, presents merely a repetition of the proper curved lines at the proper distance apart—that is to say, if any number of straight lines were drawn parallel to the axis or sides of the roller, the portions of those straight lines intercepted by the screw-curves, viz. $a\ b$, $a_1\ b_1$, $a_2\ b_2$, &c., also $c\ d$, $c_1\ d_1$, $c_2\ d_2$, &c., &c., would be all equal to one another. The ordinary practical mode of making such a drawing would be to set out the points of one portion of the curve, such as $a\ a_4$, by the method indicated on Fig. 22, to shape a piece of card-board or thin wood to fit it, and then marking out the proper distances $a\ b$, $a_1\ b_1$, $a_2\ b_2$, $b\ c$, $b_1\ c_1$, $b_2\ c_2$, $c\ d$, $c_1\ d_1$, $c_2\ d_2$, &c., and applying the shaped card to those points successively, trace by a pen or pencil as many repetitions of the curve as might be required. By a similar process, inverting the card, the elevation of the curve as it would appear through the roller were it transparent, as indicated by the dotted line in Fig. 23, could be repeated.

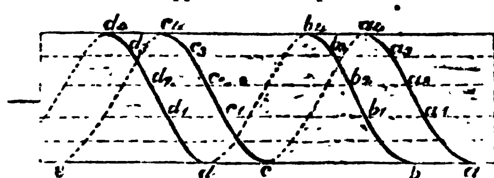


Fig. 24.

Recurring to Fig. 24, let us now suppose that all that part of the surface of the roller contained between the double threads were cut down to a certain depth, so as to leave a real solid screw-thread prominent, such as would be produced by winding a square wire obliquely round a cylinder. Our method of drawing this would be as follows (Fig. 25) :—The transverse section shows two circles, one the boundary of the outside of the prominent thread, and the other the roller on which it is wound, or the bottom of the cutting made in the solid roller. Let half of both these circles be divided into

any number of equal parts, and let the length *a b*, which the thread traverses during half a turn, be divided into the same number of equal parts; then, by drawing parallels from

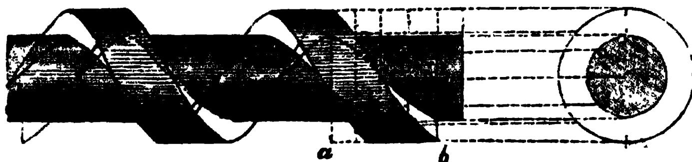


Fig. 25.

the division points of the two circles until they meet perpendiculars from the division points of *a b*, we get the half limbs of the outside and inside curves; and shaping a card to each of those, we can repeat them successively. A little consideration will show what portions of these lines would be visible in a side-view, and what parts would be concealed. It is usual to show by full lines only the visible portions, the invisible parts being marked, when necessary, by dotted lines.

Other curves can be treated in the same way as screw-curves, the general principle being the same to all—namely, finding the projection of a number of points through which the curve must pass, and tracing through those points a continuous curved line as nearly as the eye can judge or the hand execute. The greater the number of points projected, the more nearly accurate will be the drawing of the curve. But a little practice in drawing, and careful observation, soon enable a draughtsman to trace projections of curves with sufficient accuracy without requiring very many points for their determination. The drawing of a screw-curve, such as we have described, is as difficult as any of the projections that ordinarily occur in mechanical drawing. We have, therefore, dwelt upon it at some length, being convinced that the student who sees his way clearly through this example, will master most others without great difficulty.

Drawing Unfolded Surfaces.—Besides the method of drawing objects by projection, it is sometimes useful to represent surfaces developed or unfolded. This may be clearly understood by conceiving that a pattern of some kind—say a series of squares, as *a* in Fig. 26, is wrapped or folded round the roller *b*. We shall suppose that the piece of flat paper, or fabric, containing the pattern, will fold exactly round the roller so that the edges meet.

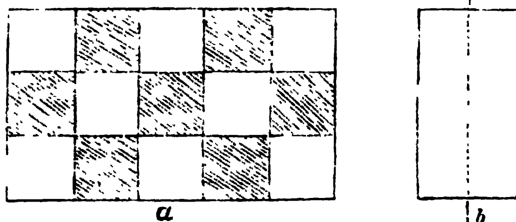


Fig. 26.

The projection of this pattern, as it would appear in a side elevation of the roller, may be drawn as in Fig. 27. Observing that the pattern is divided into three parts in height and five in width, we draw an elevation and a plan of the roller, as in Fig. 27, dividing the elevation into three equal parts in height, and dividing the circumference of the circular plan into five equal parts by the points *a*, *b*, *c*, *d*, *e*. Tracing up lines from such of these points as would be visible, looking on one side of the roller, viz. *a* and *b*, so as to cross the horizontal lines on the elevation, we get at once the elevation of the pattern as it would appear in projection when folded round the

roller. This would really be a process of envelopment, for we have first drawn the pattern on a flat surface and then enveloped the roller in it. The process of development is just the converse; for it consists in developing or unfolding a surface already on the body, and then spreading it out flat. Suppose, for instance, that we had the elevation of a roller, such as Fig. 28, and we desired to develop the pattern which is represented in projection on its surface. We draw any required number of horizontal lines, as a, b, c, d, e, through the marked points of the pattern, and trace perpendicular lines from the same points till they meet the circumference of the circular plan in the points f, g, h, k, l. We then draw a straight line of the length of half the circumference of the circle, and make it the base of an oblong of the same height as the roller. Dividing the base by the points $f_3\ g_3\ h_3$, into parts of the same lengths respectively as the parts of the half circumference separated by $f\ g\ h$ in the plan, and dividing the height, $e_1\ a_1$, by the points $b_1\ c_1\ d_1$ into parts corresponding with the divisions of the roller in height; and through all these points, drawing parallels and perpendiculars to the base; we get the positions of the different points $f_1\ g_1\ h_1$, &c., in the elevation, as they should be in the development, and can trace through these points the lines of the pattern. Should the pattern be complex, or made up of curved lines, we should have to trace the development of a greater number of points in order to get a correct drawing of it as it would appear when unfolded.

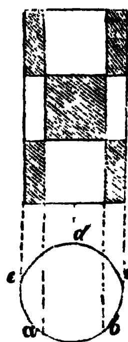
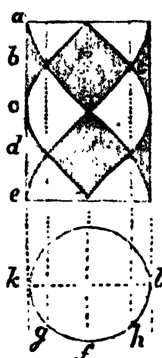


Fig. 27.



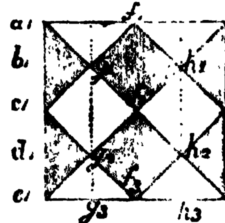
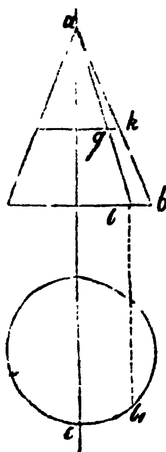


Fig. 28.



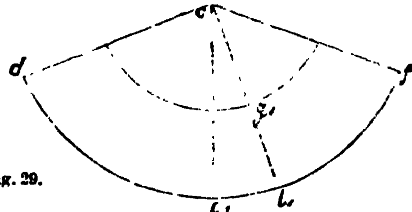


Fig. 29.

We may here observe that a conical surface is developable as well as that of a cylinder or roller; and the mechanical draughtsman has seldom occasion to deal with the development of any surfaces except these.

The development of a conical surface is effected thus (Fig. 29). Since all the straight lines drawn from a the apex of the cone to the circular base are exactly equal, we have only to draw a circle from any centre c with a radius $c\ d$ equal to $a\ b$ the length of any one of those lines. Then measuring round the circular plan of the base from any point e, and setting off the same measurement from a point d to f on the circle we have

described from the centre c, and drawing the straight lines $c\ d$ and $c\ f$, we get the development of the conical surface. A piece of paper cut to the shape of the circular sector $c\ d\ f$ would exactly fold round the cone, the edges $c\ d$ and $c\ f$ meeting. To develop any pattern, we should proceed, as in the case of the cylindrical surface, to find the development of a number of points. We shall show the method of proceeding with one point marked g in the elevation. Through the point g draw a parallel to the base $g\ k$, and a line $a\ g$ from the apex meeting the base in l. Project l down to l_1 on the plan, and measuring round the circumference from e to l_1, set off the same measurement from e_1 to l_1 on the development, and draw the line $c\ l_1$, which will be the development of the line $a\ l$. Then from the centre c with a radius $a\ k$, draw a portion of the circle cutting $c\ l_1$ in g_1, and g_1 will be the development of the point g, for it is in the developed line $c\ l_1$, and at the proper distance from the apex. A similar process may be adopted with respect to any other points, and thus a pattern of any kind might be developed.

To develop the frustum of a cone—that is, a cone with a portion of it towards the apex cut off, as in Fig. 30—we imagine the cone completed, as indicated by the dotted



Fig. 30.

lines in the elevation, and have only to develop the complete cone and cut of f the wanting part, leaving the development of the actual surface of the frustum.

Intersections.—In consequence of the facility with which materials can be shaped into cones, cylinders, or any forms which have circular sections, and for other reasons connected with strength and fitness, these forms are generally used in practical mechanics. It is therefore most important for the draughtsman to be expert in the projections and developments of such forms. Among the numerous problems in drawing these forms, there is an interesting, and often useful class depending on the various intersections of one curved surface with another. We shall give one example of this to illustrate the method that may be very generally applied in most of them. Let us suppose that a pipe of a certain diameter branches out from another of a larger diameter at a certain angle, as indicated on the longitudinal section in Fig. 31, and that we have to draw two elevations and a transverse section of this, so as to show the form of the joint, or intersection of the branch with the main-pipe.

In the first place, we should draw the centre lines or axes of the cylindrical pipes, and then easily mark off the general outlines and points of intersection in the two elevations. The points c, c_1 in the transverse section, would be determined by setting off from f_1 on the centre line, the inside radius of the small pipe, as determined by the line $f\ g$ in the longitudinal section. These points would determine c_2 in the longitudinal section, by making $a_1\ c_2$ equal to $a\ c$ in the transverse section. In a similar manner b_2 could be found. There still remains for us to determine the particular curves of the intersection lying between the points so fixed; and this we

must do by finding a number of other points in the different projections, which shall be the proper representations of points in the circumferences of the larger and smaller pipes where they intersect each other. Let us, for instance, endeavour to determine some point in the curve of intersection represented on the front

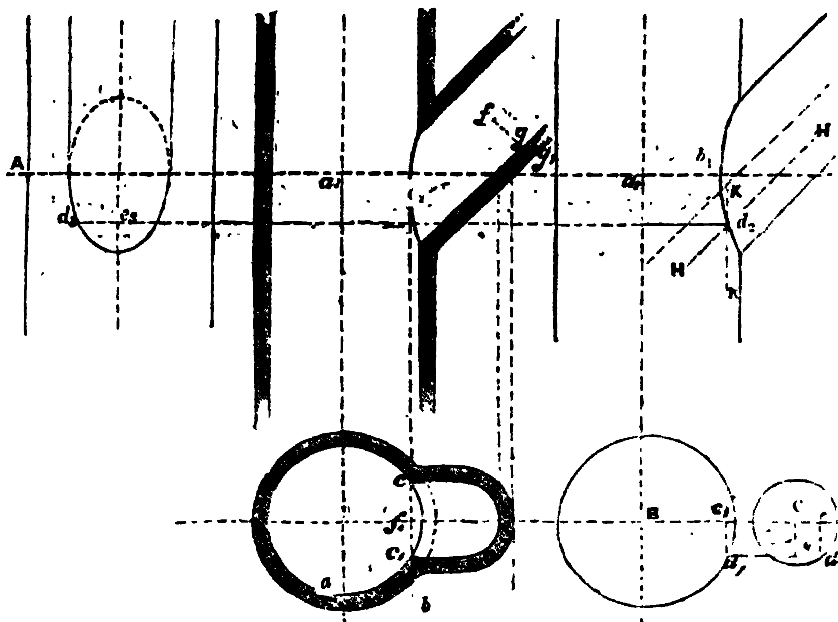


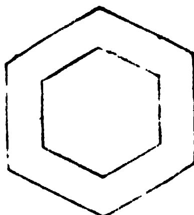
Fig. 31.

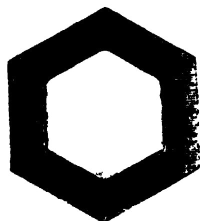
elevation; and as this must be a point on the outside of both pipes, we shall draw their outside circumferences from the centres B and C, connected by a line. Taking any point d in the circumference of C, and tracing it on to d_1 on the circumference of B, and drawing the perpendiculars $d\ e$ and $d_1\ e_1$, we draw on the side elevation H H parallel to the axis of the small pipe at the distance C e from that axis, and K K parallel to the axis of the large pipe, at the distance B e_1. The point d_2, where these lines H H and K K intersect, is the projection of a point in the intersection of the circles on the side elevation. Tracing this across by a horizontal line to the front elevation, cutting its centre line at e_2 and setting off $e_2\ d_3$ equal to $e\ d_1$, we get d_3 as the projection of the same point on the front elevation. And thus, by projecting a number of points, we should be enabled to trace through them a correct projection of the curves of intersection in their different aspects.

Hitherto we have only treated of the mode of drawing outlines; we may now proceed to discuss the various modes of rendering mechanical drawings distinct and more easily intelligible. These may be said to consist chiefly in hatching or section-lining, colouring, shading, and shadowing.

Hatching.—Hatching is merely drawing parallel lines across the parts of any section

which are supposed to be cut through the solid material. Fig. 32 is the mere outline of the section of a hollow hexagonal prism; the lines convey no particular meaning, and they would apply as well to the end elevation of a prism with another projecting from it; but if the section through the solid matter be hatched, as in Fig. 33, as if to indicate the appearance of the fibres of the cut material, it is understood that the drawing represents a section.


Fig. 32.


Fig. 33.

Hatching is also useful in some cases for distinguishing the different pieces of which the body drawn in section consists. This is effected by hatching one of the pieces by oblique lines lying in one direction, and an adjacent piece by lines lying in another direction. Thus, Fig. 34 may represent a section of a vessel with a separate cover fitted to it, and one of the bolts and nuts used for holding the cover in its place. The hatching lines of the vessel and its cover are oblique in opposite directions, and those of the bolt and its nut are vertical and horizontal respectively, so that all the separate pieces are distinguished.

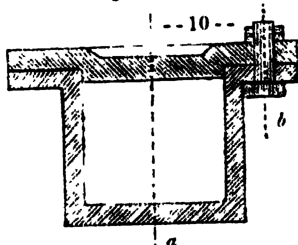
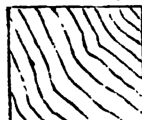
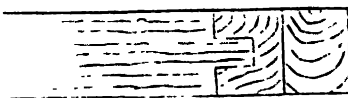


Fig. 34.

Hatching in colours, or coloured lines, is sometimes useful to indicate the material as well as the separate pieces. Thus, hatching parts of a section by black or blue lines may indicate cast or wrought-iron; by yellow or red, brass or gun-metal. Sections of wood are generally hatched as in Fig. 35, by curved lines, imitating somewhat the appearance of the rings seen in timber when cut across. Two separate pieces of timber seen in cross section, side by side, may be hatched by these lines lying in opposite directions (Fig. 36); while timber in longitudinal section may be hatched by crooked discontinuous lines, lying generally lengthways.


Fig. 35.


Fig. 36.

In working-drawings—that is to say, projections showing the parts in their proper dimensions, so that a workman can measure and transfer them to the solid material—in such drawings, colours are usefully employed to distinguish the different materials. Thus, parts tinted dark blue might represent cast-iron; while a pale blue might indicate wrought-iron or steel; yellow would mark brass or gun-metal; brownish-yellow, wood; brownish-red, copper; and so on. When any special material is to be used, its name is generally written on such part of the drawing as represents it.

Central Lines.—In almost all mechanical works, or at least in their details, there is necessarily symmetry, or repetition of equal parts on different sides of some central points. For instance, a cylinder, cone, column, and such like bodies, are symmetrical round their axes. The teeth of a wheel are symmetrical round its centre, and so with almost all the forms suited to machinery.

It is, therefore, most important in mechanical drawings to mark the central or axial lines of the different parts. As these lines do not necessarily appear on the ma-

terial, they are generally distinguished from those that mark outlines of actual form by the use of a different-coloured ink; for instance, if the lines of the drawing generally be in black, as they usually are, the central lines may be ruled in red or blue. They should always be fine but distinct lines, because they serve as a starting-point for numerous dimensions. As an illustration, we may refer to Fig. 34, which we may suppose to be the section of a round or cylindrical box, the centre or axis of which is marked by the dotted line *a*. One of the bolts for fixing the cover is also shown; and as this bolt is also cylindrical, it has an axis or central line *b*, and the position of the bolt is sufficiently indicated on the section by figuring the distance of its axis from that of the cylinder, as it is marked, 10, on the figure. We have indicated the central lines *a* and *b* by dotted lines, to distinguish them from the outlines and hatchings; on a drawing they would probably be marked in blue or red, not dotted, but plain, fine, and distinct.

Shadows and Shades.—In order to render mechanical drawings more intelligible than they otherwise would be, recourse is had to shadowing and shading. The projections which constitute mechanical drawings are not representations of the actual appearances of the objects to the eye; they are merely the traces of their outlines formed by parallel lines drawn from all parts of them to certain imaginary planes or flat surfaces, which are supposed to be transferred to the paper. In like manner, in the shadowing and shading of mechanical drawings, we must not attempt to give the natural appearance of the lights and shades visible on the objects. We must concert some system of illuminating their surfaces in accordance with our system of projecting their outlines. The object of such light and shade is to make up for the deficiency of one of the dimensions of solid objects, depth, thickness, or distance, when we represent them on paper or any flat surface.

These outlines, however accurate, can never show more than two dimensions—length and breadth; and can convey no idea of parts projecting or receding from any surface represented. Nor can light and shade applied to these outlines give the other dimensions, so that they can be measured; but they can be applied with very good effect to indicate which parts are supposed to project or recede from the general surface, and the comparative amount of such superficial variations. If we had, for instance, the outline elevation of a square block of some material (Fig. 37), we cannot judge

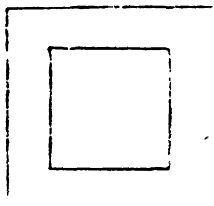
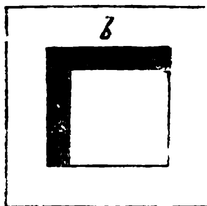


Fig. 37.



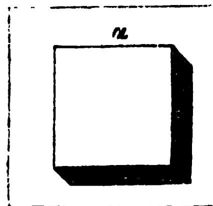


Fig. 38.

whether the outline is intended to represent a prominence or a hollow in the general surface of the block; but by a little shadowing we can give the effect of either, as in Fig. 38. In *a* we see that the inner part must project beyond the outer, in order to cast a shadow upon it; while in *b* the outer part of the surface must project beyond the inner for the same reason.

Again, shading may be very serviceable in giving a just idea of the form of an object seen in elevation. Thus *a* (Fig. 39) is the bare outline elevation of a roller

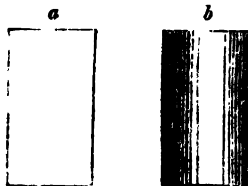


Fig. 39.

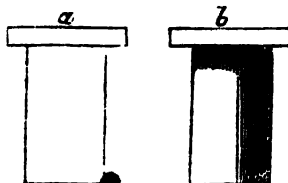


Fig. 40.

standing on one end; but it gives no notion of the roundness of the object. *b* is the elevation shaded, and at once conveying the idea of roundness.

When the lights, shades, and shadows are all introduced in their proper places, as in Fig. 40, the notion of solid form is rendered very distinct, and a very clear conception of the object represented is conveyed to the mind.

The mode in which shadows are drawn is very similar to the system of projecting the outlines themselves. If we suppose a body, such as a die, to stand on a table, and numerous parallel rays of light, indicated by the dotted lines, to strike on it at an angle, some of them would be intercepted by the die, and

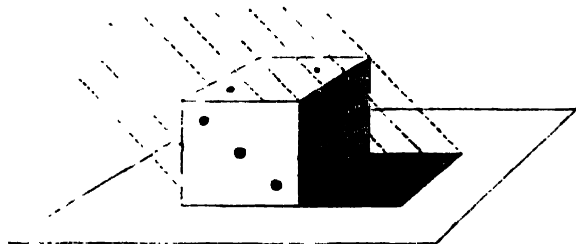


Fig. 41.

thus all the surface of the table and the die would be illuminated, with the exception of the portions from which the light is kept off by the solid body. The dark surface thus left on the table, is a shadow of the die; and the unilluminated side of the die is a shade on the die. It appears, then, that the shadow is merely a projection of the figure made on a plane by tracing oblique parallel lines from all the points in the outline of the body to that plane.

But as this projection may, like the projection of the object itself, be effected on more than one plane, and as we are at liberty to suppose the object placed in light coming in any convenient direction, we may select such an obliquity of the rays as may at the same time furnish us with the most distinct shadows, and with those most easily drawn. In selecting, then, the angle at which the rays of light shall strike, we can assume any that is not coincident with the planes of projection on which our drawing is made. But as these planes are at right angles, or square to one another, it seems natural to select half a right angle, 45°, or what is generally termed by workmen a mitre-angle, as the direction of the rays. Again, as a line drawn at such an angle may extend either from below upwards, or from the right towards the left, or from a distant point towards us, we must select the most suitable of these courses for giving us distinct shadows. As we are generally accustomed to see objects illuminated by light coming from above, it is natural that we should determine on employing rays coming from above downwards. Further, as our drawings are intended to represent the visible

surfaces of objects, we must suppose those sides that are exposed to us, illuminated; and therefore select rays coming from behind us, and striking on and past the objects. As to whether these rays shall come in a direction from left towards right, or from right towards left, is a matter of indifference. The only circumstance which can affect our choice in this last respect is, that draughtsmen generally sit with their left side towards the light, and therefore are more likely to select rays coming in the direction to which they are accustomed: that is, from left towards right. Now, in order to see clearly the effect of rays supposed to come in the chosen direction, in illuminating certain surfaces, and in producing shades and shadows, we shall suppose a piece of paper, cut to the form of Fig. 42, consisting of three equal squares, and that portions of the diagonals of these squares were drawn, lying at mitre-angles, or angles of 45°, with the sides of the squares. Now, let us suppose that the paper is folded along the two lines *a b* and *a c*, so as to turn up the two squares perpendicular to the third, and to each other, as in Fig. 43, forming three sides of a square box, with the diagonal lines drawn upon them. If the lines of projection upwards, towards the right and towards the front from the extremities of these three diagonal lines respectively, be drawn till they meet, as indicated by the dotted lines, we get a line *d e* lying obliquely to each of the three sides, of which line the diagonal lines are correct projections. Now, this line *d e* would represent a portion of one of the rays of light which, as we suppose, illuminate the objects, and all the other rays would be parallel to it. The shades and shadows, therefore, of plans, elevations, and sections, projected on the three planes, will all be determined by lines lying at angles of 45° to the boundaries of those planes. As an example of the mode of correctly delineating shadows according to this law, we may explain the mode of defining the shadows in Fig. 44.

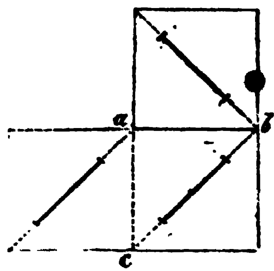


Fig. 42.

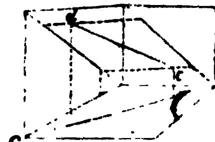
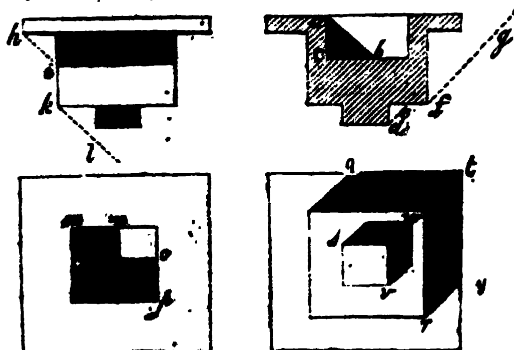


Fig. 43.



Plan 1, looking from above. Plan 2, looking from below.
Fig. 44.

1. The line *a b* drawn at 45° from the edge of the section, gives the breadth *c b* of the bottom, over which the shadow of the side extends; we therefore set off *m n* and *p o* in the Plan 1, each equal to *c b*; and drawing lines from *n* and *o* parallel to the edges, we have the outline of that shadow.

2. Drawing *d e* and *f g* at 45° from two lower edges of the section, we get the extent of the shadows observable in Plan 2, the inner shadow *s w v* of the projecting part of the bottom being limited, because the line *d e* falls within the edge of the bottom in section, while the shadow *q t y* extends as far as the edges of the Plan 2, and would extend

further if there were any surface to catch the shadow, because the line fg in section extends beyond the edge of the body.

3. Again, in the elevation, $h\,i$ gives the point i as the limit of the shadow of the upper projection, while $k\,l$, extending beyond the projecting part of the bottom, shows that all its surface must be within the shadow.

For drawings of bodies, whose boundaries are plane surfaces, and whose edges are straight lines, the shadowing is very simple; and the proper inclinations and boundaries of the shadows almost suggest themselves, without the necessity for their actual projection. But when the bodies are bounded by curved surfaces, such as cylinders, cones, spheres, and the like, the projection of the shadows is somewhat more difficult. A little consideration, however, will in most of these cases enable the draughtsman to give a sufficiently faithful representation of the desired effect. It must not be supposed that the shadowing of mechanical drawings is intended to give them any merit in an artistic point of view; for, as they are not real perspective representations of objects, but only imaginary projections, so the shadows tinted on them are not the representations of shadows actually seen on the objects themselves, but geometrical projections of shadows that would occu on certain suppositions as to the direction of illuminating rays. The only purpose of the shadowing is to give a clearer conception of the solid form intended to be shown; and the draughtsman should therefore take care to make these shadows geometrically accurate. After having solved a few problems by actual projection, he will find it easy to give tolerably faithful views of shadows by the eye. We will work out two problems in circular shadowing as examples of projection; and we have selected two kinds of shadows most commonly occurring in mechanical drawing for this purpose.

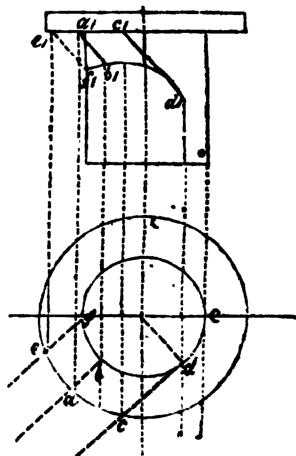


Fig. 45.

First, in Fig. 45, we have an elevation, and a plan looking from below, of a cylindrical body, with a flange or projecting rim round its upper end. The plan and elevation have a common centre line corresponding with the axis. It is evident that the shadow of the lower edge of the flange is the boundary of the shadow of the flange on the elevation. If, then, we take any point, such as a on the plan, tracing it up to a_1 on the elevation, and drawing $a\,b$ at 45°, tracing up from b a line in elevation till it meets a line drawn at 45° from a_1, we get b_1 in the elevation as the shadow of a_1. Taking a number of points, such as a, and finding their shadows in a similar manner, we should be able to trace through them a curved line $f_1\,b_1\,d_1$ as the boundary of the shadow of the flange. The extreme point f_1 is found at once by taking f on the plan, drawing $f\,e$, tracing e up to e_1 on the elevation, and drawing $e_1\,f_1$. The line $s\,d$ on the plan is supposed to touch the inner circle in d; therefore, $d\,g$ the remainder of the visible part of that circle must be all in shadow, as the body itself intercepts the rays. The line traced up from d to the elevation, and meeting the proper shadow line $e_1\,d_1$ in d_1, is therefore the boundary of the illuminated part of the elevation; and if the shadow part be filled in with a black tint, we have the appearance presented in Fig. 46 of the oblique projection

or shadow of the flange on the cylinder. It is manifest that this shadow does not convey a proper notion of the cylindrical surfaces intended to be shown. The drawing requires shading to give the full effect; and after another example of simple shadowing, we will endeavour to show how the shading may be effected.

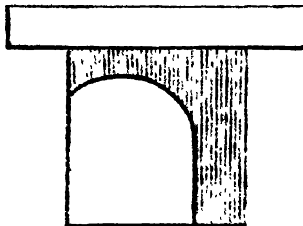
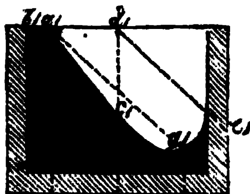


Fig. 46.



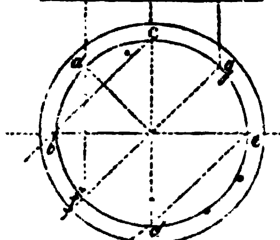


Fig. 47.

Referring now to Fig. 47, showing a plan and section of a cylinder open at top, we have to inquire as to the form of shadow in the section cast on the inside by the cylindrical side. Taking any points *b f d* on the inner circle in plan, and drawing lines from them at 45° till they meet the opposite limb of the circle in the points *e g e*; tracing up these points by lines to the elevation, and also the points *b f d* to $b_1 f_1 d_1$, and from the latter drawing lines at 45° to meet the others, we get the points $e_1 g_1 e_1$ in the boundary of the shadow. The point *a*, where a line at 45° touches the circle, being traced up to a_1 in the section, gives the commencement of the shadow; and the outline being filled in by a dark tint, presents the appearance of an interior cylindrical shadow. Like the exterior shadow shown in Fig. 46, shading is required in order to give the notion of a curved surface.

We shall, therefore, now proceed to discuss the question of shading; which consists in placing on a drawing, tints of various degrees of lightness or darkness, so as to represent the comparative amounts of reflected light from the different portions of the surface represented. We are not aware that the question of shading mechanical drawings has ever been discussed on geometrical grounds, like that of shadowing; it is, therefore, with considerable diffidence that we venture to offer some considerations which may furnish a clue to the proper variations of light and shade on projections. A skilful draughtsman has little difficulty in bringing up a very correct, and even an artistic effect of light and shade on mechanical drawings; but we believe that it is to a practised eye and an expert hand that he owes his success. In shadowing, he trusts to the same elements of success; but as the outline of every shadow can be determined with mathematical accuracy, on the supposition of parallel rays of light proceeding at certain angles to the planes of projection, on the same supposition we think the variations of light and shade can also be determined. It must be confessed that, even if the lights and shades of a projection were determined in quantity, so that we could say such a part of a surface must have twice or thrice the darkness or the light of such another part, we should still labour under the difficulty of carrying out these dimensions of light and shade. We might, however, approach them by applying repeated dark tints to the shaded parts; making the number of equal tints laid on above one another, correspond with the degree of darkness determined on. But even without attempting any mathematically accurate mode of carying out the theory in practice, we may at least derive useful hints from its investigation.

If we suppose the circle in Fig. 48 to be the plan of a cylinder, of which we wish to represent a shaded elevation, one half A D B would not be shown; the other half has its circumference divided into eight equal parts. We will suppose parallel rays of light R, R_1, R_2, R_3, R_4, R_5, R_6, to come at the proper angle (45°), and be reflected from the surface of the cylinder in the directions r, r_1, r_2, r_3, r_4, r_5, r_6, respectively. These directions of the reflected rays are of course determined by drawing them so as to make the same angles with the radii C A, C1, C2, &c., as the incident rays make with those radii, but on opposite sides of them, according to the well-known optical principle that rays of light are reflected from a surface at the same angle as that with which they strike it. Now, of all those rays, that striking the point 3 is reflected most directly to an eye situated in the line C E at a great distance from C, 3 r_3 being parallel to C E, and therefore the elevation of the point 3 should be the brightest. Again, the points 2 and 4 would appear equally illuminated, because the reflected rays 2 r_2 and 4 r_4 lie at equal obliquities to C E; but each of those points would appear less illuminated than the point 3, because of this obliquity. Farther, the points 1 and 5, whence the rays are reflected parallel to A B, mark the places where the illumination of the surface ceases; and were it perfectly smooth and polished, all beyond 1 towards A, and beyond 5 towards 6, would be perfectly dark. The portion 6 to B, receiving no light at all, would be represented in shadow. In the elevation then, if the points in the circumference of the plan be projected, and the surface darkened by lines or tints in accordance with the deficiency of reflected illumination at its different parts, we get a geometrically shaded representation of a cylinder.

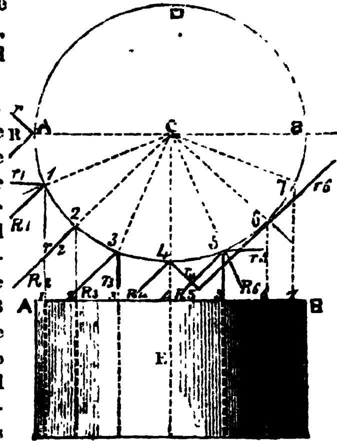


Fig. 48.

For the interior surface of a cylinder shown in section, the light and shade of the different parts may be found in a similar way, as marked in Fig. 49, where the point 5 giving the direct reflection will be the brightest, the points 4 and 6 having equal intensity of light, but each less than 5, because of the obliquity of the reflected rays; the part A 1 2 3 4 quite dark, because the light is intercepted by the edge A; the point 7, whence the ray is reflected parallel to A B, the last part of the illuminated surface; and 7 B dark, because the rays are reflected backwards. The section shows the shading in accordance with this variation of reflected lights, the lower part of the interior being completely shadowed in the form marked in Fig. 47.

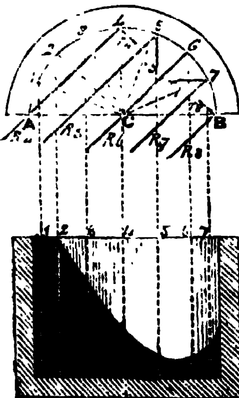


Fig. 49.

The shading of other curved surfaces might be determined in a similar way; but as cylindrical surfaces are those which most commonly occur in drawings of machinery, we consider it sufficient to have pointed out the principles upon which the shading of them may be determined. A little practice will soon enable a draughtsman to give a

rounding effect to drawings of such surfaces; and a little care given to this will often render plain and explicit a drawing which would otherwise be comparatively obscure. The lights, shades, and shadows to complete drawings need not be determined by geometrical projections, which would often involve great labour without adequate results. A draughtsman accustomed to make drawings of machinery, forms in his own mind a very accurate conception of the solidity, projections, and hollows of the various parts, and throws in the shades and shadows by eye in such a manner as to give a tolerable notion of these variations of surface. It is well, however, that the beginner should know the principles on which shading should depend, and he will then be better prepared for that success in their application which practice of eye and hand can only give.

Before concluding this part of our subject, we may say a few words respecting the instruments required by the mechanical draughtsman. The board, on which the paper is fixed, should be of well-seasoned wood, accurately right-angled, or, as it is commonly called, *square* at the angles. As wood is often apt to warp and shrink according to changes in temperature and moisture, it is often necessary to have the edges of the board adjusted by the carpenter's plane, especially if the wood be new. Where great accuracy of drawing is required, this is a matter which should always be carefully looked to.

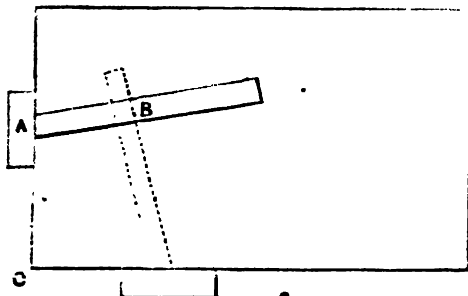


Fig. 50.

The drawing-square, or T square, consists of a stock, with a projecting lip which can slide along the edge of the drawing-board, and a blade having the edges accurately straight and parallel. The edges of the blade should be at right angles to the face of the stock; but this is not of very great importance, for even if the blade made any other than a right angle with the stock, provided the angle C (Fig. 50) of the drawing-board be a right angle, the lines drawn by the square applied to both edges of the board will be at right angles to one another. Indeed, the blade and stock are often made so that the angle can be varied at pleasure, a screw being fitted in the stock in such a manner as to hold the blade fast at the angle required. This form of instrument is called the bevil T square, and is useful when there are to be drawn numerous lines parallel and at right angles to one another, but oblique to the edges of the board.

Set-squares are triangular pieces of wood having their sides accurately straight, and making particular angles with each other.

The most useful set-squares are the three shown in Fig. 51.

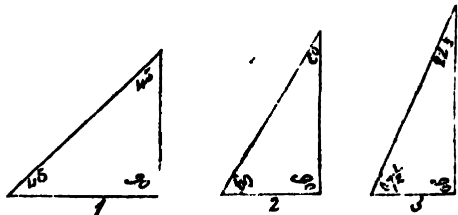


Fig. 51.

No. 1 is a triangle, having one angle a right angle, or 90°; and the other two each 45°, or half a right angle.

No. 2 has one angle 90°; another 30°, or one-third of a right angle; and the third 60°, or two-thirds of a right angle.

No. 3 has an angle 90°; another 22½°, or one-fourth of a right angle; and the third 67½°, or three-fourths of a right angle.

As an example of the use of these set-squares, we will suppose that we have to describe a hexagon round a given circle (Fig. 52). The circle being described round the centre G, the blade B of the T square being brought into any convenient position, the set square No. 2 is applied to its edge in the different positions marked by the dotted lines, and the sides *fe*, *ed*, and *dc* of the hexagon drawn, touching the circle; the other sides, *cb*, *ba*, *af*, being similarly drawn by shifting the set-square into suitable positions, which are not marked in the figure, to avoid confusion.

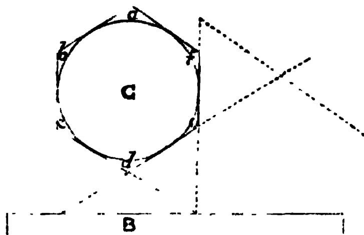


Fig. 52.

Scales are blades of ivory, boxwood, metal, or card-board, having straight edges divided into equal parts, which are generally fractions of a foot or inch. In making drawings of machinery it is generally necessary to represent the parts, not of the real sizes to which they are made, but to some known scale, so that all the proportions of the parts are accurately maintained. For instance, had we to make drawings of a steam-engine, we should select some convenient size for the drawings; we might, for example, represent the engine one-eighth of its real size, and should form a scale every inch of which should stand for eight inches of the real work, every foot for eight feet, and every inch and a half for one foot. This would be called a scale of one-eighth real size, or a scale of an inch and a half to a foot, because an inch and a half is the eighth part of a foot. Now, in making the drawing, or measuring from it when made, it would be troublesome and tedious to calculate what should be the size drawn of each dimension of the work; but having first formed a scale, such as Fig. 53, every inch and a half of

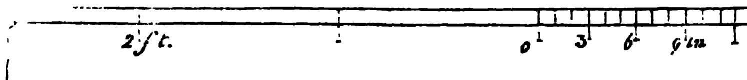


Fig. 53.

which represents a foot, one of these feet being divided into twelve inches, we can at once apply it to the drawing, and set off any required dimension in feet and inches.

The most convenient scales for mechanical drawings are those of

6 inches to	1 foot, or	$\frac{1}{2}$ real size.			1 inch to	1 foot, or	$\frac{1}{12}$ real size.		
3 ,,	1 ,,	$\frac{1}{4}$	,,		$\frac{1}{2}$,,	1 ,,	$\frac{1}{24}$	,,	
2 ,,	1 ,,	$\frac{1}{6}$	,,		$\frac{1}{4}$,,	1 ,,	$\frac{1}{48}$	,,	
1½ ,,	1 ,,	$\frac{1}{8}$	,,		$\frac{1}{8}$,,	1 ,,	$\frac{1}{96}$	,,	

For architectural drawings, and drawings of large works generally, the following scales will be found convenient, in addition to the above:—

1 inch to	10 feet, or	$\frac{1}{120}$ real size.			1 inch to	50 feet, or	$\frac{1}{600}$ real size.	
1 ,,	20 ,,	$\frac{1}{240}$	,,		1 ,,	80 ,,	$\frac{1}{960}$	,,
1 ,,	40 ,,	$\frac{1}{480}$	,,		1 ,,	100 ,,	$\frac{1}{1200}$	,,

For drawings of land-surveys, it is usual to employ chains as units of measurement; and scales are therefore made in terms of them, such as ten chains to one inch, and the like.

In plans of railway or canal works it is often necessary to measure round curves; and as these curves are generally made portions of circles for ease of setting out and execution, circular scales, divided equally round their circumferences, are sometimes employed.

Offsets are short scales which act as set-squares as well as scales.

The protractor is an instrument generally made semicircular, divided out equally round the circumference in degrees; and it is used for setting off such angles as may be required.

The other instruments, such as dividers, drawing-pens, pen and pencil-compasses, are so generally known, and their use is so simple, that we need not devote space to considering them.

For the draughtsman employed on devising, on paper, work to be executed in solid materials, we strongly recommend that he should, as much as possible, endeavour to draw all details of their full size. Scale-drawings, especially of large works, are absolutely required to show the combination of all the parts into the one machine or engine, full-size drawings of which would be of most inconvenient size. But in addition to the scale-drawings of the whole, it is the practice of the best engineers to execute full-sized drawings of details. One great advantage of this is, that the draughtsman sees better on the full size what should be the best forms, dimensions, and combinations of the parts in respect of strength, efficiency, and economy of material and labour; and another advantage consists in the facility with which the workman can read off the drawings, or transfer to the solid materials he labours on, the dimensions and forms marked on the drawings: he has only to apply his rule to the drawing and to his work, and make them agree. Farther, as it is customary for workmen to use rules divided into inches, and these again into halves, quarters, eighths, and sixteenths, the draughtsman should make his dimensions such as can be measured in those fractions. We have known numerous workmen who perfectly comprehend the eighth or sixteenth part of an inch, but had not the remotest conception of what was meant by the tenth or the twentieth part. Were the dimensions even figured by tenths, the workman would read it in quarters, eighths, or sixteenths, and would most probably err in reducing it from the one denomination to the other. It is, therefore, preferable in the first instance to draw or figure the dimensions in a denomination which the workman will employ.

STRENGTH OF MATERIALS.

The materials with which the practical mechanic has principally to deal are metals, such as steel, wrought-iron, cast-iron, copper, brass and gun-metal, lead, tin, and zinc; timber, such as oak, teak, ash, beech, and other hard woods; fir, pine, cedar, and other soft woods; stones, bricks, mortars, and cements; cordage, straps and bands, and the like. The strains to which these materials may be subjected, may be classed under five heads:—

1. Tension, or a strain applied in the direction of the fibres of a body, so as to pull it asunder, as in the case of ropes, bands, tie-rods, and chains. The strength of a body to resist a strain of this kind, is called its cohesion, or the force with which its parts are held or knitted together.

2. Compression, or a strain applied also in the direction of the fibres so as to crush a body, as in the case of a column. Compressive strain is exactly opposite to tensive strain; and the power of a body to resist it seems to depend chiefly on its elasticity.

3. Transverse strain, or force applied perpendicularly to the fibres so as to break the body across, as in a beam loaded in the middle. In resisting a strain of this kind, the cohesion and elasticity of a body are also exercised.

4. Torsion or twisting, or a force applied at the end of a lever so as to turn one end of the body round its axis, while the other end is fixed, as in a shaft or axle.

5. Clipping or shearing, or a power applied to divide or cut a body across its fibres.

All the possible strains to which materials, whatever be their forms and arrangements, can be subjected, are of one or more of these classes. We therefore propose to consider them separately, and endeavour to apply the results of experiment and observation to their discussion, endeavouring to avoid, as far as possible, mere abstract mathematical investigations. We cannot, however, altogether dispense with mathematical aid, but we shall endeavour to place the reasoning as much as possible on such a footing that a student having a moderate acquaintance with the general principles of mechanics, may have little difficulty in following it. We must premise that the whole subject is as yet in an unsettled condition in some respects, for it has happened that very few writers have discussed it as a whole; and of those who have discussed portions of it, some may be said to have done so with too little regard to its practical application; while others have erred in the opposite direction, giving mere empirical rules without theoretical reasons for their establishment. We shall endeavour to steer a middle course between these extremes, trying to show as clearly as possible the reasons for conclusions that have been arrived at, and quoting from the best authorities the results of experiments in a form that may render them conveniently applicable in practice.

1. **Tension.**—We may consider that every body subjected to tension consists of numerous fibres laid side by side, and extending over the whole length of the body. Even if the body be not of a fibrous constitution, we may conceive its particles to be arranged in longitudinal rows, each particle being held to the next by some force which we call attraction of cohesion. A row of particles so held together exactly corresponds with our notion of a fibre; and we are therefore warranted in treating all bodies as fibrous while we discuss their strength to resist tension.

Let us suppose that we have a rope capable of sustaining a hundredweight, and no more, without breaking. We should call the absolute cohesive strength of that rope one hundredweight, or 112 lbs. The length of the rope by which the weight is suspended has evidently no influence on its strength; for if any one part of its length be capable of sustaining the strain, and if it be uniform, every other part will be equally capable of sustaining it. It is true that were it hanging vertically, additions to its length would increase the weight, stretching its upper portion, because each portion of added length becomes an added weight. But considering the weight of the rope itself as part of the stretching force, we say that the given rope has an absolute cohesive strength of 112 lbs., whatever be its length.

Now, if any number of such ropes were suspended in a row (Fig. 54), each might have a hundredweight attached to it without breaking, and the absolute cohesive strength of all together would be expressed by the number of hundredweights sustained; that is, by the number of ropes, reckoning each as having the strength of a hundredweight. Whether we deal with ropes, however, of hemp, or rods of wood or of iron, or of any other material, if we know the strength of one such rod and the

number of them sustaining equal weights, we know the total weight sustained by them all; that is to say, their combined strength. Let us assume, instead of ropes, that we experiment with square bars of iron measuring an inch each way, or having a square inch of sectional area. Whether a number of these bars, such as nine of them, be suspended in a row, of which A (Fig. 55) is the plan, with intervals between them; or in a close row with no intervals as in B, and forming a bar nine inches wide by one inch thick; or in a square, C, measuring three inches each way; there can be no difference in their combined strength, or in the total weight carried by the whole number of bars, however they may be arranged.

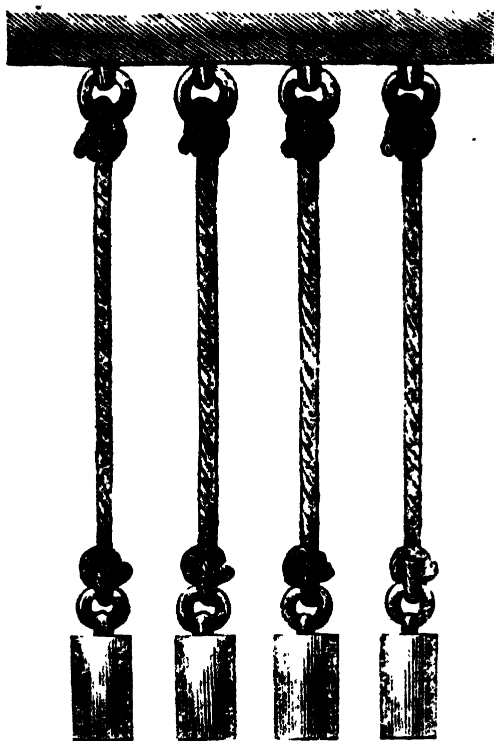


Fig. 54.

If, then, we know the weight sustained by a bar or rod of any material having a sectional area of one square inch, we can estimate the weight that can be sustained by any other bar of the same material, by calculating the number of square inches in its sectional area, and allowing the known strength for each square inch. For example: let us suppose that we find by experiment the average strength of a bar of wrought-iron, 1 square inch in section, to be 30 tons, or 67,200 lbs.; let us ascertain the strength of a bar 10 inches wide and 10 inches thick. Since the sectional area is

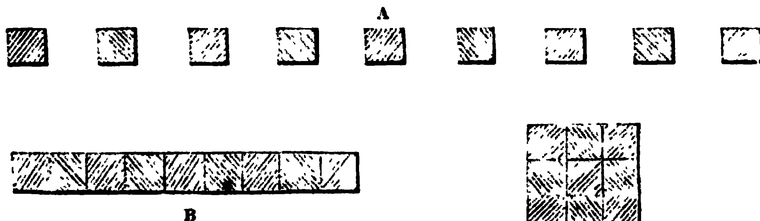


Fig. 55.

10 in. × 10 in., or 100 square inches, the strength is 30 tons × 100 sq. in., or 3000 tons; or 67,200 lbs. × 100 sq. in., viz. 6,720,000 lbs. The strength of a round bar

1 inch in diameter would be estimated in the same way; for as the area of a circle 1 inch in diameter is 0·7854 sq. in., the strength of the round bar 1 inch in diameter would be 30 tons × 0·7854 sq. in., about 23½ tons; or 67,200 lbs. × 0·7854 sq. in., about 52,780 lbs.

Farther, as the areas of circles are in proportion to the squares of their respective diameters, the strength of a round bar of iron of any diameter is 23½ tons, or 52,780 lbs., multiplied by the square of the diameter in inches. Thus, a round bar of 10 inches in diameter has a strength of 23½ tons × 100 = 2350 tons; for the area of a circle 10 inches in diameter is 100 times that of a circle 1 inch in diameter. All this mode of calculation manifestly proceeds on the simple principle, that every equal fibre or row of particles is supposed to act equally in resisting strain; and that as the number of such fibres or rows is proportional to the area of section, the strength of different bars or rods are therefore proportional to their areas of section.

So far the theoretical principle of cohesive strength appears very simple and easily applicable in practice. It now becomes our business to inquire how far practical results agree with the theoretical principle. Many engineers have devoted great care to experiments on this subject; and although the results present considerable differences, yet all of them seem to bear out the principle we have laid down. As an example of the near approach which practical results make to the theoretical law, we may select some experiments made on round copper bars, as shown in the following table. The first column contains the diameters of the bar in inches; the second the sectional areas in square inches, or decimal parts of a square inch; the third contains the weight in tons, and decimal parts of tons, which were required to break the bars; and the fourth column contains the weights in tons per square inch of sectional area required to break or tear the bars asunder. The numbers in the last column are calculated by a simple proportion; thus, taking the bar 1 inch in diameter, the area of which is 0·7854 square inches, and the breaking-weight 17 tons, we say—

Sq. in.		Sq. in.		Tons.		Tons.
0·7854	:	1	::	17	:	21·645;

the weight that would be required to break the bar were it one inch square.

Table I.—*Experiments on Cohesive Strength of Cylindrical Copper Bars.*

Diameter of bar: inches.	Area: sq. inches.	Breaking-weight: tons.	Breaking-weight reduced to 1 sq. inch: tons.
1⅛	0·9940	22	22·133
1	0·7854	17	21·645
⅞	0·6013	12·8	21·287
¾	0·4417	9	20·376
			Mean 21·36 tons.

It will be seen that, although the sectional areas of the bars differ very considerably, the breaking-weights, reduced to one square inch of area, do not vary greatly from their mean value.

Table II. contains the results of experiments on best hempen bower cables used in the navy. The first column contains the circumferences, or girths, of the cables in inches (the dimensions of cables being generally stated in terms of their girths); the

second column contains their sectional areas in square inches; the third expresses the number of threads in each; the fourth column marks the actual breaking-weights; the fifth the breaking-weights per square inch of sectional area; and the sixth the breaking-weights per thread.

TABLE II.—*Experiments on the Cohesive Strengths of Best Hempen Bower Cables.*

Circumf.: inches.	Sectional area: sq. inches.	No. of threads.	Breaking-weight: tons.	Break. weight per sq. inch: tons.	Break. weight per thread: lbs.
23	42	2736	114	2·714	93⅜
18	26	1656	63	2·423	85¼
14½	17	1080	40	2·353	83
				Mean 2·5 nearly	87

The strengths in these cases also are very nearly as the sectional areas or numbers of threads. We shall finally quote the results of some experiments on the strength of wrought-iron bars of various qualities, as marked in Table III., where the breaking-weights per square inch are given as calculated from the experiments. The first column contains the description of bar, the second contains the dimensions, the third gives the amounts in inches which each foot of the bars stretched, and the fourth gives the breaking-weights per square inch of sectional area.

TABLE III.—*Experiments on the Cohesive Strengths and Extension of Wrought Iron Bars.*

Description and form.	Dimension before extension: inches.	Extension per foot: inches.	Breaking-weight per sq. in. of section: tons.
South Wales, cylindrical.	1·38 diam.	1·81	29·3
Ditto ditto.	1·50 ,,	1·86	29·8
Stafford, square.	0·75 side.	1·22	27·2
Ditto ditto.	1·08 ,,	1·24	27·5
Welsh, square.	1·00 ,,	2·38	29·0
Scrap, square.	1·00 ,,	2·50	29·0
Common, cylindrical.	2·00 diam.	1·50	31·8
Stafford, square.	1·00 side.	1·10	31·0
			Mean 29·3

By numerous experiments, such as those quoted, data have been determined from which we can readily calculate the dimensions of a bar of any material to carry a given weight, or conversely the weight which a given bar of material will carry. These data have been determined by the actual breakage of the bars tested; but in practice, where durability and adequate strength to meet contingencies of strain are required, it is necessary to give the materials we employ considerably greater dimensions than such as would merely preserve them from breakage, or to load them with considerably less weights than those which would tear them asunder. Farther, it is found in practice that materials subjected to considerable strains for long periods of time gradually lose their tenacity; and this degradation of quality must be provided against when permanence and stability are required.

In the case of metal bars it is advisable not to load them beyond one-third of their actual breaking strain; or, in other words, to provide them of three times the strength

sufficient to resist tearing asunder. In the case of ropes, timber, and such other materials as are of less uniform consistency than metals generally are, or are more liable to degradation, it is advisable to provide at least four times the breaking strength.

Table IV. contains data for the strength of a number of materials on which experiments have been made; and the numbers given may be safely used in calculation, as they are reduced to one-third or one-fourth of the breaking strength.

The strengths are given in square inches, and also in circular inches for convenience of calculation; and the following are the rules for calculating the strength of a given bar, or the dimensions required to bear a given load.

I. Given, the dimensions of a bar of any material to find its cohesive strength, or the constant load it will safely bear in the direction of its length.

1. When the bar is square.

Rule.—Multiply the number of inches in the side by itself and by the number opposite the given material in column 2 of Table IV.

Example 1.—Required the cohesive strength of a square bar of English wrought-iron, of which each side measures $2\frac{1}{4}$ inches.

$2\frac{1}{4}$ expressed decimally	.	2·25
Multiplied by itself	.	2·25
		1125
		450
		450
		5·0625
Multiply by tons in table	.	8·3
		151875
		405000
		42·01875

Neglecting the decimal fraction, we find that we may safely load the bar with a weight of 42 tons.

2. When the bar is oblong, or of some other form having straight sides.

Rule.—Multiply the width by the thickness, or generally find the sectional area in square inches, and multiply by the number in column 2 of the table.

Example 2.—Required the cohesive strength of a bar of wrought copper of the form and dimensions given in Fig. 56.

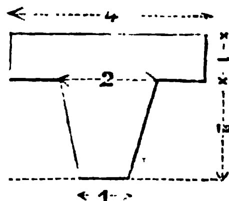


Fig. 56.

The upper part of the figure is an oblong or rectangle, of which the width is 4 inches and the thickness is 1 inch; its area is therefore 4 × 1, or 4 square inches.

The lower part of the figure is a trapezoid, of which the thickness at one edge is 2 inches, and at the other 1; the mean thickness is therefore $1\frac{1}{2}$ inch, and the width being 2 inches, its area is $1\frac{1}{2} \times 2$, or 3 square inches. The total area of the section is therefore 7 square inches: and 7 multiplied by 5 tons, the strength of wrought copper in column 2 of table, gives the strength of the bar 35 tons.

3. When the bar is cylindrical.

Rule.—Multiply the diameter in inches by itself, and by the number in column 3.

Example 3.—Required the cohesive strength of a fir pole 4 inches in diameter. $4 \times 4 \times 1{\cdot}26$ give 20·16, say 20 tons.

4. When the bar is of elliptical section.

Rule.—Multiply the greater diameter by the less, and by the number in column 3.

Example 4.—Required the cohesive strength of an elliptical lead bar, of which the greater diameter is 2 inches and the less $1\frac{1}{2}$ inch: $2 \times 1\frac{1}{2} \times 500$ give 1500 lbs.

5. When the bar is cylindrical and hollow like a pipe.

Rule.—Multiply the outer diameter by itself, and also the inner by itself, subtract the one product from the other, and multiply the remainder by the number in column 3.

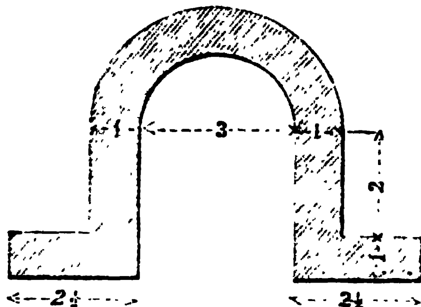


Fig. 57.

Example 5.—Required the cohesive strength of a brass tube 2 inches diameter outside and $1\frac{1}{2}$ diameter inside.

$$2 \times 2 = 4$$
$$1\tfrac{1}{2} \times 1\tfrac{1}{2} = 2\tfrac{1}{4}$$
$$1\tfrac{3}{4} \times 2{\cdot}12 = 3{\cdot}7 \text{ tons.}$$

6. As a general rule, whatever be the form of section, find the area either in square inches or circular inches, and use as a multiplier the number in column 2 in the one case, and that in column 3 in the other. Or find the strengths due to the several parts of the section, and add them together for the total strength.

Example 6.—Required the cohesive strength of a cast-iron bar of the form and dimensions given in Fig. 57.

Taking first the hollow semicircular part,

Outer diameter 5 in. $\times 5 = 25$
Inner diameter 3 in. $\times 3 = 9$

Take ½ for semicircle)16

8 circ. in. $\times$ 2·12 tons (col. 3) $=$ 16·96 tons.

Two upright sides each 2 in. by 1 in. have an area $2 \times 1 \times 2$ . . . . . . $= 4$ sq. ins.

Two lower flanges each $2\frac{1}{2}$ in. by 1 in. have an area $2 \times 2\frac{1}{2} \times 1$ . . . . . . $= 5$

Square inches 9 $\times$ 2·7 tons (col. 2) $=$ 24·3 tons.

Total cohesive strength 41·26 tons.
say 41 tons.

II. The converse operations are for calculating the dimensions of a bar required to carry a given load.

7. When the bar is square or oblong.

Rule.—Multiply the number in column 4, by the given number of tons, and the product gives the number of square inches of sectional area.

Example 7.—Required the area of a bar of wrought-iron capable of sustaining ten tons.

Number from col. 4 . . . 0·120
Multiply by 10

1·2 square inch.

This sectional area may be attained by making the bar square about $1\frac{1}{10}$ inch of a side; or making it oblong, such as 2 inches by $\frac{5}{8}$ inch thick, or any dimensions such that their product is at least 1·2 square inch.

8. When the bar is cylindrical.

Rule.—Multiply the number in col. 5 by the weight in tons, and take the square root of the product; it will be the diameter in inches.

Example 8.—Required the diameter of a cylindrical bar of wrought-copper to carry 35 tons.

Number from col. 5 . . . 0·255
Multiply by 35

9· nearly.

The square root of 9 is 3 inches, the diameter of the bar.

9. For bars having sections of various forms and proportions, the calculation must be trials to a certain extent. The number of square inches required is found as in Example 7, and these may be disposed in any suitable form that may be required.

TABLE IV.—*Data for Calculating the Cohesive Strengths of Bars of different Materials, determined from the Averages of numerous Experiments. The numbers are given roundly, as no material error will arise in practice from their use.*

Col. 1.	Col. 2.	Col. 3.	Col. 4.	Col. 5.
Name of material.	Strength per square inch of sectional area.	Strength per circular inch of sectional area.	Area in sq. inches to bear 1 ton.	Area in circ. inches to bear 1 ton.
	Tons. lbs.	Tons. lbs.	sq. ins.	circ. ins.
Ash	2·2 or 5,000	1·73 or 3,900	0·45	0·57
Beech	1·5 „ 3,400	1·18 „ 2,700	0·66	0·84
Box	2·7 „ 6,000	2·12 „ 4,700	0·37	0·47
Fir	1·6 „ 3,600	1·26 „ 2,800	0·62	0·79
Mahogany	1·1 „ 2,400	0·86 „ 1,900	0·93	1·18
Oak	1·4 „ 3,100	1·10 „ 2,400	0·72	0·92
Pear-tree	1·3 „ 3,000	1·02 „ 2,360	0·75	0·96
Teak	2·0 „ 4,500	1·71 „ 3,500	0·50	0·64
Brass	2·7 „ 6,000	2·12 „ 4,700	0·373	0·476
Copper (cast)	2·8 „ 6,300	2·20 „ 4,900	0·356	0·454
Copper (wrought)	5·0 „ 11,200	3·93 „ 8,800	0·200	0·255
Iron (cast)	2·7 „ 6,000	2·12 „ 4,700	0·373	0·476
Iron (wrought), English	8·3 „ 18,600	6·52 „ 14,600	0·120	0·153
Ditto (Swedish)	10·7 „ 24,000	8·41 „ 19,000	0·093	0·118
Lead (cast)	0·3 „ 650	0·24 „ 500	3·446	4·390
Steel (blister or cast)	20·0 „ 45,000	15·7 „ 35,000	0·050	0·064
Steel (shear)	18·5 „ 41,500	14·54 „ 32,500	0·054	0·069
Tin (cast)	0·7 „ 1,600	0·55 „ 1,260	1·400	1·783

Before we leave the subject of tensive strength, we will discuss a case which frequently occurs in practice, and where the cohesive force of materials is employed in a vessel to resist a bursting or exploding strain arising from the pressure of a fluid within it. Were a boiler, pipe, or other vessel intended to contain a fluid exerting considerable pressure, made of any form except circular in section, the internal pressure would change the form of section. Thus suppose the section were of a square form, as in Fig. 58, the internal pressure acting equally on every part of the casing, as marked by the arrows, would bulge the flat sides, as indicated by the dotted lines; and, finally, were the material sufficiently pliable, would extend it into a circular form. After having attained a circular section, no farther change of form would be effected, and the pressure of the internal fluid might be increased until it attained sufficient magnitude to burst or rend asunder the material. Vessels destined to sustain considerable internal pressure are generally made circular in section; and a few simple considerations will enable us to ascertain the amount of strain which the pressure in such vessels throws upon the material of the casing, and hence to compute the strength of material that should be employed in their construction in order that they may sustain a given pressure.

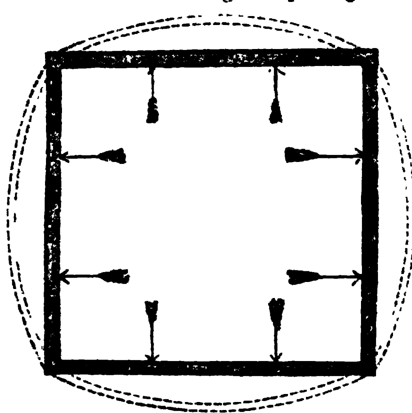


Fig. 58.

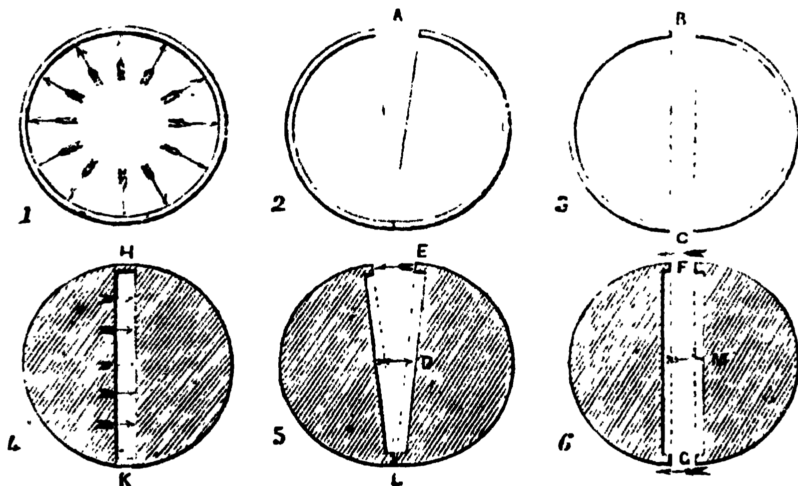


Fig. 59.

Let us suppose that a steam-boiler is of cylindrical form as in section 1 (Fig. 59), having pressure exerted on it equally on every part of its surface, as indicated by the

arrows. One effect of this pressure, if it exceeded the strength of the material containing it, would be to rend it asunder at some point A, section 2; or it might rend it at two opposite points, B and C, section 3, so as to force one half of the casing away from the other. Now, if the material were of perfectly uniform consistency and strength throughout, there is no reason why one point A, or any two points B and C, should be selected for breakage more than any other points; but, as in practice materials are never perfectly uniform, so in theory we may suppose the casing to be somewhat weaker at A, or at B and C, than elsewhere, and trace the influence of the pressure to effect a breakage at such points. Were we to suppose the whole of the vessel to be filled up with solid matter, except a thin film of fluid in the middle, as shown in section 4, exerting a pressure on every point of the flat side of the solid filling-up exposed to it, we should involve no change in the conditions, for the two points of the circumference, H and K, terminating the film of fluid, would have to resist exactly the same bursting-strain as if the rest of the casing were filled with fluid instead of solid matter. Farther, as this section would apply at any part of the length of the cylinder, we may take a belt or portion of its length, one inch in breadth, and trace the effect on it—an effect which would be repeated equally on every such belt one inch wide throughout the whole length of the cylinder. Knowing the pressure which the internal fluid exerts on every square inch of the casing, we see by section 4 that the bursting pressure on a belt of casing one inch wide is to be measured by the pressure on a surface represented by the line H K,—that is, on a surface having for length the diameter, and one inch in width. If this pressure burst the casing at one point E, section 5, so as to cause one half to turn round the opposite point L as a fulcrum or centre; then, by the well-known principle of the lever, that a uniform pressure acting at every point of the arm of a lever has the same effect to turn it round its fulcrum as if it were all collected into one force acting at the middle point of the arm, we see that the bursting pressure is equivalent to a force acting at D (as marked by the arrow), while the cohesive force of the casing acts at E, double the distance from the fulcrum L. Hence we conclude that the bursting force, as resisted at one point of the circumference, is half the pressure on the diameter, or the pressure on half the diameter or radius. Again, if the vessel open at two points F and G, section 6, we see that the whole bursting force, marked by the central arrow, being resisted by two equal forces at F and G, each of those forces need be only half the bursting force; so that in this case we find that the effect to open the circumference is half the pressure on the diameter as before.

To apply these considerations in practice, let us suppose that a boiler of English wrought-iron, 6 feet in diameter, has to sustain an internal pressure of 100 lbs. per square inch, and ascertain the necessary thickness of the plate to resist this force. Taking a belt of it 1 inch wide,

Half the diameter, 3 feet or	36 ins.
With a width of	1 in.
Has a surface of	36 sq. ins.
Multiplying by pressure per sq. in.	100 lbs.
The pressure on half the diameter is	3600 lbs.

which is the bursting strain on any belt of the circumference 1 inch wide

By Table IV. we find that English wrought-iron having 1 sq. in. area of section, sustains 18,600 lbs.; and by a simple proportion, 18600 lbs. : 3600 lbs. : : 1 sq. in : 0·2 sq. in

nearly, we find that the sectional area of the belt of circumference must be 0·2 sq. in.; or that, as it is 1 inch wide, its thickness must be 0·2 in., or $\frac{2}{10}$ or $\frac{1}{5}$ of an inch. In practice, boilers are made of numerous plates riveted together; and as the material is cut away by the rivet-holes, it may be much weaker at these places than elsewhere. It would, therefore, be advisable to make the strength of the plate at least double that calculated, which would give a thickness of about $\frac{3}{8}$ of an inch.

In the case of hydraulic presses which are made of cast-iron, and have to sustain enormous pressures, it is necessary to make the material of great thickness. There is, however, a limit to the pressure which such cylinders can sustain; for it is found that after a certain thickness of iron has been attained, the actual substance of the iron becomes compressed, so that the inner surface is considerably extended, while the outer surface sustains scarcely any bursting force. Additional thickness of metal does not therefore contribute proportional increase of strength. Indeed, the practical difficulty of casting thick masses of iron sound and solid in texture, renders it almost impossible to construct hydraulic presses of any very great force. It is doubtful whether, in any case, it is advisable to exceed a thickness of 6 or 7 inches when cast-iron is the material employed. The same remarks apply to a certain extent in the case of ordnance: but in making large guns or mortars, a considerable advantage results from the process of manufacture. The gun is cast solid, and afterwards the heart is bored out. Most of the unsound and spongy portions of the metal are thus removed, and the shell left is much sounder than it would be, were it cast hollow as hydraulic presses generally are. Vessels for containing fluids exerting pressure, are likewise subject to a bursting strain in the direction of their length, which is resisted by the cohesion of the casing. Taking the case of the boiler 6 feet in diameter, and 0·2 in. thick, we may readily calculate what pressure it would bear before it gave way in the direction of its length. The whole boiler may be taken as a hollow or tubular bar of wrought-iron, capable of sustaining a certain weight hung to it, equivalent to the pressure on one of its circular ends, tending to force or blow it to a distance from the other end

Outer diameter . . .	72·4 ins. × 72·4 =	5242 nearly
Inner diameter . . .	72·0 ins. × 72·0 =	5184
Area of ring in circ. ins.		58
Cohesive strength of 1 circ. in. wrought-iron		14600
Total strength of circumference		846800 lbs.

As the area of the end is 4072 sq. ins., the boiler would bear a pressure of more than 200 lbs. per square inch, before parting lengthwise. Indeed, it will be found that when uniform thin material is employed, a cylindrical vessel subjected to internal fluid-pressure is almost exactly twice as strong lengthwise as it is circumferentially. When the thickness is considerable, this proportion does not hold good. For instance:—

Let r = radius, t = thickness, a = cohesive strength of 1 square inch of casing, p = pressure to burst at circumference, and P = pressure to divide the casing lengthwise.

$$pr = at \therefore p = \frac{at}{r}$$

When t is small compared with r, the sectional area of casing is nearly $2\pi rt$; and as πr^2 = area of end,

$$\pi r^2 P = 2\pi rat \therefore P = 2\frac{at}{r} = 2p.$$

It is a singular fact that many metals, when drawn into wire, become stronger in

respect of tensive strain. This may probably be owing to the wire-drawing process causing the particles to arrange themselves in continuous fibres. From numerous experiments made on iron wire less than ¼ inch in diameter, it appears that the strength of wire is about one-fourth or 25 per cent greater than that of iron bar; so that while the cohesive strength of wrought-iron bars may be taken at 8 tons per square inch of sectional area, that of iron wire may be taken at 10 tons; or taking bar at 6¼ tons per circular inch, wire will have a strength of 8 tons per circular inch.

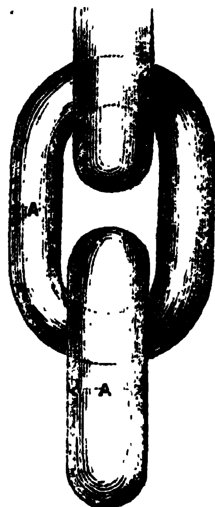


Fig. 60.

The following are the rules for computing the strengths of hempen ropes and chains. The dimensions of the ropes are stated in terms of girth or circumference, and those of the chains in terms of the diameters of the metal constituting the links. Thus the chain which has a diameter of ¾ inch at A (Fig. 60) is called a ¾-inch chain.

We have formed the rules so as to compute the strains at a lower rate than they are generally stated. It may be true that a new chain or rope will bear a much greater strain than that which we have allowed to it; but the chain loses strength by use—not so much from wear as from the particles of iron assuming a crystalline instead of a fibrous texture; and the rope loses strength by wear as well as by the gradual decay of its fibres. It is preferable, therefore, to err rather on the safe side, especially when it is considered that great damage may often result from the breaking of a chain or rope, and, what is still more carefully to be guarded against, serious injury to life and limb.

1. To find the diameter of a chain to carry a given weight.

Rule.—Multiply the weight (tons) to be carried by 30; the product will be the square of the diameter of the chain reckoned in 16ths of an inch.

Example.—Required the diameter of a chain to carry 10 tons. $10 \times 30 = 300$, which is near 324, the square of 18; therefore the diameter of the chain must be $\frac{18}{16}$ inch, or 1⅛ inch.

2 To find the weight which a chain of given diameter will carry.

Rule.—Divide the square of the diameter (reckoned in 16ths of an inch) by 30; the quotient will be the number of tons carried.

Example.—Required the strength of an inch chain: 1 inch is $\frac{16}{16}$ of an inch, and $16 \times 16 = 256$; dividing by 30 we have about 8½ tons.

3. To find the circumference of a rope to carry a given weight.

Rule.—Multiply the weight (tons) by 11; the product will be the square of the circumference in inches.

Example.—Required the size of a rope to carry 18 tons: $18 \times 11 = 198$, which is near 196 the square of 14 inches, the circumference required.

4. To find the weight which a given rope will carry.

Rule—Divide the square of the circumference in inches by 11; the quotient will be the weight carried in tons.

Example.—Required the strength of a 4-inch rope: $4 \times 4 = 16$, and 16 divided by 11 gives 1½ tons.

2. **Compression.**—The strength of materials to resist compressive strains, appears to depend chiefly upon some forces among their particles acting in the opposite direction to cohesive attraction. Indeed, the particles of every solid body appear to be ranged in such positions with respect to one another, and so balanced by cohesive attraction keeping them together on the one side, and by some repulsion resisting their nearer approach on the other, that considerable force is generally required to alter their relative positions. The intensity with which these forces act in any body appears to be measured by their hardness or strength to resist external forces. But while cohesive attraction seems to follow a simple and regular law in any material—the amount of attraction or the strength to resist a tensive strain being proportional to the sectional area, or, in other words, to the number of particles upon which the attraction is exerted—experiments have as yet shown no very regular law as to strength of materials to resist compression. Fortunately, in practice, the other strains to which materials are subjected are generally so much more likely to affect them than mere compressive strain, that when we make the parts of our work sufficiently strong to resist the former, we are tolerably safe in respect of the latter.

Some valuable experiments have been made upon the strengths of building materials—wood and iron—to resist compression; and attempts to deduce laws from these experiments have not been wanting. We are not aware, however, that any satisfactory results have attended those efforts; and we fear that any rules founded on our present scanty information in respect of this subject, would tend rather to mislead than assist the practical mechanic.

If we suppose that a cubical piece of any material (Fig. 61) is loaded by a certain weight which it is just able to bear without being crushed, we may readily imagine that a number of such cubes—say nine, for instance—would bear nine times the weight which the one bears; and if these cubes were arranged either in a row (Fig. 55) at intervals, or close together, or in a square, they should still all together bear nine times the weight that one of them can bear. When the material we deal with is of uniform consistency, we are therefore warranted in reckoning that its strength to resist compression, precisely as that to resist tension, is proportional to the area over which the compressive force is spread. Thus, a block of stone 4 feet long and 3 feet broad, should be able to sustain six times the compressing force which a block 2 feet long and 1 foot broad can sustain; because $4 \times 3 = 12$, the area of the one, is six times $2 \times 1 = 2$, the area of the other.

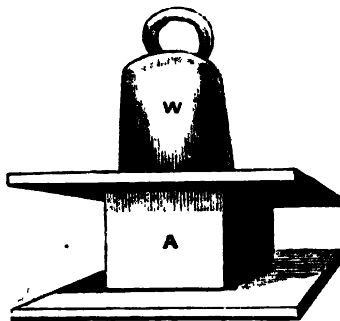
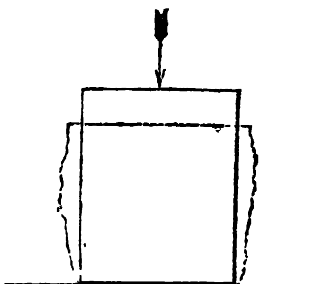


Fig. 61.

But if we endeavour to trace the practical effect of crushing any material, we shall find it difficult to establish the truth of this simple law, without making great allowances for irregularities in the consistency of the material and in the application of the compressive force. Some materials of a soft consistence, such as clay or lead, on being subjected to vertical compression, merely spread out sideways. Thus, a piece of lead compressed by a heavy weight bulges in width, and thickness, to the form marked by the dotted lines (Fig. 62). Other materials of a fibrous texture, such as wood, become splintered by compression, owing to some of the fibres being driven or wedged into the

interstices between others, and thus bursting them asunder. Hard and brittle materials, such as cast-iron, glass, stones, or bricks, are riven and splintered in most irregular and unexpected ways, some portions being actually pulverized in the process, or cracking away and occasionally flying off with considerable force from the rest.


Fig. 62.

These results, however, occur when the compressive strain is carried beyond what the material is capable of sustaining without disintegration; for all practical purposes, where we make the strength more than adequate to meet the load, we may very safely follow the simple law, and reckon the strength to be very nearly as the area of section at the weakest part. This, however, only holds good when we pay no regard to the height of the mass pressed upon. As we increase the height we must increase the strength, for several reasons, which we shall endeavour to illustrate. Suppose we take a cylindrical piece of cast-iron one inch in diameter and six inches high, and placing it upright, load it on the end until it will bear no more. As we place more and more weight upon it, the particles become pressed closer and closer together, and the mass yields or subsides a little in the direction of its length. After being compressed to a certain extent, the particles cannot get into closer contact; but some of them must insinuate or wedge themselves into the interstices between others, and thus the mass will tend to bulge sideways; or, being of a hard unpliable consistency, the sides will become fractured, and burst away. The mass thus weakened will finally all crumble under the pressure. Now let us take a cylindrical piece of the same material and of the same diameter; but, instead of being six inches high, let it be merely a thin disc one-eighth or one-sixteenth of an inch thick. Here, although the area of section pressed upon be the same as before, yet the height being so much less, the number of particles that can be forced to one side or the other is much less, the number of interstices to receive pressed particles is less, the amount of bulging is diminished, and the whole resistance is enormously greater. As an example of the effect of increased height in diminishing strength to resist vertical pressure, we may quote some experiments made on pieces of cast-iron, all having square bases a quarter of an inch each way, and various heights; whence it appears that as the height was increased, the strength to resist compression was diminished, so much that the force to crush the piece $\frac{8}{8}$ inch or 1 inch high was little more than two-thirds of that required to crush the piece $\frac{3}{8}$ inch high.

Height. in.	Crushing force. lbs.
$\frac{3}{8}$	9006
$\frac{4}{8}$	8845
$\frac{6}{8}$	8362
$\frac{7}{8}$	6430
$\frac{8}{8}$	6321

Farther, when the height is considerable, as in the case of columns, a slight want of uniformity in the material, or a slight want of equality in the distribution of the load, will cause one side to be affected more than the other, and thus produce flexure in the case of pliable materials, such as wood or wrought-iron, or oblique separation in the case of stone. An extreme load pressing on a column just tottering on the verge of ruin will certainly find out the weakest place, and there begin the demolition; and having once begun it, will very rapidly complete it. Thus a knot in a wooden post, or a vein in a marble pillar, may become important elements in determining the mode of

their fracture, and the amount of compression required to effect it. It appears from some experiments made with wooden columns, that when the height is more than eight times the diameter, there is considerable risk of the column bending under an excessive load. With timber of uniform strength, and a load well-balanced upon the column, when its height does not exceed this proportion, the wood will generally splinter under a crushing force. In columns of stone or brickwork destined to sustain a heavy load, the ratio of height to diameter may be increased considerably above that of eight times, without the danger of their breaking by a sloping fracture. In such cases, however, the eye seems instinctively to judge of proportion, for a very tall slender column conveys a notion of instability, and is consequently wanting in that grace which depends mainly on the fitness of the object for the work it has to perform.

Cast-iron is found to be capable of supporting a very considerable vertical load without flexure or crushing; and columns of this material are therefore with safety made of tall and slender proportions. Even here, however, there is a limit beyond which the eye becomes dissatisfied; and accordingly, when the column is required to be of considerable height, it is generally made of greater diameter, the inside being hollowed out to save weight and material, without sensibly affecting the strength. Where, from inequalities of the load, there is any risk of unequal pressure on the summit of a cast-iron column, and of consequent tendency to bend, the material used in constructing the column is much more advantageously applied in the shell of a hollow column, or in the ribs of one the section of which is a cross, than in one solid. We only advert to this now, as we shall have an opportunity of recurring to it when we discuss the question of transverse strain.

Some very valuable experiments have been made upon the crushing of building materials, such as stones and bricks. The compressive forces in most of them have been applied to cubical pieces of 1 inch or 2 inches. In some where the pieces had a square base 1 inch broad and a height of 2 inches, the force required to crush has been found less than two-thirds of that required for the pieces only 1 inch in height. The increase of height, therefore, considerably diminishes the strength to resist compressive force. In connection with this subject it may be interesting to inquire how high a brick building might be carried without becoming crushed by its own weight. We may suppose a massive brick pier or column divided into numerous separate columns each 1½ inch square in section, and each supported along its whole height by those around it, against lateral pressure or flexure. The lowest part of each of those columns would thus have to support the vertical weight of all the column above it; and if that weight exceeded the load required to crush a brick cube of 1½ inch, the lowest part of the column would of course give way. Now it has been found by experiment that the load necessary to crush a 1½ inch cube of brick, varies from 1200 lbs. up to 3000 lbs., according to the different qualities of the brick. It would perhaps be safest to take the lower number, 1200 lbs. The weight of 1 foot of brickwork having a 1½ inch square base, is about 2 lbs.; therefore it would require a height of 600 feet to crush the base. An architect wishing to secure the permanence of his building, would certainly not venture to raise it one-third of this height without having recourse to the ordinary expedient of spreading the base or foundation by extensive footings, so as to diffuse the crushing pressure over a much larger surface of material.

Table V. contains the average results of experiments made on the resistance of various materials to crushing force. We would, however, caution the mechanic against

placing too much reliance on these results, as we think the number of experiments made scarcely warrant their establishment as practical data.

TABLE V.—*Resisting Power of Materials to Crushing Force.*

Name of material.	Dimension of cubical piece.	Crushing pressure.
Elm	1 inch	1300 lbs.
American pine	,,	1600 ,,
White deal	,,	1900 ,,
English oak	,,	3800 ,,
African oak	,,	5000 ,,
Chalk	1½ inch	1100 ,,
Soft brick	,,	1200 ,,
Red brick	,,	1800 ,,
Hard-burnt brick	,,	3000 ,,
Fire-brick	,,	3800 ,,
Grindstone grit	,,	8000 ,,
Portland stone	,,	10300 ,,
York paving	,,	12800 ,,
White marble	,,	13600 ,,
Cornish granite	,,	14300 ,,
Compact limestone	,,	17300 ,,
Peterhead granite (red)	,,	18600 ,,
Purbeck granite (red)	,,	20600 ,,
Hard freestone	,,	21200 ,,
White Italian marble	,,	21800 ,,
Aberdeen granite (blue)	,,	24500 ,,
Cast lead	¼ inch	480 ,,
Cast tin	,,	960 ,,
Wrought copper	,,	6400 ,,
Cast copper	,,	7300 ,,
Wrought iron	,,	8000 ,,
Brass	,,	10000 ,,
Cast iron	,,	10000 ,,

3. **Transverse Strain.**—The effect of a load on the end of a beam projecting from a wall, or on the middle of a beam supported at both ends, is to throw a transverse strain on the beam; and if the load be excessive, to break the beam transversely, or across its fibres. Of all questions respecting the strength of materials, this is certainly the most important. In architectural structures, the stability of roofs, floors, and walls supported on beams or girders; in civil engineering, the safety of girder-bridges and rails; in mechanical engineering, the strength of beams, levers, framing, and the like,—all depend mainly upon the correct solution of questions relating to transverse strain. It is to this branch of the subject, therefore, that the consideration of engineers has been chiefly directed; and a vast number of experiments have been made with a view to ascertain practical data from which the transverse strength of all useful materials may be calculated.

The theoretical investigation of transverse strain is by no means a difficult one; but the application of data in computation involves some very complicated questions, which the practical mechanic generally solves more by the eye, as he sees proportions in his

drawing or model, than by any very accurate calculation. Such a mode of meeting the difficulty implies experience and observation; and without these we fear no amount of theoretical knowledge, and no expertness in calculation, will enable one to decide on the strength of his structure. In machinery especially, the strains to which parts may be subjected are so various in magnitude, owing to their respective movements, and the local qualities of materials differ so greatly, that the mere calculated strain and strength offer very little aid in guiding to the most suitable proportions. When the architect determines on the dimensions of an iron girder destined to carry a heavy wall, he can calculate with tolerable precision the weight to be supported and the strength of girder required to carry it without danger. The girder is placed, the wall is built, and twenty years after the load is the same, and the strength of the girder but little diminished.

But when the mechanic makes a pattern of a beam for a steam-engine, although he may readily calculate the strain which the pressure of steam on the piston throws upon the beam while the engine works steadily, he has little notion of the sudden though transient strain which may be thrown upon it by the sudden stoppage of some part of the machinery, or the occurrence of some slight obstacle to the movement. It not unfrequently occurs that a little water in the cylinder of a steam-engine, causes the fracture of some of the strongest parts. The machinery being all in motion, the piston descends upon a film of water which has no ready means of escape, and which being almost totally incompressible, offers as determined an obstacle as solid iron would do. In such a case, the momentum of all the moving parts must be suddenly destroyed; some of the beams, rods, or levers, through which the piston is connected with the rest of the engine, must give way; or the cylinder itself, which holds the water, must yield to the strain, and break under it. That we may form some idea of what damage such a strain as we have described may effect, we have only to reckon that in a large steam-engine there are 15 or 20 tons of iron, moving probably at an average velocity of 100 feet per minute, to be suddenly arrested. In destroying the momentum, as much force is expended as would be measured by the blow of a 68 lb. cannon-ball striking its mark at a very near range. Nor do these sudden and unexpected strains constitute the only difficulty under which the mechanic labours when he computes the strength of his work. He cannot always depend upon the internal soundness of the material with which he deals. Cast-iron is especially treacherous in this respect; and often it happens that a casting, externally sound, has some sponginess or air-bubbles under the skin, which are discovered only in the event of fracture. It is, therefore, his business not only to contrive devices for regulating the movements of his machinery, and for affording relief in cases of undue pressure, and to use every precaution against unsoundness in his materials; but also to provide such strength as shall meet the contingencies which a slight derangement may frequently bring about. It may be asked then, why, if the mechanic has to apply such excess of strength to meet contingencies, he should take any trouble in calculating or ascertaining practical data from which he may compute? The answer is plain. While he makes every part strong to excess, yet he has to maintain a proportionate strength in all. To make one part of a beam strong enough to sustain ten times its usual strain might be very proper; but to make another part of it capable of sustaining twenty times its load would be absurd, for then either the weaker part is only half as strong as it should be, or the stronger is twice as strong as required. The mechanic, then, must have a clear conception of how strains affect materials of different forms, and how far the change of one dimension or another may affect the strength, before he can venture to design or execute a machine justly proportioned in all its parts, and sufficiently

strong throughout to meet the contingencies of its action, without undue waste of material and labour in its construction.

The most simple case of transverse strain is that to which a beam projecting from a wall is subjected when a weight is suspended from its outer end. It is clear that the longer the beam, the greater the strain; and that if the beam give way anywhere, presuming it to be of uniform strength throughout, it will be at a point close to the wall where it is fixed, because at that point the weight acts with the greatest leverage. On tracing the breaking effect of the weight, we see that in bringing the end of the beam down to the position marked by the dotted lines, some action must take place among the fibres of the material at A B, where the weight acts with greatest power. This

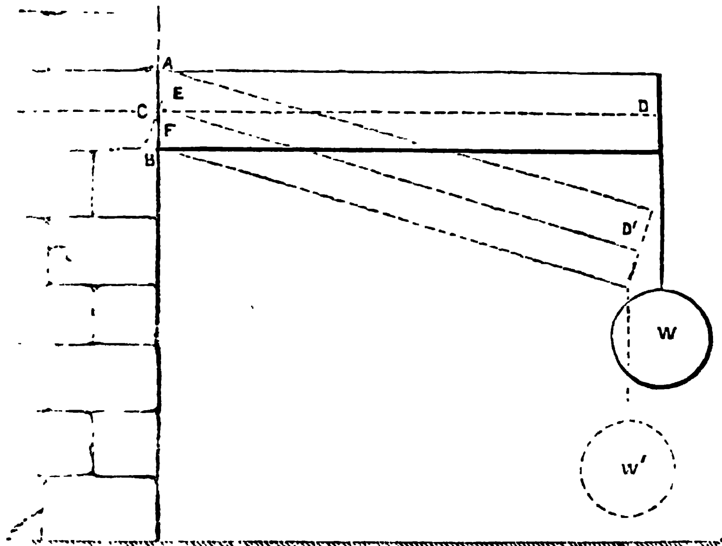


Fig. 63.

action must be that of tension among the upper fibres, and of compression among the lower ones; and there must be some point C in the beam where the extension and the compression of the fibres meet, and where there is neither of these actions. Such a point marks the position of what is called the neutral axis; that is, a line extending along the beam horizontally, and separating that part of the material which is extended or torn asunder from that part which is compressed or squeezed together. It is manifest that, once the tearing asunder of the fibres commences, it must continue while the load continues to act, because the number of fibres resisting it are diminished, and those that remain have therefore more load to bear.

What we have to discover, however, is the strength of the beam while it is entire, or how much load W at a certain distance from the wall it will bear without the destruction of the fibres. Supposing a horizontal line C D drawn through the neutral axis to form the long arm of a bent lever of which the fulcrum is C, one short arm C A,

and the other C B, we have a certain power or weight W acting at the extremity of the long arm, and a number of resistances, viz. those of the fibres to extension, acting along the arm C A; also a number of resistances, viz. those of the fibres to compression, acting along the arm C B.

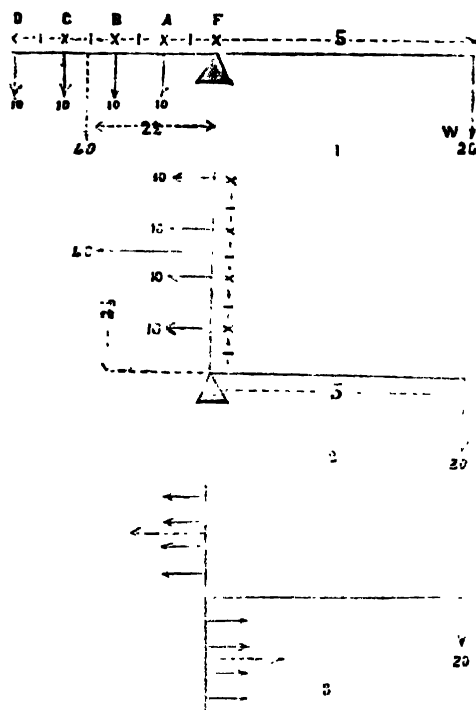


Fig. 64.

Now, when on one arm of a lever a number of equal forces, equally distributed over the arm, act against a given weight at the other end of the lever, the effect of all these forces is the same as if they were all collected into one force at the middle point of the arm on which they act (Fig. 64). This is true whether the lever be straight as 1, or bent with one arm as 2, or bent with two arms as 3. As a numerical example, let us suppose that the short arm of 1 is 4 feet long, and at each foot there hang 4 weights A, B, C, D, each of 10 lbs., as marked by the arrows; while the length of the other arm is 5 feet. A weight of 20 lbs. hung to the longer arm balances the four weights of 10 lbs. distributed along the shorter, because

A = 10 lbs. at 1 foot leverage is balanced by	2 lbs. at 5 feet leverage.
B = 10 lbs. at 2 feet ,, ,,	4 ,, ,,
C = 10 lbs. at 3 feet ,, ,,	6 ,, ,,
D = 10 lbs. at 4 feet ,, ,,	8 ,, ,,
Total A, B, C, D distributed . . .	20 ,, ,,

And 20 lbs. at 5 feet leverage would be balanced by 40 lbs. at $2\frac{1}{2}$ feet; that is, the total of the 4 weights A, B, C, D hung at their middle point, as marked by the dotted arrow. The same law will be found true, whatever the number of equally-distributed weights, and whether the lever be straight or bent. Applying this to the question of the beam Fig. 63, let E be the middle point of A C, and F the middle of B C; then the load W acting with leverage C D has to resist the united tensive strength of the fibres along C A acting at the leverage C E, and the united compressive strength of those along C B acting with the leverage C F. Those resistances will necessarily be equal to one another, because the neutral axis C is the fulcrum, on each side of which they equalise themselves; and the tensive strength of C A, multiplied by its leverage C E, is therefore equal to the compressive strength of C B multiplied by the leverage C F; or, as C A is

double of C E, and C B double of C F, and as doubles of equal things are themselves equal, the tensive strength of C A multiplied by the length C A is equal to the compressive strength of C B multiplied by the length C B; and each of those products is equal to the weight W multiplied by its leverage C D, the length of the beam, because each is double of half the effect of W to break the beam.

To show how this reasoning may be applied numerically, let us suppose that a beam of oak 1 inch thick and 6 inches deep, fixed in a wall so as to project 10 feet, is broken by a load hung at the end; and that while in the act of breaking the neutral axis is observed to be 2 inches from the under side, and therefore 4 inches from the upper side of the beam, we may, from knowing the tensive strength of oak, estimate the weight required to break the beam. Taking this tensive force per square inch (or force required to tear asunder a square inch) at 12,000 lbs., we have in the case before us 4 square inches having a tensive force of $4 \times 12,000 = 48,000$ lbs. acting with leverage of 4 inches against leverage of 10 feet or 120 inches; and as 4 inches is the $\frac{1}{30}$th of 120 inches, the weight at D must be the $\frac{1}{30}$th of 48,000, viz. 1600 lbs.

By such calculations as this we could compute the transverse strength of materials, from knowing their tensive strength, provided we knew the position of the neutral axis of fracture. The determination of this, however, is a point of very great difficulty; for it must, in the first place, depend upon the relative proportions of compressive and tensive strength: and as we are much in the dark as to the former, we cannot institute a comparison between it and the latter. In the second place, no material to which we can apply transverse strain resists completely, and then instantaneously gives way. While we add weight after weight, the extension of fibres at one side and the compression of those at the other goes on; the beam bends or becomes deflected; greater and greater strain is thrown on the outer fibres; the tensive and compressive strength of each varies as it is more and more extended or compressed; the neutral axis changes its position; fibres, which were during part of the process compressed, begin to be extended; and the condition of the material at and near the point of fracture becomes generally so altered as it approaches destruction, that we cannot clearly estimate its resistance, even if we were possessed of the most accurate knowledge of its strength to resist compression or extension. Owing to these circumstances, it becomes necessary to make distinct experiments on the transverse strength of bodies, and by means of them to establish certain data which we may apply in calculation. But while experiment and observation furnish the data or facts on which to calculate, we must still trust to reasoning for our methods of applying these facts, and fortunately the reasoning is of a simple character.

As to the neutral axis of fracture, without requiring to determine its position in any beam, we assume that, for beams of the same material, its distance from the upper or lower sides is proportional to the depth of the beam. Thus, if in a beam of ash 6 inches deep, it is 2 inches from the lower side and 4 inches from the upper; in a beam of ash 12 inches deep it will be 4 inches from the lower and 8 inches from the upper side; in one of 1 inch deep, it will be $\frac{1}{3}$rd of an inch from the lower and $\frac{2}{3}$rds of an inch from the upper; and so on in regular proportion. We can see no reason why this should not be the case; and, so far as we can trace the circumstances which determine the position of the neutral axis, we see every reason for believing that it is so. This assumption greatly simplifies the rest of our reasoning; for it enables us to get rid of all calculation as to the actual position of the neutral axis, and to proceed with the comparison of beams as to transverse strength independently of it. We will suppose that we have three beams of the same material, all of equal lengths and breadths, but of the following

depths: A, depth 2 inches, B depth 4 inches, C depth 6 inches, and that we desire to compare their transverse strengths. Let us assume, for the sake of simplicity, that the neutral axis is midway in the depth; then in A we have above the neutral axis a set of fibres extending over 1 inch in depth, and acting with a leverage of 1 inch to resist the breaking load. In B we have fibres extending over 2 inches, and therefore double the number of those acting in A, acting with 2 inches leverage double the leverage of those in A. We see, then, that the strength of B must be twice 2, that is 4 times the strength of A. Again, in C we have 3 inches depth of fibres and a leverage of 3 inches; therefore C has a strength of 3 times 3, or 9 times that of A. And so with any other depths, the strength being always as the depth multiplied by itself, or as the square of the depth. If we had assumed the neutral axis to be in any other position, such as ⅓rd of the depth from the lower side, we should still have found the same law to obtain: for so long as the beams we are considering are of the same material and of similar form, the neutral axis must proportionally divide their depth, and leave proportional numbers of fibres to act with leverage proportional also to the depth.

We, therefore, have established the simple law, that in beams of the same form, length, breadth, and material, the transverse strengths are as the squares of the depths; and knowing the strength of a beam 1 inch deep, we can estimate the strength due to any other depth by multiplying that of the 1-inch beam twice by the depth of the other in inches; that is, by the square of the depth.

Example.—An iron beam 1 inch deep breaks with a load of 2 tons: required the load that will break a similar beam 4 inches deep. The square of 4 is 16, and $2 \times 16 = 32$ tons, or 2 tons $\times 4 \times 4 = 32$ tons.

Now, let us ascertain what relation the transverse strengths of beams bear to their breadths. Suppose that a beam 1 inch broad carried a load of 1 ton, then another exactly like it will also carry 1 ton, a third will also carry 1 ton; and the three placed side by side or united into one beam 3 inches broad, will of course carry 3 tons. Beams, then, of a certain material of equal lengths and depths, have transverse strengths which are proportional to their breadths; and if we known the strength of a beam 1 inch broad, we compute that of a beam having any other breadth, by multiplying the strength due to 1 inch of breadth by the number of inches in the given breadth.

Example.—A fir beam 1 inch broad carries 10 cwt.: required the load carried by a fir beam 6 inches broad of like length and depth. 10 cwt. $\times$ 6 ins. $=$ 60 cwt., or 3 tons.

We may now combine the laws as to breadth and depth into one, and thus compute the transverse strength of a beam as depending on both dimensions. This law is, that the transverse strengths of beams of the same material and similar in form, are as the breadths multiplied by the squares of their depths; and if we know the strength of a beam 1 inch broad and 1 inch deep, we compute the strength of a beam having any other breadth and depth by multiplying the strength of the 1-inch beam by the breadth in inches, and twice by the depth in inches of the other.

Example.—Suppose a beam 1 inch broad and 1 inch deep bears 5 cwt.: required the strength of a beam 4 inches broad and 6 inches deep.

5 cwt. $\times$ 4 ins. $\times 6 \times 6 =$ 720 cwt. or 36 tons.

The strength of a beam is irrespective of its length; but the actual weight which it can carry depends upon the length, inasmuch as the leverage with which the weight acts, so as to fracture the beam, is greater, the greater the length. A weight of 12 tons hanging at the end of a beam 1 foot long, is exactly equivalent to a weight of 6 tons at 2 feet distance, of 4 tons at 3 feet, of 3 tons at 4 feet, of 2 tons at 6 feet, of 1 ton at 12

feet, of ½-ton at 24 feet, and so on,—the weight multiplied by its distance being always a constant quantity. If, then, we know the load which a beam 1 foot long will carry at the end, we estimate the weight carried at any other distance by dividing that carried at 1 foot by the distance in feet; and combining this element with those of breadth and depth, we have the general rule embracing all the dimensions of a beam, to multiply the weight carried by a beam 1 inch broad, 1 inch deep, and 1 foot long, by the breadth in inches, and twice by the depth in inches, and divide the product by the length in feet.

Example.—A beam 1 inch square and 1 foot long, carries 5 cwt. at its end: required the load carried by a beam 4 inches broad, 6 inches deep, and 18 feet long.

$$5 + \frac{4 \times 6 \times 6}{18} = 40 \text{ cwt. or 2 tons.}$$

So far we have discussed the strength of beams of square or rectangular sections, but similar reasoning will apply to those of other forms. When the section is circular, the breadth being equal to the depth, we have to multiply the strength of a cylindrical beam 1 inch diameter and 1 foot long, three times by the diameter in inches, and divide by the length in feet.

Example.—A cylindrical beam 1 inch diameter and 1 foot long, bears 3 cwt., required the load supported by a cylindrical beam 4 inches diameter and 8 feet long.

$$\frac{3 \text{ cwt.} \times 4 \times 4 \times 4}{8} = 24 \text{ cwt.}$$

When the section is of any other form, such as the T shaped iron girder (Fig. 65), from our ignorance as to the position of the neutral axis of fracture, we are at a loss how to reckon the effect of the fibres at different parts of the section, and cannot therefore estimate the strength from any data obtained by experiments upon beams of square or rectangular section. But if we know the strength of a beam of certain dimensions, and of the form in question, we may pretty nearly estimate that of a beam of the same kind whose dimensions, as to breadth and depth respectively, are proportionally greater or less. For instance, to compare two T shaped beams of which the dimensions are as follow:—

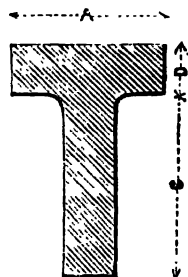


Fig. 65.

No. 1.—A = 3 ins., B = 1 in., C = 4 ins., D = 1 in., length 10 feet, is found to bear 2 tons: required the strength of No. 2, where A = 6 ins., B = 2 ins., C = 6 ins., D = 1½ in., length 15 feet.

$$\text{Strength of No. 1 is as } \frac{3 \text{ ins.} \times 4 \times 6}{10} = 4{\cdot}8$$

$$\text{Strength of No. 2 is as } \frac{6 \text{ ins.} \times 6 \times 6}{15} = 14{\cdot}4.$$

And by the simple proportion,

Strength No. 1.		Strength No. 2.		Tons.		Tons.
4·8	:	14·4	: :	2	:	6, load carried by beam No. 2.

In the absence, then, of sufficient knowledge of the effect of material at different parts of the section in contributing to strength, for every different form of section we are under the necessity of making such experiments as shall fix data for calculation, to be applied to cases where the section is similar, but the dimensions different. A number

of such experiments have been tried from time to time, and attempts have been made to settle, by theoretical reasoning as well as by practical results, the best form of section to be used; that is to say, the form which gives the greatest strength with the least material. We are not aware, however, that for any material, the subject has been so far investigated as to warrant us in laying down any absolute proportions; and the form of section must therefore be determined, in a great measure, by balancing a number of circumstances, and adopting such arrangements as shall combine the results most advantageously.

In a recent engineering work, a triumph of skill and perseverance over difficulties at first sight apparently insurmountable—we mean the Britannia Tubular Bridge across the Menai Straits—an enormous amount of preliminary investigation was conducted before the form and dimensions of the structure were finally determined. Models were made of all convenient forms, and tested against each other: one was found too weak in one place, another too weak in another place. Fresh models were made, with the weak parts strengthened by additional materials, and the mass of material in strong parts removed: the most suitable and convenient form was thus decided. A much larger model was made and tested as to deflection and fracture; and after a large comparison of results, the actual dimensions and details of construction of the full-sized structure were determined, and carried out with merited success.

Hitherto we have referred only to the circumstance of a beam projecting and loaded at one end. This, however, is by no means the most ordinary condition under which materials are exposed to transverse strain. Beams are usually supported at both ends, and carry their load in the middle. We have, therefore, now to ascertain what relation exists between the strength of such a beam and that of a beam projecting and loaded at one end. If we suppose a beam built into a wall and projecting equally on both sides (Fig. 66), each end being loaded with an equal weight, it is clear that the wall supports double the weight suspended at each end. Now, if we conceive the forces acting on such a beam inverted or turned upside down, we establish the conditions of a beam supported at both ends, and loaded in the middle (Fig. 67) with double the weight which each of the end supports has to bear. Now, if, in the first case, where the beam projected both

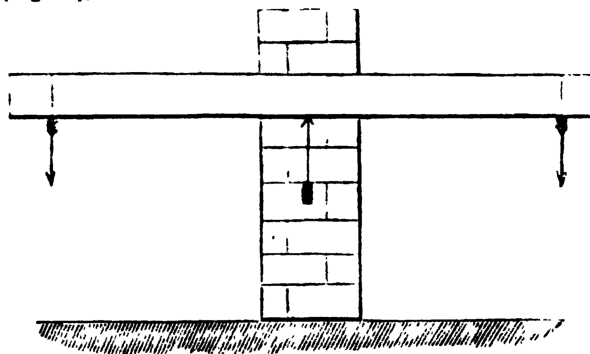


Fig. 66.

ways from the wall, each end were loaded with the weight capable of breaking the beam, that is up to its transverse strength; in the second case, the beam, which is twice the length of each projecting arm of the first, may be loaded with double the weight which hung from each end of the first, and this weight will measure its transverse strength. We, therefore, come to the conclusion that the transverse strength of a beam, supported at both ends and loaded in the middle, is double that of a beam half the length at one end and loaded at the other. We have already shown that if we

double the length of a projecting beam, we can load it with only half the weight; therefore, if each projecting arm of the beam were made equal in length to the beam supported at both ends, the former would bear only one-fourth of the weight which the latter can bear. For equal lengths, therefore, the strength of the beam supported

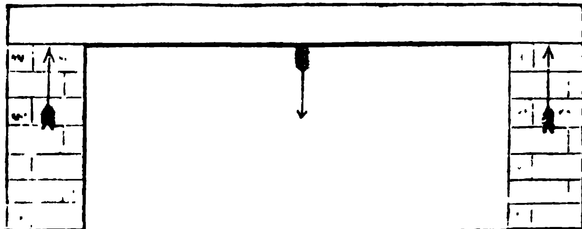


Fig. 67.

at both ends and loaded in the middle, is just four times that of the beam fixed at one end and loaded at the other; or, conversely, the strength of the beam projecting is only one-fourth of that of the beam supported at both ends. Experiments upon transverse strain have been generally made upon beams supported on both ends; and tables of practical data founded on these experiments have been formed. Such tables generally contain the transverse strength of beams, having square and also circular sections one inch in diameter, and having a length of one foot between the supports, their load being supposed to be placed in the middle of their length. When the load is placed out of the middle it may be increased, because its leverage to break the beam is diminished the farther it is from the middle point.

The mode of reckoning this diminution of strain may be best illustrated by an example. Suppose we found that a beam 10 feet long bore 42 cwt. at its middle point, and desired to ascertain how much it would bear suspended 2 feet from the middle—that is, 7 feet from one end and 3 feet from the other (Fig. 68)—we should proceed as follows:—Square half the length of the beam, or multiply 5 by itself, giving 25, and this by 42 cwt., the load sustained at the middle, product 1050; now multiply 7 by 3 (the two portions into which the beam is divided), product 21, and divide the former product 1050 by this, giving a quotient 50 cwt., the load which the beam would carry 2 feet from the centre.

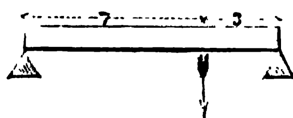


Fig. 68.

The simple principle of this computation is, that the load hung from any point of a beam is to the load which may be hung from any other, as the product of the two lengths into which the second point divides the beam is to the product of the two lengths into which the first point divides it. Thus, in the case we have given above, knowing that the beam bears 42 cwt. at the middle, or when it is divided into two lengths each 5 feet, we say

$$7 \times 3 = 21 : 5 \times 5 = 25 :: 42 : 50.$$

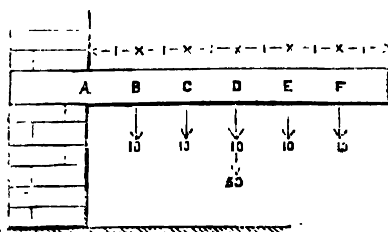


Fig. 69.

It frequently happens that beams have to bear a load not hung at any one point, but distributed uniformly over their length; as in the case of roofs, floors, and girder-

bridges. Here it may be readily seen that the strain is only half that which it would be if the whole load were collected at the middle. If we suppose a beam projecting 6 feet from a wall, loaded at intervals of a foot by weights each 10 cwt. (Fig. 69), then, by the principle of the lever, the effect of each of the weights to break the beam at A may be reckoned as follows:—

B = 10 cwt.	acting at	1 foot from A	has the effect of 10 cwt.	at 1 foot leverage.
C = 10	,, ,,	2	,, ,, 20	,, 1 ,,
D = 10	,, ,,	3	,, ,, 30	,, 1 ,,
E = 10	,, ,,	4	,, ,, 40	,, 1 ,,
F = 10	,, ,,	5	,, ,, 50	,, 1 ,,
Total 50 cwt. distributed equally.			150 cwt.	at 1 foot leverage.

Or, as 150 cwt. at 1 foot leverage are equivalent to 50 cwt. at 3 feet, we find that the total strain is the same as if the total weight were collected at D, the middle point. Were we to assume a greater number of weights at smaller intervals, we should still find the same result; and the more numerous the weights and smaller the intervals we assume, the more nearly do we approach to the case of a beam uniformly loaded over its whole length; whence we conclude that the effect of the distributed load is the same as if it were collected at the middle of the beam, and therefore just half of what it would be if hung at the end. Or conversely, if the beam bears a certain load at its extreme end, it will bear double that weight distributed over its whole length. The same law applies in the case of a beam supported at both ends and loaded uniformly throughout its length: the strain of the load is reduced to half what it would be if collected at the centre; or the beam will bear twice as much distributed weight as it can bear at its middle point.

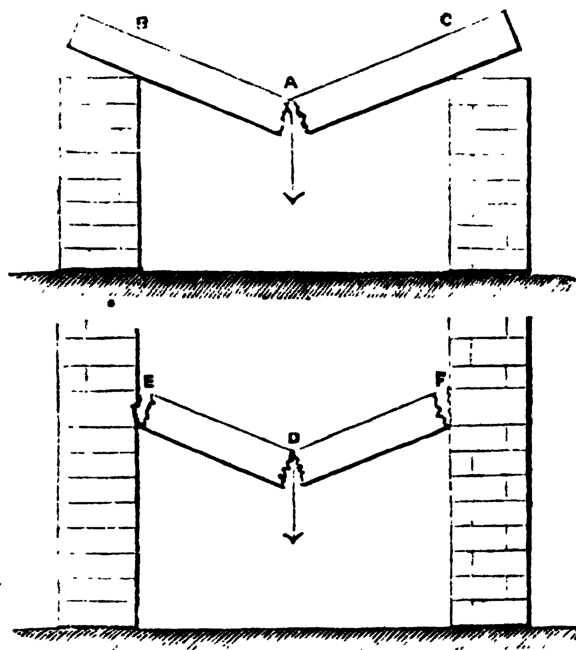


Fig. 70.

When a beam is not merely supported at each end, but fixed firmly there, its strength is increased by one-half. It would appear at first sight, that by fixing the ends of a beam we should double its strength, for the following reasons: when the beam is merely supported, an extreme load in the middle has only to effect one fracture at A (Fig. 70), the two ends B and C being free to rise. But when the ends are fixed, the

load has not only to break the beam at the middle point D, but also at E and F; and to do this it must be double what it would be if the ends were free, as may be very simply computed thus. Suppose, in the first case, it required 12 cwt. to effect the single fracture at A, then in the second it would require likewise 12 cwt. in the middle to effect the fracture at D, and 6 cwt. in the middle to effect each of the fractures at E and F; making a total of 24 cwt. in the case of the beam with fixed ends.

Many writers have taken this view, neglecting a circumstance which must in such cases occur, and which greatly modifies the distribution of strain on the middle and end. A single glance at the figure shows that the amount of tension and compression on the fibres at the middle fracture must be double that at either of the end fractures; and hence each half of the weight required to produce either of the end fractures, or the total weight due to the middle fracture, must be 4 times either of those due to the end fractures. If then we supposed the total breaking-weight divided into 6 equal parts, 4 of those parts would act to break the beam in the middle, and 2 to break it at the ends. But the 4 parts required to effect the middle fracture must make up the breaking-weight due to a beam merely supported without being fixed at the ends; and the other 2 parts—that is, half as much more—make up the additional weight required when the ends are fixed. Taking the numerical example as before, if it required 12 cwt. to effect the single fracture at A, it would require as much to break the beam at D, and one-fourth, or 3 cwt., to break it at each of the points E and F, making a total of 18 cwt., which is the sum of 12 and 6, the ordinary breaking-weight increased by its half.

As this principle of computation accords better with experiment than the former, it is important that its demonstration should be clear, inasmuch as numerous theorists have adhered to the principle, that by fixing the ends of a beam its strength is doubled. According to them, the circumstances correspond with those of a beam (Fig. 71) resting on supports A and B, and projecting each way one half of its length beyond. A load of 1 at each end, balanced by two loads, each 1, in the middle, throws a breaking strain of 2 upon the beam at each prop; and an additional load of 2 in the middle will measure the breaking strain there, so that the total middle load is 4, or double the ordinary breaking strain.

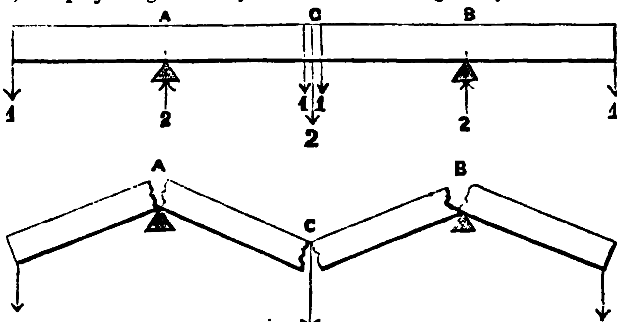


Fig. 71.

This is no doubt true, because if we suppose the fracture effected, the amount of compression and extension of the fibres at each of the points A, C, and B is the same; and therefore each fracture requires the same load to effect it. But in the case of the beam with ends not balanced but fixed, as we have already explained, the end fractures demand only half the amount of extension and compression due to the middle fracture.

When a load acts on a beam not perpendicularly or square to its length, but at some other angle, it throws less strain upon it, because the actual leverage of the weight is not

the length of the beam as measured from A to B (Fig. 72), but the length of a line measured from A to C perpendicularly to the vertical line in which the weight acts.

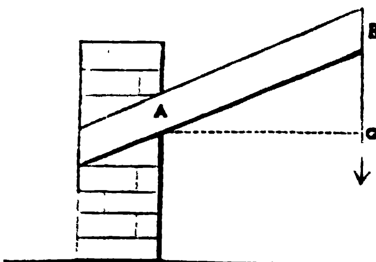


Fig. 72.

So when a beam is supported at both ends, but lies obliquely, the transverse strength is to be reckoned as that due to a beam of the length indicated by the horizontal line D E (Fig. 73) measured between the supports.

Hitherto, for the sake of simplicity, we have discussed the question of transverse strain as applied to materials perfectly inflexible, such as break but cannot bend. We have, however, no practical experience of materials of this character, although stones, slates, or even cast-iron, approach very nearly to it. Timber beams, and those made of wrought-iron, bend considerably before they become fractured by transverse strain; and as in such beams deflection from their straight or horizontal condition may be inconvenient and unsuitable, it becomes important to estimate the amount of deflection which they will exhibit under certain loads, so that they be made slightly curved in the opposite direction before the load is placed upon them. If it were found, for instance, that a beam loaded with a certain weight deflected so far from the straight line that its middle point B (Fig. 74) sunk a certain distance—say 6 inches—below the horizontal line; then if the beam, instead of being made straight, were made somewhat arched, or *cambered*, as it is technically called, the load being placed upon it would still deflect it, and thus bring its surface to a horizontal line, if the camber or amount of arching C D were properly estimated.

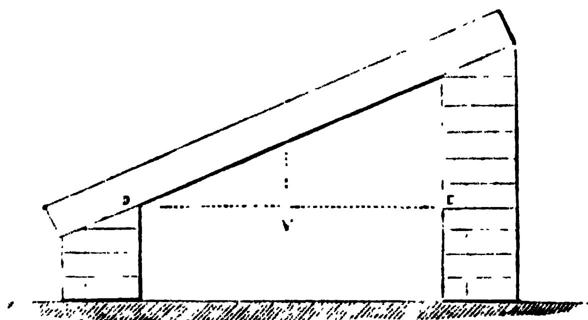


Fig. 73.

The complete investigation of the question of deflection would involve us in mathematical reasoning of rather a complex character, which would scarcely be in place here; and, indeed, practical results as to deflection present so many irregularities, and so many deviations from any apparent law, that it is questionable whether theory would prove a very safe guide. Some writers on this subject have determined theoretically, that the amount of deflection of a beam of certain length increases in the same proportion as the load, and that the deflection under a certain weight varies as the square of the length. If this law were true, a beam 20 feet long with a certain weight on it would be deflected four times as far as one 10 feet long, because the one has twice the length of the other, and the square of 2, or 2 multiplied into itself, is 4. This law, however, has not by any means been found to be true in practice. More accurate investigators have furnished a law which, while it appears to be theoretically correct, presents

results very nearly according with those of experiment. This law is, that the deflection of a beam (having a rectangular section) varies directly as the weight and as the cube of the length (or the length multiplied 3 times into itself), and inversely as the breadth and the cube of the depth.

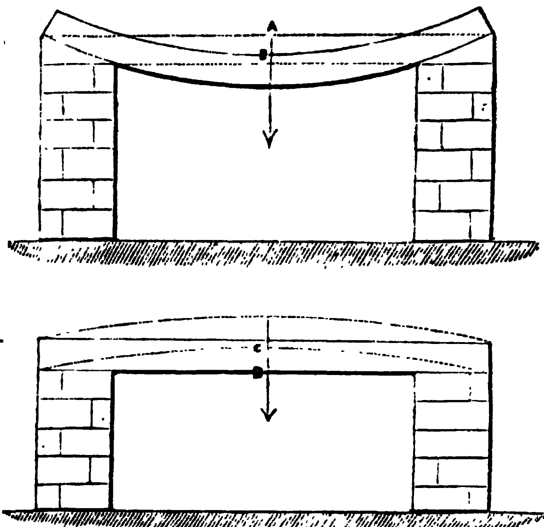


Fig. 74.

If, then, we knew the deflection of a certain beam, we might estimate that of another of the same material, but varying in all its dimensions. Suppose, for example, that a fir batten 1 inch broad and 2 inches deep, with a load of 1 cwt., deflected $\frac{1}{16}$th of an inch in a length of 3 feet, and we desired to know the deflection of a fir beam 3 inches broad, 10 inches deep, and 15 feet long, under a weight of 1 ton, or 20 cwt. In the case of the batten, which is 2 inches deep, since $2 \times 2 \times 2 = 8$, the deflection is only $\frac{1}{8}$th of what it would be were the depth 1 inch, because it is inversely as the cube of the depth. The deflection, then, of a batten 1 inch deep, under a load of 1 cwt., would be $8 \times \frac{1}{16} = \frac{1}{2}$ inch, the length being 3 feet. Further, the cube of 3, or $3 \times 3 \times 3$, is 27; and were the batten only 1 foot long instead of 3 feet, the deflection would be $\frac{1}{27}$th; the deflection then of a batten 1 inch broad, 1 inch deep, and 1 foot long, would be $\frac{1}{27}$th of $\frac{1}{2}$ an inch, or $\frac{1}{54}$th of an inch, with a load of 1 cwt. Having thus got an estimate for a beam with all the dimensions reduced to unity, we can apply it to the beam, according to the law we have stated. This law is arithmetically applied by multiplying the deflection found above, $\frac{1}{54}$th of an inch, by the weight 20 cwt., by the cube of the length 15 feet (or 3 times by 15 feet), and dividing the product by the breadth 3 inches; and the cube of the depth 10 inches or the deflection is $\frac{\frac{1}{54}\text{th in.} \times 20 \times 15 \times 15 \times 15}{3 \times 10 \times 10 \times 10} = \frac{6}{12}$ths of an inch, or nearly $\frac{1}{2}$ an inch.

As to the deflection of beams of various forms and materials, and subjected to strains under various conditions, although numerous experiments have been made, yet they have been conducted with too little reference to each other for us to develop any law of general application. In Barlow's Treatise on the Strength of Materials, the question of the deflection of wrought-iron, as applied in the construction of rails, is treated at considerable length; but the conditions of strength in malleable iron must differ very considerably from those in other materials, because their comparative tensive and compressive strengths differ very widely. To show how cautious we ought to be in applying to any particular material the results deduced from experiments on

other materials, we may instance the peculiar difference between cast and wrought-iron in respect of tensive and compressive strength. The direct cohesive strength of wrought-iron is about 3 times that of cast-iron; or, to bear a certain tensive strain, the area of section in cast should be about 3 times that of wrought-iron. As to compressive strain, on the other hand, although it may be true that wrought-iron will sustain much greater compression than cast-iron before it becomes entirely crushed and disintegrated, yet the greater softness of the wrought-iron permits it to yield under pressure, and to become compressed to a considerable extent; while cast-iron scarcely yields perceptibly until it entirely gives way.

Now, if we apply these considerations to a beam subjected to a transverse strain, as when it is supported at both ends and loaded in the middle, we see that at the middle of the beam, just before fracture takes place, there must be a portion of the material above the neutral axis compressed, and a portion below it extended by the action of the load. If the resistance of the material to extension and to compression under those conditions be equal, the neutral axis would be in the middle of the beam; and its resistance to fracture would then be the greatest possible, because the total leverage of the compressed and extended portions on each side of it would be greater when it is in the middle than when it is elsewhere. But if it happened that the material was more easily compressed than extended, it would be nearer the lower side; while if it were more easily fractured by extension than compression, it would be nearer the upper side. Desiring, however, in either case to secure the advantage of having it in the middle, and thereby giving both the compressed and the extended portions the greatest possible leverage, we should somewhat alter the form of transverse section, so as to increase the area of the weaker part, or add to it some fibres, diminishing the area of the stronger, and upon the whole not altering the quantity of material or total area of section, but only modifying it in such a way as to bring the neutral axis to the middle of the depth. In a rectangular beam of malleable iron C (Fig. 75), the neutral axis is below the middle of the depth, because the upper portion yields more readily to compression than the lower to extension. In a beam of cast-iron B the neutral axis is above the middle, because the upper portion resists compression more than the lower resists extension. To bring the neutral axis to the middle in both, we should for malleable iron remove portions of thickness from the lower edge, and add them to the upper, as in A; for cast-iron we should take from the upper and add to the lower, as in D. We thus find that the modification of form requisite for increasing the transverse strength of cast-iron of certain depth, without adding to the mass of material, is exactly the opposite of that suited to malleable iron. The usual section of cast-iron girders, supported at both ends and loaded in the middle, is the inverted T, swelled a little at the upper edge (Fig. 76). The dimensions of a girder one foot deep are nearly those marked in the diagram. This may not be precisely the best form for combining

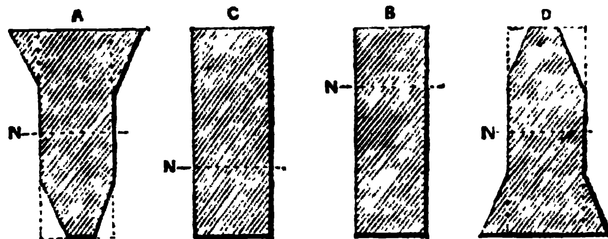


Fig. 75.

the greatest strength with economy of materials, but it approaches it; and, besides, it possesses practical advantages which deserve consideration. The flange, or increased width at the lower side, not only affords the additional strength required there in order to meet the tensive strain, but presents a projecting ledge for receiving the beams or arches which the girder may have to bear. The increased thickness at the upper side not only provides additional material to resist compressive strain at the greatest possible distance from the neutral axis, but serves to stiffen the girder so as to prevent it from bending or buckling sideways, either from contraction in cooling from its molten state, or from excessive strain in its place. The thickness of the metal in the middle part and in the lower flange is made as nearly equal as possible, because it is found practically that, in making iron castings, unequal thicknesses of metal cause unequal contractions or shrinkings in the metal as it cools, and thus tend to distort the work.

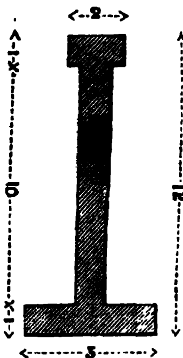
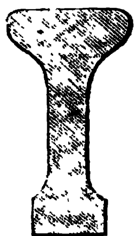
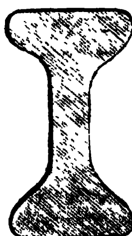


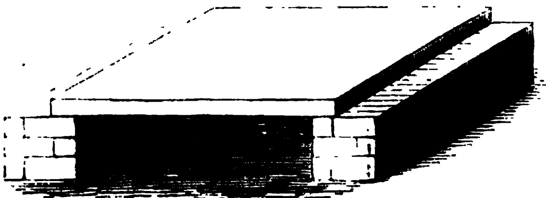
Fig. 76.

The best form of section for malleable-iron rails is nearly the opposite of that for cast-iron girders, a ┬ not inverted (Fig. 77). As, however, the upper part of the rail gets worn and uneven by the friction of the traffic, it is sometimes thought desirable to have the opportunity of inverting it, and thereby wearing both the upper and lower sides before the rail is rejected as worn out. The section is therefore made symmetrical above and below, as well as on both sides (Fig. 78).


Fig. 77.


Fig. 78.

As we have shown above, the strength of any beam to resist transverse strain increases very greatly as the depth is increased. It is therefore of great importance in all cases of transverse strain to give as much depth as possible. This is often effected not by increasing the total depth of the material, and thus adding greatly to its weight, but by introducing ribs or flanges, and thus dispersing a given quantity of material in a better form. If, for instance, we had to provide a square cast-iron plate of sufficient strength to resist a great weight pressing on its middle (Fig. 79), while it is supported on two piers at the sides: instead of making the plate of solid iron, sufficiently thick to sustain the strain, we should probably make the upper part a thin flat plate (Fig. 80); and round the edges, as well as across the diagonals, provide projecting ribs of the greatest depth in the middle (Fig. 81); and thus attain sufficient strength and stiffness, while we should save a considerable quantity of material.* This mode of attaining strength is particularly useful with such a material as cast-iron; for when it is formed in thick masses, the cooling of the outer crust, while


Fig. 79.

the inner part of the mass remains fluid, sets or fixes the outside, and the subsequent contraction of the inside causes a sponginess or looseness of texture which greatly diminishes strength. It is, therefore, of the utmost importance to attain the required strength without great thickness, as well for the sake of securing solidity and firmness of material, as for avoiding air-bubbles and flaws, which are apt to occur in thick castings; and which if they do occur in thin castings do not take so much from the strength, or, at all events, are more likely to be visible and can be allowed for. In malleable-iron bars, when they are required for strength and stiffness, the T form is usually employed. In plates, corrugating or wrinkling adds greatly to the stiffness, because it provides depth of material transversely.

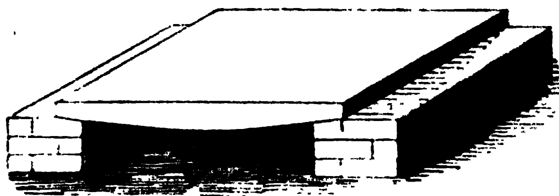


Fig. 80.

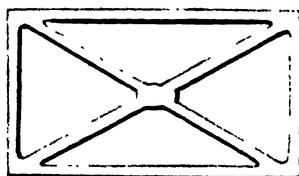


Fig. 81.

The question of how to dispose material in order to secure the greatest strength with the least weight and cost, is indeed the main subject of mechanical contrivance as to form. We have already instanced the contrivance of the Britannia Tubular Bridge as an example of skill in device going hand-in-hand with experiment. Were the material employed in one of the great tubes of this bridge all compressed into one solid bar, having a section of dimensions proportional to those of the tube, we question if it could support its own weight without breaking, even if the span were reduced to half that of the tube; while with one-fourth of the span the deflection, from its own weight, would be enormous (Fig. 82). In this tube the material is disposed in such a manner as to attain the greatest strength with the least weight, and the most suitable form for the purposes of the traffic. The additional material required in the upper part of the section (as in the case of other wrought-iron girders), is arranged in the sides of cells or subsidiary tubes; and due regard has been paid to the securing of lateral stiffness to resist the pressure of strong winds against its immense side-surface, as well as to the attainment of vertical stiffness to resist the strain and shake of a heavy passing load. In some other cases where malleable-iron is used in the construction of bridges, a girder has been formed of plates. The upper part is a tube bent over to an arch-form (Fig. 83), and the lower portion a flange or ledge, as well for strength as for receiving the ends of the beams that carry the roadway; and the upper and lower portions are connected by a longitudinal fin, with several transverse ribs for stiffening and more firmly connecting the whole together. This ingenious arrangement of parts is due, we believe, to Mr. Brunel the engineer.

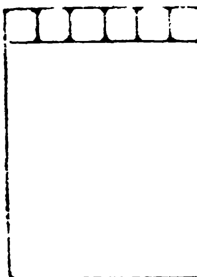


Fig. 82.

Not only, however, is there scope for modification of form of the transverse section so

as to secure strength with economy of material; the longitudinal section and plan of a beam are also susceptible of modifications.

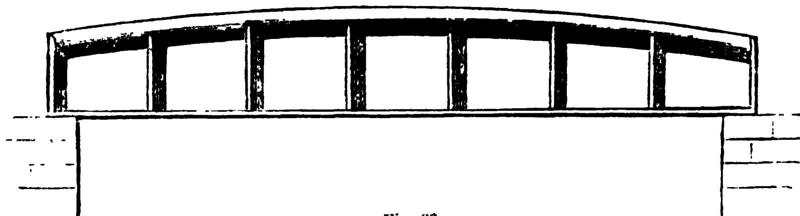
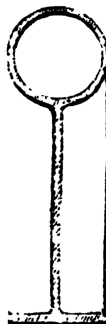


Fig. 83.

In the case of a beam projecting from a wall (Fig. 84), with a weight suspended at its extreme end, the strain is greatest at A, close to the support, and diminishes towards the end, because the leverage with which the weight acts to fracture the beam diminishes. Thus, at C, midway, the strain is only half that at A, at B it is $\frac{3}{4}$ths, and at D it is $\frac{1}{4}$th. If, then, the breadth of the beam be uniform throughout its length, its depth may be with safety diminished towards the extremity. As the strength is proportioned to the square of the depth, the depths at B C D may be made such that their squares are respectively $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ of the square of A. This may be done by removing material from the upper or lower side of the beam, so as to give it a curved outline above or below; and still the strength of the beam is maintained. The curve for such a beam is what is called a parabola, the peculiar property of which is, that a square of the length of each of the vertical lines or ordinates A, B, C, D, is proportional to its distance from the apex or extremity E. Let us take, as an example, a beam of cast-iron projecting 12 feet, and 12 inches deep at A by 3 inches broad. The weight of such a beam of uniform depth throughout would be about 12 cwt. But by tapering it off as we have indicated, its weight would be reduced to 8 cwt., $\frac{2}{3}$rds of what it was. Thus, not only is a saving effected in the cost of the beam, but



Section.

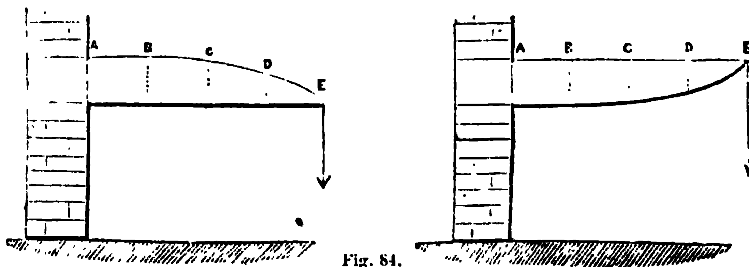


Fig. 84.

as its own weight is an important part of the strain at A, this element of strain is considerably diminished, and the weight hanging from the end may be proportionally increased. The depths at the different points would be as follow:—

At A, close to the support, 12 inches.
,, B, $\frac{3}{4}$ths of the length from A, about $10\frac{1}{2}$, because $10\frac{1}{2}$ squared is about $\frac{3}{4}$ths of 12 squared. $10\frac{1}{2} \times 10\frac{1}{2} = 110\frac{1}{4}$, and $12 \times 12 = 144$, $\frac{3}{4}$th of which is 108.
,, C, $\frac{1}{2}$ the length, or 6 feet from A, about $8\frac{1}{2}$ inches.
,, D, $\frac{3}{4}$th the length, or 9 feet from A, 6 inches.

If the depth of the beam cannot conveniently be varied, the breadth may be diminished at a distance from the point of support; because the strength of a beam being as its breadth, and the leverage of the weight being lessened as we recede from the breaking-point, the breadth may be lessened in like proportion. Sometimes it is convenient to vary the depth and breadth also, and thus maintain a similar section throughout the whole length; that is, a section of like figure, but of varying dimensions.

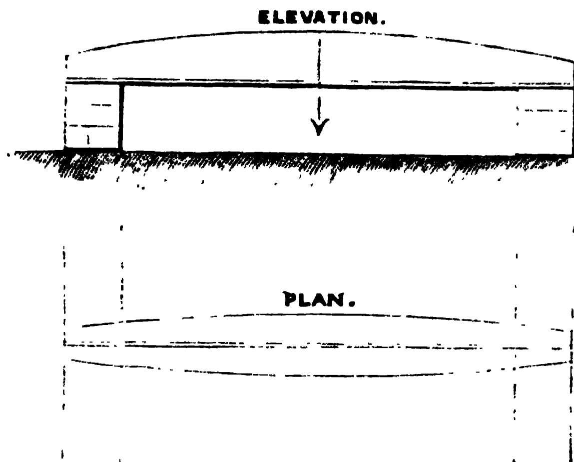
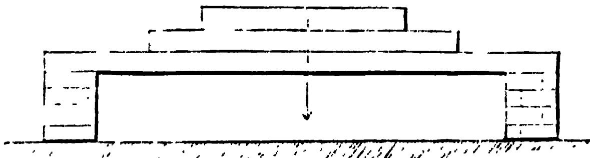


Fig. 85.

In the case of beams supported at both ends, and loaded in the middle (Fig. 85), the same principle is applied; the dimensions, in respect of depth and breadth, being made greatest in the middle and diminished gradually towards the ends. This economy of material is easily effected in cast-iron girders by making the pattern from which the casting is moulded of the requisite form. When timber beams are used, the increased depth towards the middle is attained by piling several beams on one another (Fig. 86). As the lower side of a beam resting at both ends is subjected to a tensive strain, great additional transverse strength may be secured by straining a rod or chain B B (Fig. 87) from end to end of the beam, and blocking it off from the lower surface by means of one or more wooden or iron struts A or C C. By this arrangement the wooden beam, when loaded between the supports, is subjected to a compressive strain only, the whole of the tensive strain being thrown upon the rod or chain.


Fig. 86.

Again, by fixing a strut, or king-post, C (Fig. 88), upon the upper side of the beam A A, and connecting it by pieces B B to the ends, the compressive strain is thrown on B B and the tensive strain only on A A. Indeed, there is no limit to the contrivances

by means of which the direction and amount of the strains can be varied so as to economise material and secure stability. We have only instanced a few as examples of arrangements, a complete account of which would fill many volumes.

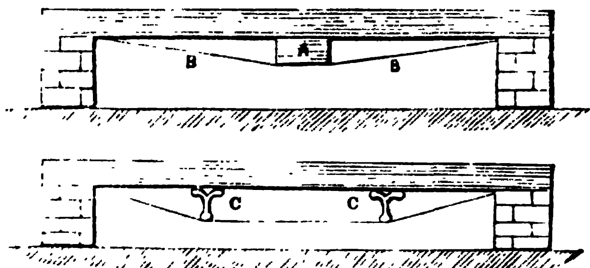


Fig. 87.

The table of transverse strengths gives the load which may be safely placed on the middle of beams of different materials supported at both ends. The table applies to beams 1 foot long between the points of support. The weights are those for beams the transverse section of which in the middle is 1 inch square, or 1 inch broad and 1 inch deep; and these apply to all beams of rectangular section. The weights for beams of circular section, may be taken at two-thirds of those for square section. The following are the rules for computing the strengths of beams of various dimensions, according to the data furnished by the table:—

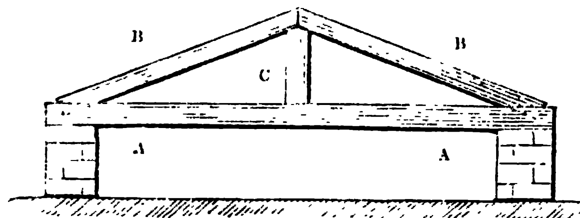


Fig. 88.

I. For beams supported at both ends and loaded in the middle. Given the length, breadth, and depth, to find the load.

Rule.—Multiply the number in the table by the breadth (in inches), twice by the depth (in inches), and divide by the length (in feet).

Example 1.—Required the transverse strength of a beam of teak 8 inches broad, 12 inches deep, and 14 feet between bearings.

Number from table opposite teak	270 lbs.
Multiply by breadth	8 ins.
	2160
Multiply by depth	12 ins.
	25920
Again by depth	12 ins.
Divide by length	14)311040
	22217 lbs.

Very nearly 10 tons, or 22,400 lbs.

Example 2.—Required the transverse strength of a wrought-iron bar 1½ inch broad, 4 inches deep, and 6 feet long.

$$\frac{1300 \times 1\frac{1}{2} \times 4 \times 4}{6} = 5200 \text{ lbs., nearly } 46\frac{1}{2} \text{ cwt.}$$

For bars of circular section.

Rule.—Multiply the number in the table three times by the diameter (in inches), divide by the length (in feet), and deduct one-third of the result from itself.

Example 3.—Required the transverse strength of a beech roller, 2 inches diameter, and 4 feet long.

$$\frac{170 \times 2 \times 2 \times 2}{4} = 340$$

$$\text{Deduct one-third} = 113$$

$$227 \text{ lbs., about } 2 \text{ cwt.}$$

II. When the beam is fixed down at the ends.

Rule.—Find the strength as before (I.), and add to it its half.

Example 4.—Required the strength of a cast-iron bar 2 inches broad, 6 inches deep, 9 feet long, fixed at both ends.

$$\frac{850 \times 2 \times 6 \times 6}{9} = 6800 \text{ lbs., about } 3 \text{ tons.}$$

$$\text{Add one-half} \quad 3400 \quad ,, \quad 1\frac{1}{2} \text{ ton.}$$

$$10200 \quad ,, \quad 4\frac{1}{2} \text{ tons.}$$

III. When the beam is supported loosely at both ends, and has the load uniformly distributed over its length.

Rule.—Find the strength as before (I.), and double it.

Example 5.—Required the strength of a deal rafter 3 inches broad, 11 inches deep, 10 feet long, loaded uniformly throughout its length.

$$\frac{130 \times 3 \times 11 \times 11}{10} \times 2 = 9438 \text{ lb., about } 4\frac{1}{4} \text{ tons.}$$

IV. When the beam is fixed at both ends, and has a load uniformly distributed over its length.

Rule.—Find the strength as in I., and triple it.

Example 6.—Required the strength of a round wrought-iron bar 2 inches diameter, 10 feet long, fixed at ends, the load being uniformly distributed.

$$\frac{1300 \times 2 \times 2 \times 2}{10} = 1040$$

$$\text{Deduct one-third} = 347$$

$$693 \times 3 = 2079 \text{ lbs.}$$

V. When the beam is loaded by a weight not in the middle.

Rule.—Find the strength as before (I.), multiply twice by half the length, and divide by the length of each part into which the beam is divided by the point of suspension.

Example 7.—Required the strength of an ash beam 6 inches broad, 8 inches deep, 14 feet long, to carry a weight 3 feet from one end. Half the length is 7 feet, and the two parts are 11 and 3.

$$\frac{250 \times 6 \times 8 \times 8}{14} \times \frac{7 \times 7}{11 \times 3} = 10182 \text{ lbs., about } 4\frac{1}{2} \text{ tons.}$$

VI. When the beam is fixed at one end and loaded at the other.

Rule.—Find the strength as in I. and divide it by 4.

Example 8.—Required the strength of an English oak beam 13 inches square, projecting 10 feet from a wall, to bear a load at the end.

$$\frac{190 \times 13 \times 13 \times 13}{10} = 41743 \text{ lbs., strength by I.}$$

Divide by 4 = 10433 lbs.

These cases include most of the circumstances which occur in practice. The converse operations for finding the dimensions of a beam when we know the weight it has to bear, do not furnish us with the breadth and depth separately (except in the case of cylindrical and square beams when the depth and breadth are equal), but give us a result which is the product of the breadth by the square of the depth. We must, therefore, be guided by other circumstances in determining one of these dimensions; and having determined the one, we easily find the other. For instance, if we were required to provide a cast-iron girder of rectangular section, such that when placed on two walls 10 feet apart it should carry a load of 2 tons in the middle, we should reason thus:—By the table, a cast-iron rectangular beam 1 inch broad and 1 inch deep, and 1 foot between supports, bears 850 lbs.; one 10 feet between supports must be made ten times as strong to bear 850 lbs.; and to bear 2 tons, or 4480 lbs., it must be about five and a quarter times as strong, because 4480 lbs. is about five and a quarter times 850 lbs.

The strength, then, for the given length and weight must be $5\frac{1}{4} \times 10 = 52\frac{1}{2}$; or, more correctly, $\frac{10 \times 4480}{850} = 52{\cdot}7$ times that of a beam 1 inch broad and 1 inch deep. But as the strength is as the breadth multiplied by the square of the depth, the number 52·7 must be the product of the breadth in inches, multiplied by the square of the depth or twice by the depth; and if we determine one of these dimensions, we can easily ascertain the other, thus—

Assume breadth,		then depth,		because nearly
1 in.	. .	$7\frac{1}{4}$ in.	. .	$1 \times 7\frac{1}{4} \times 7\frac{1}{4} = 52{\cdot}7$
2 in.	. .	$5\frac{1}{8}$ in.	. .	$2 \times 5\frac{1}{8} \times 5\frac{1}{8} = 52{\cdot}7$
3 in.	. .	$4\frac{1}{8}$ in.	. .	$3 \times 4\frac{1}{8} \times 4\frac{1}{8} = 52{\cdot}7$
4 in.	. .	$3\frac{3}{4}$ in.	. .	$4 \times 3\frac{3}{4} \times 3\frac{3}{4} = 52{\cdot}7$
5 in.	. .	$3\frac{1}{4}$ in.	. .	$5 \times 3\frac{1}{4} \times 3\frac{1}{4} = 52{\cdot}7$

And so on, the one dimension being assumed according to circumstances of convenience or for other reasons. When it is determined that the section of the beam shall be square, the product found as above is the cube of the breadth or depth, or it is the breadth multiplied 3 times by itself; and therefore the breadth or depth is found by taking the cube root of this product. In the case given, the cube root of 52·7 is about $3\frac{3}{4}$, and we therefore conclude that if the beam be of square section, its breadth or depth must be $3\frac{3}{4}$ inches, because $3\frac{3}{4} \times 3\frac{3}{4} \times 3\frac{3}{4} = 52{\cdot}7$. When the beam is cylindrical, the diameter is also determined. Since the strength of a cylindrical beam is about $\frac{2}{3}$ of a square one of the same material, we must make up the other $\frac{1}{3}$ by adding to its dimensions; and as $\frac{1}{3}$ is one-half of $\frac{2}{3}$, we must add to the product its half, and then take the cube root as for a square beam. In the case given, adding to 52·7 its half, or 26·35, we have 79·05, which is nearly the cube of $4\frac{1}{4}$; and we must therefore make the diameter of a cylindrical beam $4\frac{1}{4}$ inches, or more correctly 4·3 inches.

As a proof of the accuracy of our calculation, we have only to reverse the operation, and calculate the load which the beam of the dimensions we have thus determined will bear. If the result corresponds with the given weight, we know that we are right.

In the example we have taken, we have to find the strength of a cylindrical cast-iron beam 4·3 inches diameter and 10 feet long. By the rule for bars of circular section, case I.,

$$\frac{850 \times 4{\cdot}3 \times 4{\cdot}3 \times 4{\cdot}3}{10} = 6758 \text{ lbs.}$$

$$\text{Deduct } \tfrac{1}{3} = 2253$$

$$4505$$

Nearly 2 tons, or 4480 lbs., the load which the beam was intended to carry.

The following rules give the mode of calculating the *strength product* under various circumstances of strain, and then the method of dealing with the *strength product* when the form, proportions, or one of the sectional dimensions of the beam is given. The cases are numbered in the same order as those in the calculation of the load.

I. Beams supported at both ends and loaded in the middle: given the length and load to find the *strength product*.

Rule.—Multiply the load (in lbs.) by the length (in feet), and divide by the number in the table.

Example 9.—Required, the *strength product* for a beam of teak 14 feet between bearings, to carry 10 tons in the middle.

$$10 \text{ tons} = 22{,}400 \text{ lbs.}$$

$$\frac{22{,}400 \times 14}{270} = 1161, \text{ strength product.}$$

8 ins. broad and 12 ins. deep would suit this case, because $8 \times 12 \times 12 = 1152$, nearly 1161.

II. When the beam is fixed down at the ends.

Rule.—Find the strength product as in I., and deduct from it its ⅓rd part.

Example 10.—A cast-iron bar fixed at both ends 9 feet apart has to carry 10,200 lbs.

$$\frac{10{,}200 \times 9}{850} = 108$$

$$\text{Deduct } \tfrac{1}{3} = 36$$

$$72 = \text{strength product.}$$

2 ins. broad and 6 ins. deep, for $2 \times 6 \times 6 = 72$.

III. When the beam is loosely supported, and the load uniformly distributed.

Rule.—Halve the strength product.

Example 11.—A deal rafter 10 feet long is loaded uniformly with 9438 lbs.

$$\frac{9438 \times 10}{130} \times \tfrac{1}{2} = 363$$

3 ins. broad and 11 ins. deep, for $3 \times 11 \times 11 = 363$.

IV. When the beam is fixed at both ends, and has the load uniformly distributed.

Rule.—Divide the strength product by 3.

Example 12.—A wrought-iron bar 10 feet long fixed at ends is loaded with 2080 lbs. uniformly distributed.

$$\frac{2080 \times 10}{1300 \times 3} = 5\tfrac{1}{3}, \text{ the strength product.}$$

If the bar be round, we must increase this by one-half, making it 8; and as 8 is the cube of 2, or $2 \times 2 \times 2$, the round bar must be 2 ins. diameter.

V. When the beam is loaded not in the middle.

Rule.—Multiply the strength product successively by the two lengths into which the weight divides the beam, and divide twice by half the total length.

Example 13.—An ash-beam 14 feet long, has 10,182 lbs. suspended from it 3 feet from one end (consequently 11 feet from the other, half the length being 7 feet).

$$\frac{10{,}182 \times 14}{250} \times \frac{11 \times 3}{7 \times 7} = 380$$

6 inches broad and 8 inches deep, for $6 \times 8 \times 8 = 384$.

VI. When the beam is fixed at one end and loaded at the other.

Rule.—Multiply the strength product by 4.

Example 14.—An English oak beam projects 10 feet and carries 10,433 lbs. at the end.

$$\frac{10{,}433 \times 10}{190} \times 4 = 2196.$$

13 inches square, for $13 \times 13 \times 13 = 2197$.

The following rules embody the methods of dealing with the strength product when found as above.

1. Given the depth to find the breadth.

Rule.—Divide the strength product twice by the depth (in inches), the quotient is the breadth (in inches).

Example 15.—Beam 6 inches deep to give strength product 72.

$$\frac{72}{6 \times 6} = 2 \text{ inches, the breadth.}$$

2. Given the breadth to find the depth.

Rule.—Divide by the breadth and take the square root of the quotient.

Example 16.—Beam 8 inches broad, strength product 1161.

$$\frac{1161}{8} = 145, \text{ the square root of which is nearly 12, for } 12 \times 12 = 144.$$

3. When the beam is square in section.

Rule.—Take the cube root of the strength product.

Example 17.—Square beam, strength product 2196.

Cube root of 2196 = 13 nearly, for $13 \times 13 \times 13 = 2197$.

4. When the beam is cylindrical.

Rule.—Increase the strength product by its half and take the cube root.

Example 18.—Cylindrical beam, strength product $5\frac{1}{3}$

Add half of $5\frac{1}{3} = 2\frac{2}{3}$

8

Cube root of 8 = 2.

For beams of other forms than those which have square, rectangular, or conical sections, we cannot furnish any rule. Practical men, from long experience and from a habit of observing the proportions suitable to certain strains, can form a tolerably accurate conception of the strength due to particular forms under various conditions. Before any mechanical work is executed, the appearance of the parts on the drawing recommends itself to a practised eye, or otherwise, according as the details are studied in consonance with the just proportions of strength or otherwise. In providing not only against absolute fracture, but also against deflection, vibration, and all such ele-

ments of degradation and weakness, the strength must be made very much in excess of that which the mere calculation of breaking-strain would indicate; and the different modes of providing adequate firmness and permanence without extravagant use of materials, are matters of ingenious contrivance for which no rule could be furnished.

Table of transverse strengths of beams, 1 foot long between supports, lying loose at both ends, loaded in the middle, having a section 1 inch square:—

Name of material.	Permanent load.	Name of material.	Permanent load.
Ash	250 lbs.	Oak (English)	190 lbs.
Beech	170 ,,	Pitch pine	190 ,,
Deal	130 ,,	Teak	270 ,,
Elm	120 ,,	Iron (cast)	850 ,,
Oak (African)	230 ,,	Iron (wrought)	1300 ,,

4. **Torsion.**—As in machinery motion is generally conveyed from one part to another by means of shafts or spindles rotating on their axes, it becomes a matter of considerable importance to determine the strength of materials to resist a twisting or wrenching force. If we suppose an iron shaft fitted with a wheel at each end, one of which is driven by some prime mover, such as a steam-engine, and the other conveys the power or motion to some machinery, it is clear that the whole power so conveyed has to pass through the shaft; and the resistance of the machinery at the one end to the prime force applied at the other, subjects the shaft to a torsive or twisting strain.

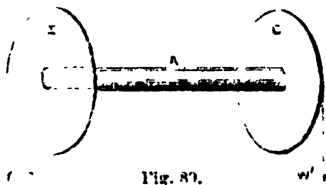


Fig. 89.

That we may simplify the view which we should take of this kind of strain, we shall suppose a shaft A (Fig. 89) fitted with a wheel at each end Z and C, round the circumference of each of which, in opposite directions, a rope passes, suspending a weight W. The wheels and weights being equal are balanced, and the shaft is not caused to turn in either direction, but both weights tend to twist the shaft in itself. If instead of one of the weights W we were to substitute a fixed pin or hook to which the rope might be attached, the rope would still be subjected to the same strain as if it had the weight suspended from it; and as it would react on its wheel with a force precisely equal to its tension, the twisting effect on the shaft would not be altered. We may, therefore, suppose the wheel and weight entirely removed from one end of the shaft, and that end fixed firmly in such a manner that it cannot turn or revolve on its axis, while it is throughout its whole length subjected to a twisting strain from the action of the weight at the other end. The power with which this weight tends to twist the shaft depends both on its magnitude and on the size of the wheel round which its rope passes. The radius, or half-diameter, of the wheel is the leverage of the weight; and by increasing it we increase the torsive strain in like proportion. For ease of calculation we will suppose the radius, or half-diameter, of the wheel to be 1 foot; and if we can find the torsion of a certain weight acting at a radius of 1 foot, we can reckon that of the weight at any other radius; or having found the weight to produce a certain torsion at 1 foot radius, we can easily reckon the radius at which the same weight would produce some other amount of torsion.

Thus, by doubling or trebling the radius we double or treble the torsive strain; by

doubling or trebling the weight we have to place it at one-half or one-third the radial distance from the centre of the shaft, in order to subject it to the same torsion; and so generally in simple proportion. The torsive strain, therefore, is as the weight multiplied by its radial distance. One ton at a radial distance of 3 feet is equivalent to 3 tons at a radial distance of 1 foot, because 1 ton $\times$ 3 feet $=$ 3 tons $\times$ 1 foot.

We have now to inquire what effect the dimensions of the shaft, or the quantity of material in it, has in resisting a certain torsive strain; how large the shaft must be to resist a given strain, or what strain a shaft of given dimensions can resist. The replies to these questions must depend upon the form of section and the nature of the material employed; but if we can ascertain, by direct experiment, the torsive strength of a shaft of certain material, form, and dimensions, we must endeavour to find a principle on which to reckon the strength of a shaft of the same form and material, but of different dimensions. We may imagine two plates of metal or other material placed face to face, having such corresponding projections and hollows in their surfaces that the projections of the one fit into the hollows of the other. We may suppose these plates pressed together, and some force applied at their edges, so as to push the one along the face of the other. It is manifest that the force for this purpose must depend upon the amount of roughened surface in contact, upon the pressure squeezing the plates together, and the peculiarities of the roughness which their surfaces present. Instead of two separate plates put artificially together, we may conceive that at any place where a solid body might be shorn across, the particles of the body or the grains of which it is composed fit into the interstices of each other; and the two parts into which the body may be divided or shorn are held firmly together by the cohesion of the particles over the whole surface where the division takes place (Fig. 90). In order to effect a separation by pressing the body in opposite directions, as with shears, we should have to overcome the cohesion, and the resistance which the asperities present to the slip across the rough surface of separation. The actual amount of cohesive attraction and the resistance from asperity, must mainly depend upon the peculiar nature of the material acted on; but the relative amount of resistance which two pieces of the same material of different dimensions present, seems to depend simply upon the number of particles separated; that is to say, on the area of the section or quantity of surface separated. Thus, to separate crosswise a piece of iron 2 inches square, we should expect to

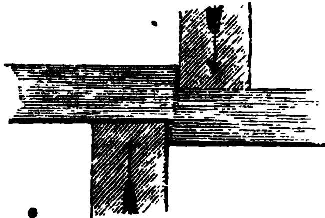


Fig. 90.

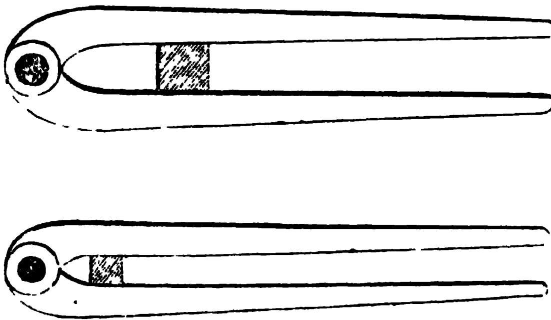


Fig. 91.

apply four times as much force as would be required to separate a piece of the same iron 1 inch square, because the piece 2 inches square has an area of section, or a surface of separation, four times as great as the piece 1 inch square. But, farther, if we suppose the separation to be effected by means of a shears (Fig. 91) so arranged that the 2-inch piece is placed twice as far from the pivot or fulcrum of the lever of which the shears form the arms as the 1-inch piece, we should have to apply double the force to divide the 2-inch piece, on account of the disadvantage of leverage; and, on the whole, in such a case we should require eight times the force to divide the 2-inch piece that we require for the 1-inch piece. If now, instead of shearing across a square bar, we apply this principle to the fracture of a cylindrical bar or shaft by torsion, the centre of the shaft becomes the fulcrum of the lever which we may assume to be one foot long; and each portion of the sectional area of the shaft resists separation with a force proportional to its area, and to its leverage or distance from the centre C (Fig. 92). If we take any small square of the section, such as A, and suppose a ring traced inclosing it, it is clear that the resistance of every part of that ring of particles, of the same magnitude as A, offers the same resistance, because it has the same leverage, or is equally distant from the centre; and that, therefore, the total resistance of the ring is as its area multiplied by its leverage A C. But if the ring be very narrow, its area is nearly its circumference multiplied by its breadth, and its circumference is as its radius A C; therefore its resistance is as its breadth multiplied by its radius twice or the square of its radius.

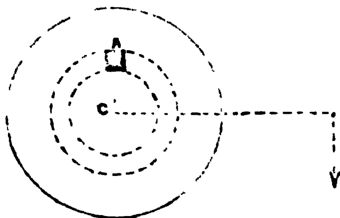


Fig. 92.

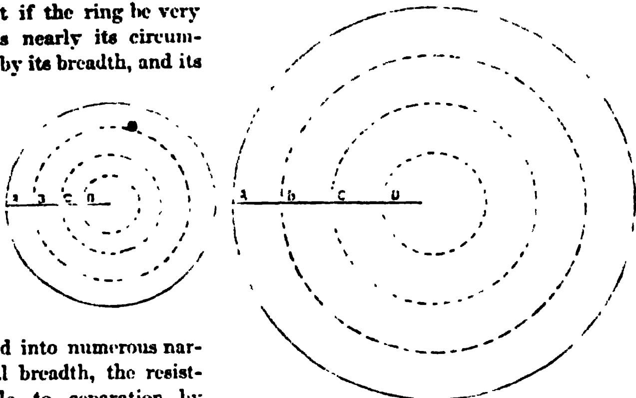


Fig. 93.

Were we to suppose, then, the whole circle divided into numerous narrow rings of equal breadth, the resistance of the whole to separation by torsion would be the sum of the resistances of all the rings, the resistance of each being as its breadth multiplied by the square of its radius. Now, if we compare a circle of one radius such as 12 ins. with another such as 6 ins., dividing each radius into an equal number of parts as 4, and tracing the rings marking them, for the sake of distinctness, by large and small letters respectively, we find that the breadth of ring A is double that of A, and that the radius of A is also double that of A; therefore the resistance of A to torsion is 2 × 2 × 2, or 8 times that of A. So we find the resistance of B to be 8 times that of B, and so on; therefore the resistance of the whole of the larger circle is 8 times that of the smaller. Were the radius of the one 3 times that of the other, we should find, by a like process,

that its resistance would be 3 × 3 × 3, or 27 times; and generally that the resistance of any cylindrical shaft to torsion is as the cube of its radius, or of its diameter (the double of its radius).

This general proposition is almost self-evident; for if we admit that the resistance of any part of the circular section is as its area multiplied by the leverage at which it acts, the resistance of the whole circular section must be as its area multiplied by the mean or average leverage of all its parts. The areas of circles are as the squares of their diameters, and in every circle the mean leverage is proportional to the diameter; therefore the resistance to torsion is as the area or square of the diameter multiplied by the diameter—that is to say, as the cube of the diameter.

The reasoning we have used is founded on the assumption that the effect of excessive torsion is to divide or shear a shaft across at some place where the material is accidentally weaker than elsewhere. But in shafts of uniform strength throughout their length, and of fibrous texture, this is by no means the effect of destructive torsion. In a wrought-iron shaft, for instance, the fibres are made to twist, so that in some cases the outer surface presents the screw-form of a cord or rope; and ultimately they separate and present a fracture where parts seem to have been dragged out of their place by this twisting action. In producing such a twist of fibres there must be a considerable elongation, particularly of the outer fibres; for it is longer round the thread of a screw than in a direct line from one end of a bar to the other; and the greater the diameter of the thread, or the farther it is from the centre, the longer is its course. If, then, we confine our attention to any set of fibres similarly situated, in two bars of different diameters subjected to torsion, we see, in the first place, that in the larger bar the number of fibres in the portion of the section referred to is greater than the number in a similar portion of the smaller bar, in proportion as the square of the one diameter, or total area of the larger, exceeds that of the smaller. Again, in the larger bar the leverage of the set of fibres to resist the screw elongation, or the amount of the elongation which they have to undergo, exceeds that of the smaller in proportion as the one diameter exceeds the other. Therefore, upon the whole, we must conclude, as before, that the resistance of the larger to torsion is to that of the smaller as the cube of the one diameter is to that of the other.

The same principles of reasoning apply equally to other than circular forms of section The torsive strengths of square bars follow the same law; and those of any other forms, provided they be similar or have all their corresponding parts proportional. In comparing bars of circular with those of square section of equal diameter (Fig. 94) we readily see that the torsive strength of the square must exceed the other, not only on account of the square having the larger area, and therefore the greater number of fibres or points of cohesion, but also because the additional area is situated at places farther from the centre, and therefore resisting with more leverage than any parts of the circular section. On the other hand, we observe that while all the fibres of the circular section are supported against being forced aside by those around them, the fibres in the angles of the square section are not so supported, and may be expected to yield more readily to the twisting force. Upon the whole it has been found that the torsive strength of the square section is

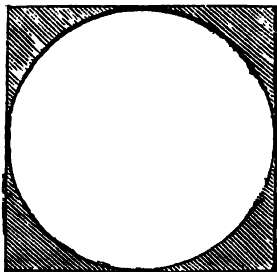


Fig. 94.

about one-fourth greater than that of the circular. As the distance of any fibre from the centre contributes much to its resisting power, it may be advantageous to remove some of the material of a shaft from the inside, where it is of little value to resist torsion, and place it on the outside, where it acts with greatly increased leverage. On this principle, hollow or tubular shafts present the advantage of economizing material and securing strength with lightness; for each of the fibres in the ring section of a tube has so much greater leverage than those towards the centre of a shaft, that their number, and consequently the sectional area of the ring, may be considerably diminished without lessening the torsive strength.

From a number of experiments made with a view of ascertaining the torsive strength of different materials, we are enabled to compile a table from which the strength of a shaft of given materials and dimensions may be calculated; or, conversely, the dimensions may be computed of a shaft destined to sustain a given torsive strain. The table, col. 2, gives the weight in pounds placed at the end of a lever 1 foot long, which a cylindrical shaft, 1 inch in diameter, will permanently sustain without injury; and the following rules embody the methods of computing the torsive strengths of other shafts, according to their diameter.

I. Given the diameter of a cylindrical shaft and the length of lever at which the twisting weight acts, to find what weight it will permanently sustain.

Rule.—Multiply the number in the table three times by the diameter (in inches), and divide by the length of lever (in feet).

Example 1.—Required the strain that can be applied at the end of a lever 2 feet long to turn a cast-iron shaft 4 inches in diameter.

$$\frac{330 \times 4 \times 4 \times 4}{2} = 10{,}560 \text{ lbs., nearly } 4\tfrac{3}{4} \text{ tons.}$$

Example 2.—A rope passes round a pulley 3 feet in diameter, fixed to a gun-metal spindle 2½ inches in diameter; required the weight that may be attached to the rope.

Since the diameter of the pulley is 3 feet, the leverage of the weight is 1½ ft.; and

$$\frac{170 \times 2\tfrac{1}{2} \times 2\tfrac{1}{2} \times 2\tfrac{1}{2}}{1\tfrac{1}{2}} = 1771 \text{ lbs., or nearly 16 cwt.}$$

II. When the shaft has a square section.

Rule.—Add ¼th to the weight.

Example 3.—Required the weight that may be hung to a lever 5 feet long, fixed to a wrought-iron (English) square shaft 2 inches broad.

$$\frac{335 \times 2 \times 2 \times 2}{5} = 536 \text{ lbs.}$$

$$\text{Add } \tfrac{1}{4} = 134 \text{ ,,}$$

$$670 \text{ lbs. nearly 6 cwt.}$$

III. Given the weight and lever, to find the diameter of a cylindrical shaft.

Rule.—Multiply the weight (in lbs.) by the length of lever (in feet), divide by the number in the table, and extract the cube root for the diameter (in inches).

Example 4.—A cast-steel spindle has to withstand the torsion of 5 tons at the end of a lever 8 feet long: required its diameter.

$$5 \text{ tons} = 11{,}200 \text{ lbs., and } \frac{11{,}200 \times 8}{590} = 152 \text{ nearly.}$$

The cube root of 152 is about 5⅓ inches.

IV. When the shaft is square.

Rule.—Multiply the weight by the lever, divide by the tabular number, deduct $\frac{1}{5}$th of the result, and extract the cube root.

Example 5.—A square shaft of English wrought-iron has to sustain 6 cwt. at the end of a lever 5 feet long: required the breadth.

$$\frac{6 \text{ cwt.} = 672 \text{ lbs.} \times 5}{335} = 10$$

$$\text{Deduct } \tfrac{1}{5}\text{th} = 2$$

$$8, \text{ cube root 2 inches.}$$

Table of torsive strengths of cylindrical shafts 1 *inch diameter; weights acting at* 1 *foot leverage.*

Name of material.	Permanent load.	Name of material.	Permanent load.
Brass	150 lbs.	Lead	34 lbs.
Copper	135 ,,	Steel (blister)	560 ,,
Gun-metal	170 ,,	Steel (cast)	590 ,,
Iron (wrought, English)	335 ,,	Steel (shear)	570 ,,
Iron (wrought, Swedish)	320 ,,	Tin	47 ,,
Iron (cast)	330 ,,		

The table and rules apply only to cases where a simple regular strain is applied. When shafts or spindles are intended to convey motion to machinery, they are generally subjected to great irregularities of torsive strain; and though they may only convey upon the whole a certain power, yet at different periods of their revolution they may be subjected to strains ranging from nothing up to many times the average strain due to the power they convey. If we consider for a moment the conditions under which the crank-shaft of a steam-engine rotates, we shall see the great variation of strain to which it is exposed. At two points of each revolution, what are technically called the dead-centres, the steam power has no effect to turn the shaft round; but, on the contrary, the momentum stored up in the fly-wheel turns the shaft, and through it gives movement to the parts of the engine. At two other points, when the connecting-rod of the engine is acting in the most favourable position, the shaft receives a torsive strain through the crank greater than the total pressure of the steam on the piston; and the amount of this strain depends on the total steam-pressure on the piston, the obliquity of the connecting-rod, and the length of the crank. But all machinery is besides subject to accidental irregularities of strain. For instance, in the engines of a steam-vessel propelled by paddle-wheels, sometimes in a heavy sea, while the engines and paddles are moving at full speed, a wave strikes one paddle and suddenly immerses it to a great depth in water, so as at once to retard its rotation. The shock thus communicated to the engine through the shaft which drives the paddle is enormous, and occasionally more than 100 times the average force passing through the shaft. It is, therefore, essential to make all shafts intended to communicate power of much greater strength than what is due to the mere average strain passing through them. The power that is communicated through any shaft is generally reckoned in horse-power; and as power consists of two elements—pressure or weight moved, and the velocity or speed with which it is moved—it is necessary to ascertain not only the power passing through a certain shaft, but also the speed with which that shaft rotates, or the number of revolutions it makes in a

given time, such as a minute, before we can compute the strain to which it is subjected, and the dimensions of which it should be made. The more quickly a shaft rotates in communicating a certain power, the less is the torsive strain to which it is subjected, for power is pressure or strain multiplied by velocity; and if to produce a certain power the velocity be increased, the strain must be proportionally diminished. The shafts of engines, and machinery connected with them, are generally made of wrought or cast-iron. As wrought-iron is more flexible and tenacious than cast-iron, it is less subject to fracture by sudden variations of strain, and is therefore preferable to cast-iron for shafts, and may be made considerably lighter. Again, the shafts which communicate the first effort of the power from the steam-pressure to the fly-wheel are subjected to much greater variations of strain than those which communicate the power afterwards from the fly-wheel shaft to other machinery. It is, therefore, advisable to give the first shafts, or prime movers as they are called, greater strength than is necessary for second movers. Practical men generally make the prime movers about twice as strong as the second movers; and the following rules embody the modes of computing dimensions of shafts for conveying given power at given velocity.

1. For wrought-iron prime movers, given the horse-power, and the number of revolutions per minute, to ascertain the diameter.

Rule.—Divide the power by the velocity, extract the cube root of the quotient, and multiply by 7, for the diameter in inches.

Example 1.—Required the diameter of a wrought-iron prime-moving shaft for 100 horse-power, making 20 revolutions per minute.

$$\frac{100}{20} = 5, \text{ cube root } 1{\cdot}7 \text{ inches, and } 7 \times 1{\cdot}7 = 11{\cdot}9, \text{ nearly } 12 \text{ inches.}$$

2. For cast-iron prime movers.

Rule.—Divide the power by the velocity, and take $7\frac{1}{2}$ times the cube root of the quotient.

Example 2.—Required the diameter of a cast-iron prime-moving shaft for 50 horse-power at 25 revolutions per minute.

$$\frac{50}{25} = 2, \text{ cube root } 1{\cdot}26 \times 7\tfrac{1}{2} = 9{\cdot}45 \text{ or } 9\tfrac{1}{2} \text{ inches.}$$

3. For wrought-iron second movers.

Rule.—Divide power by velocity, and take $5\frac{1}{2}$ times the cube root of the quotient.

Example 3.—Required the diameter of a wrought-iron second mover for 40 horse-power at 15 revolutions per minute.

$$\frac{40}{15} = 2{\cdot}6, \text{ cube root } 1{\cdot}4 \times 5\tfrac{1}{2} = 7{\cdot}7 \text{ or } 7\tfrac{3}{4} \text{ inches.}$$

4. For cast-iron second movers.

Rule.—Divide power by velocity, and take 6 times the cube root of the quotient.

Example 4.—Required the diameter of a cast-iron second mover for 30 horse-power at 36 revolutions per minute.

$$\frac{30}{36} = 0{\cdot}6, \text{ cube root } 0{\cdot}87 \times 6 = 5{\cdot}22 \text{ or } 5\tfrac{1}{4} \text{ inches.}$$

For shafts under very regular strain, and distantly connected with the prime mover, the multipliers may be for wrought-iron 5, and for cast-iron $5\frac{1}{2}$.

The dimensions computed by the above rules are those of the weakest part of the shafts. When shafts are of great length, or have great weights, such as fly-wheels, and the like, fixed upon them, they are subject to the deflection due to transverse strain. This deflection in a revolving shaft is an important element of weakness, for it continually changes in direction as the one side or other of the shaft is uppermost, and thus subjects the material to all the degradation of alternate strains in opposite directions. In breaking a piece of wire, it is not unusual to bend it backwards and forwards until its tenacity is quite destroyed. The same kind of action occurs in a shaft too light for its length, or for the weight it bears, and gradually lessens its tenacity and deprives the fibres of their continuous texture. For such cases the strengths should be made considerably in excess of what they are computed to be. In this, as in numerous other instances, experience must be the guide of the practical mechanic in the absence of set rules.

5. **Clipping or Shearing Strain.**—The strain which a body undergoes when it is divided across by means of shears, or any instrument consisting of two blades that pass one another like those of a scissors or shears, is of a kind very distinct from any of the other strains which we have discussed. Under tensive strain, the fibres are torn asunder by a force in the direction of their length; under compressive strain they are forced out of their place, and have their lateral cohesion destroyed; under transverse strain they are subjected partly to compression and partly to extension; under torsive strain they are extended in a screw or spiral direction; but under the clipping strain the particles are forced across each other, as we have described in endeavouring to investigate torsive strain on the principle of shearing.

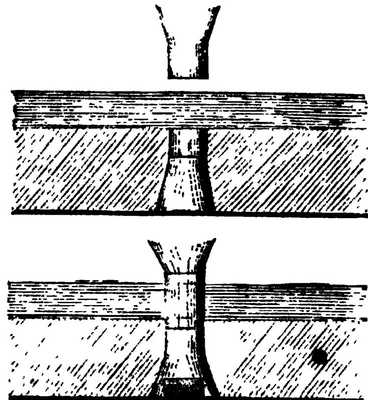


Fig. 95.

Besides actual clipping by instruments for the purpose, there are various circumstances under which materials may be exposed to a strain of a similar character. In punching holes in a plate of metal, a similar action takes place. The punch (Fig. 95) is a piece of steel, the end of which is made of the size and shape of the hole to be punched, the edges being keen and not rounded off; the matrix is a block of steel, or iron faced with steel, having a hole such as is to be punched, with edges also keen. The punch and the matrix are so adjusted that the former shall pass exactly into the latter; and a

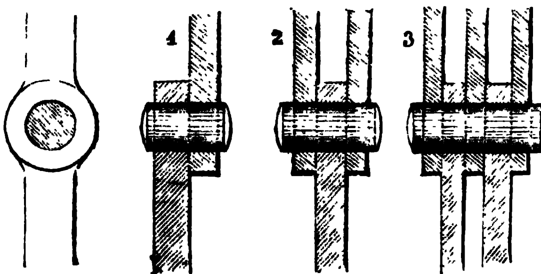


Fig. 96.

plate of metal being placed on the matrix while the punch is up, has a hole pierced in it by its descent—the piece of metal being punched or pressed down into the hole of the matrix. When the edges of the punch and matrix are keen, and they accurately fit each other, the metal of the plate is shorn cleanly round the circumference of the hole; and the force required to effect the operation appears to be the same as would be required to shear a plate of the same material and thickness as that punched, over a length equal to that of the circumference of the hole.

Another example of shearing strain occurring in practice, is that to which the pins uniting the limbs of a chain are subject (Fig. 96). If the joint consist of single limbs united by a pin, a strain applied to the limb longitudinally tends to shear the pin across at one place.

If the links be double and single alternately, the pin is exposed to be shorn at two places; where the links are three and two alternately, the shearing must take place at four places; and, generally, whatever be the number of links embracing each other at both sides, the number of places at which the pin must be shorn is double the smaller number of links. We naturally conclude that, if a certain power be required to shear a pin at one place, it would require double the force to shear it at two, triple at three, and so on, an additional force being required for each additional separation of the substance made. Practically, this is found to be true. Carrying the same reasoning to the consideration of the strains required for shearing pins of different sizes, we conclude that if we double the area of section—that is, the surface where the separation is effected or the number of fibres shorn—we have to double the force necessary to shear it. We therefore conclude that the strength to resist shearing, or the force required to shear, is proportional to the area of the body shorn at the place where the separation is effected: and here, also, our reasoning is borne out by experiment. A pin of iron 2 ins. in diameter requires 4 times the force to shear it that a pin 1 in. in diameter requires, because the area of a circle 2 ins. is 4 times, or 2 × 2 times, the area of a circle 1 in. in diameter.

It is not uncommon in machinery to couple two rods A and B in the manner indicated in Fig. 97. The ends of the rods being turned truly cylindrical, and a socket C bored to fit on them, the ends are inserted in the socket, and keys D and E are driven into slits cut through the socket and the rods. On a great longitudinal strain being applied to the rods, so as to pull them out of the socket, fracture may occur in one of the following ways:—

1. Either of the rods, or the socket itself, may give way under the effort of direct tension; and if so, they must yield at the weakest place, which is manifestly where the keys pass through them, part of their sectional area being occupied by the keys, which add nothing to the cohesive strength.

2. Either of the keys may be shorn across at the two places where they leave the socket and enter the rods.

3. Either of the rods may have the material between the key and its end drawn out so as to leave an opening F, the material forced out being punched or shorn at both sides by the strain on the key.

4. Either end of the socket may have the material at G drawn out in a similar way.

Now, if the material be of the same quality throughout, the strength to resist fracture in the 2nd, 3rd, and 4th ways may be equalized, because they are all shearing strains of the same kind, and we have only to make the several areas of the parts that

would be shorn all equal; that is to say, the sectional area of each key should be equal to that of the end part F of each rod, because the shearing would separate two surfaces in either case; and the area of the part G of the socket need be only half that, because four surfaces would be separated. But we have as yet established no relation between the strength to resist tensive strain and that to resist shearing, and cannot therefore, without further information, compute the proportional strength of the effective transverse sections of the rods and socket where the keys pass through them. The truth is, that very few experiments have been made upon the strength of materials to resist a shearing strain; such as have been made with malleable iron seem to show that it is precisely equal to the strength to resist tension, and we believe that with other materials this law may be very safely assumed. As the tensive strength is also proportional to the area of section, the areas to resist equal tensive and shearing strains should be equal. Accordingly, we should make the transverse sections of the rods and socket such that the area of each part, on either side of the key, shall be equal to the transverse section of the key; for the key would be shorn at two places, and the rod or socket would be pulled asunder at the two places on either side of the key. On examining the figure, it will at once suggest itself that the keys should be made narrow and broad (measuring the breadth in the direction of the rods lengthways), so as to trench as little as possible upon the sectional area of the socket and rods. Although for round or square pins or bars the law that the strength to resist shearing is very nearly the same as the cohesive strength, yet there is little doubt that by increasing the depth of a bar exposed to be

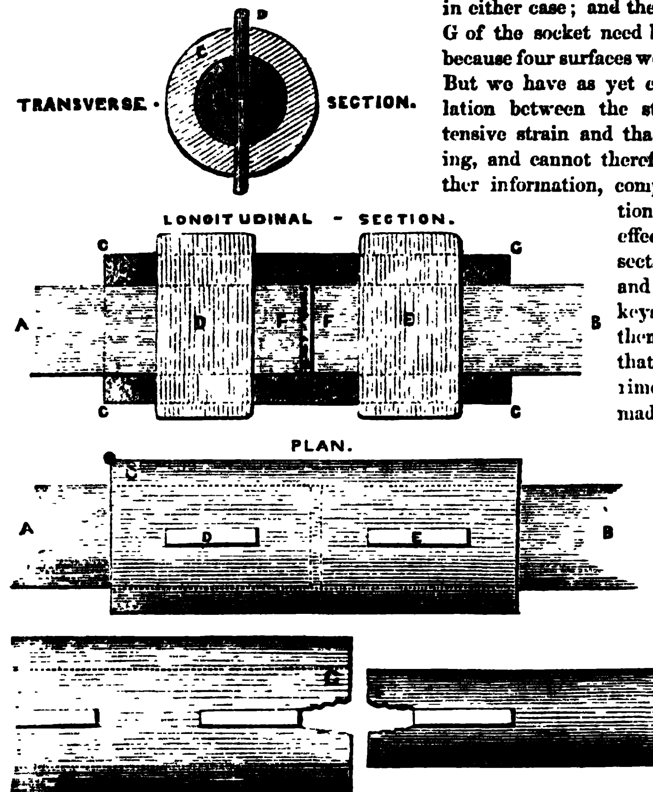


Fig. 97.

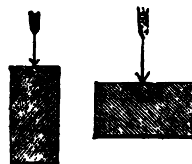


Fig. 98.

shorn, the breadth being diminished so as to retain the same area, the strength is materially increased. If we placed a bar of iron having 2 ins. × 1 in. of sectional area under a shearing strain edgeways, it would certainly resist more than when placed sideways. We are not aware, however, that experiments have been made to a sufficient extent to warrant us in laying down any law as to this.

In practice no material error will arise from the assumption of the simple law we have already stated; and if strengths to resist shearing be calculated according to it, let the mechanic, as far as he can, contrive to bring the shearing strain against the greatest depth, and he will certainly be safe against fracture from this cause.

In punching a hole in metal, the surface of the separation produced is the circumference of the hole multiplied by the thickness or depth; and the resisting force should be equivalent to the tensive strength of a bar of the same material having a sectional area equal to the surface of separation. Thus, in estimating the force required to punch a circular hole 1 inch in diameter through a plate of malleable iron ¾th of an inch thick, we should reckon the circumference of a circle 1 inch in diameter is 3·1416 inches; multiplying by the thickness, ¾th of an inch, we have the area of the surface of separation, 2·3562; and therefore the strength to resist the punching is that of a bar of iron having a sectional area of 2·3562 square inches to resist fracture by tensive strain. The weight required to tear asunder a bar of iron 1 square inch in sectional area, is found by experiment to be about 26 tons; and the weight to tear asunder 2·3562 square inches would therefore be about 61 tons, which we reckon as the force required for punching the hole such as we have described.

The subject of shearing strain is, upon the whole, somewhat obscure. The experiments have been few, and made rather with a view to the solution of other questions than the determination of a law as to this strain. We would therefore caution the practical man against placing too much dependence on any computations made in respect of it. Where he is in doubt, and has no case practically carried out to which he may refer, it is far better that he should make experiments for himself; or, in the absence of data which he might deduce from them, at least he should make his work rather in excess of strength than otherwise. Great evils may often result from the execution of work where the dimensions are too closely approximated to the results of calculation as to strains and strength. These evils are of an uncertain character—they may occur unexpectedly, under circumstances that aggravate the mischief they may cause; great expense may be incurred, limbs may be broken, life may be endangered or destroyed, and all because material has been grudgingly employed or disproportionally applied. But when strength is given in excess to any structure intended to be permanent, the first evil, that of additional cost, is the only one—it is known as to amount, is undergone, and appears not again—and the artificer has the satisfaction of thinking that if difficulties or dangers arise during the use of the work he has contrived and formed, it must be from adventitious causes for which he cannot be held blameable.

Indeed, the whole subject of strength of materials is in an unsatisfactory state. Every new experiment made, every new work completed, adds somewhat to our practical knowledge in this department of mechanics. But so long as the materials we use vary in quality, are wanting in uniformity, and are subject to the changes which moisture, temperature, and other circumstances impose upon them, so long must we remain incapable of estimating correctly their strengths for any particular purposes, and so long must we be prepared to make large allowances for all such variations and for contingencies which do not always readily suggest themselves.

SOURCES OF MECHANICAL POWER.

The principal sources of power may be classed under four heads.

1. Muscular power of men and animals.
2. Natural movements of air and water.
3. Weight and elasticity of bodies.
4. Heat, electricity, magnetism, and chemical action.

1. **Muscular Power of Men and Animals.**—It is not our province to discuss the mode in which the muscles of men and animals are put in action. This is a question which belongs to the physiologist. Suffice it to say, that the mechanism of animal structure is of that most perfect kind, which characterizes all the works of Him who designed and executed it. The simple effort of *will*, or whatever that particular mental faculty be called, appears to communicate itself through the nerves which pervade the whole animal frame like a network of electric wires, and to operate immediately on the muscular tissue. The muscles, so far as we understand their nature, appear to have no innate force of their own, but are merely organized instruments put in action by the nervous influence transmitted to them by the nerves, just as the index of an electric-telegraph is perfectly inert until it is deflected one way or other by the subtile magnetic influence transmitted to it through the electric-wire. Notwithstanding the immense variety of movements of which the different parts of the animal frame are capable, the action of the muscles which effect those movements is of one kind only—contraction in length, accompanied by expansion in diameter.

The eyes are caused to rotate in their sockets, and their focus as optical instruments is varied; the lips, the tongue, the throat, and the jaws are moved so as to give voice to musical notes and articulate language; the shoulders, arms, wrists, hands, and fingers are put in motion so as to enable man to lift heavy weights, or to manipulate with consummate delicacy; the back, body, the legs and feet are moved so that man may walk, run or leap, row, or perform feats of strength and agility. And yet every one of

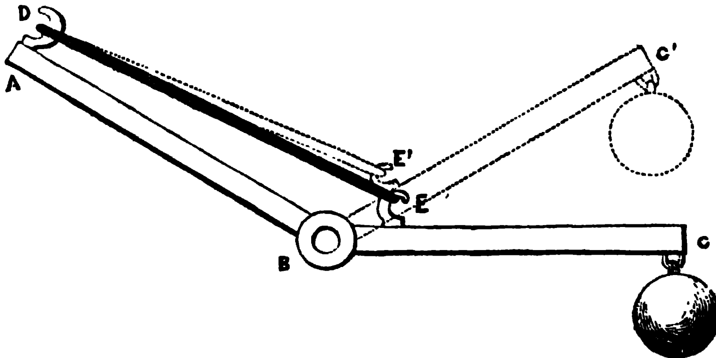


Fig. 99.

these motions is effected by the longitudinal contractions of muscles under the nervous influence of volition. The frame-work of this exquisite machinery is the skeleton, consisting of numerous bones joined together. The form, length, strength, and situation of every bone have been devised with most minute care to suit their particular objects.

Each bone plays the part of a beam, and has attached to it a tendon or strong cord near its joint or fulcrum. This tendon is the end or continuation of a bundle of fleshy fibres, which constitute a muscle, fixed at the opposite end to some other part of the framework; and as these fibres contract in length under the nervous influence, the tendon or cord is pulled, and the bone or lever caused to move round its joint or fulcrum.

We may suppose A B (Fig. 99) to represent a fixed bar, having a joint at B fitted with a lever B C, from the extremity of which a weight is suspended; and D E a cord fixed at D to the bar A B, and at E to the lever B C. By shortening the cord D E we cause the lever to turn about its fulcrum B, to some such position as that marked by the

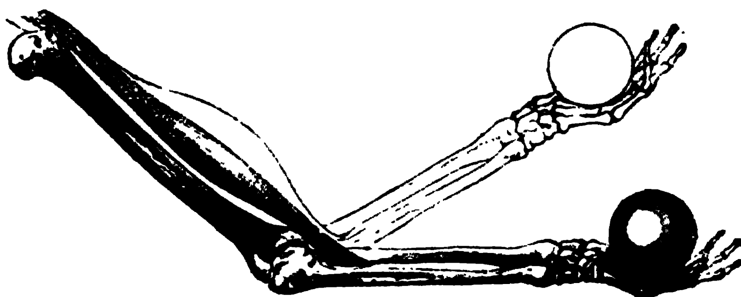


Fig. 100.

dotted lines E' C'. So in the human arm (Fig. 100), the upper bone is the bar jointed at the elbow; the radius bone terminates by the hand, in which a weight may be placed; the cord is the biceps muscle, fixed at the upper end, and terminating at the lower end in a tendon, which is fixed to the radius. The longitudinal contraction of this muscle pulls the tendon, and thus causes the radius to turn upon the elbow-joint, and raise the weight at its extremity.

Besides the direct movement of the bones as levers, such as we have described, there are numerous modified motions, sideways or rotatory, which are all effected by the contraction of muscles specially arranged for their respective purposes. For instance, in order to turn the hand upside down, it is necessary to make the arm rotate or turn upon its axis. This is effected by muscles placed spirally round the bones of the arm. By diminishing the length of one of these, the bones are caused to twist round. And so for every movement of every part of the frame, a special muscle is provided so as to effect the required motion by simple contraction.

The force or contracting energy of some of these muscles is enormous, as may readily be believed, when it is remembered that they are attached to the short end of a losing lever, and yet are capable of sustaining great weights at the extreme end, where the weight acts with immense leverage.

Some of the muscles are not placed exclusively under the control of the will, but act regularly without any effort of volition. Such are the muscles which effect the action of the heart and organs of respiration. Some muscles, again, seem to be put in action by an effort of will of which we are not conscious. Such are those of the eye to direct and focus it, and those of the tongue, mouth, and throat in speaking. Although we will to speak or articulate certain sounds, we never think of the particular muscles that we must put in action in order to effect the articulation desired. So in walking and other

actions which have become familiar by habit, no thought is given to the special muscular movements required. It seems to be enough to will to walk, and the muscles obey. It is only when fatigue and exhaustion render their exercise painful, that we have to employ special acts of volition in order to effect our purpose.

In making use of muscular power as a moving force, it is important to call into exercise those muscles which require the least effort of will for their movement, and to apply them in the way least calculated to fatigue. The animals chiefly employed—the horse, the ox, and the camel—can only be used in particular ways, as for bearing and dragging loads. The horse and the ox are best adapted for dragging, as neither of them is capable of sustaining great weights. The camel, on the other hand, moves with facility under a very considerable load placed upon its back. The elephant is employed in tropical countries both for carrying and dragging, and appears to have enormous strength for either purpose. When human power is employed it may be exercised in various ways. The operation in which man seems capable of exerting the greatest power for the longest period is that of rowing. In moving the oar against the resistance of the water, almost every muscle of the body is called into operation. The spine, bent forwards in the back-stroke, is thrown back as the pull is taken, the shoulders and arms are also thrown back, the legs are stiffened against the foot-board, and thus immense power is applied to the oar without excessively straining any one or two particular muscles. In excavating earth or clay, conveying loads up steep inclinations by a wheelbarrow, and generally in the operations for which navigators are employed, a large amount of power is developed. The muscular frames of these men, and their long practice in particular operations requiring great strength, enable them to continue for long periods at work without much fatigue; and to one watching minutely their mode of working, while there is little appearance of effort, yet there is manifest a regulated play of the muscles, and a sort of contrivance for doing the work with the least possible movement,—which indicate that every one of these muscular movements is a result of habit, without the need for a special effort of will.

In pumping, as in rowing, numerous muscles are exercised, and accordingly this kind of labour can be continued for a long period. There is, however, a monotony about the operation, and a want of visible effect to satisfy the mind of the workman, which tend to make him weary of his work. In rowing a boat, excavating, lifting or moving a load, there is a visible progress made, and the workman has a satisfaction in observing the change effected by his exertion, and can reckon his advance and the amount still remaining to be done; but in the continuous discharge of a body of water from a pump, drawn probably from an unseen source, and flowing into an unseen reservoir, or into one so large that additions to its contents produce no visible effect, there is nothing to reckon by, no progress apparent; and the workman accordingly becomes fatigued, more from want of mental satisfaction than from actual weariness of body. It is true that cases frequently occur where seamen and passengers on board a leaky vessel work for many days continuously at the pumps. But in such a case, life is at stake; and the excitement of danger, fear of death, and hope of safety, are impulses to exertion capable of overcoming much bodily fatigue and mental lassitude. In all operations demanding manual labour, it is good economy to let the workman see the effect of his efforts: let his mind be satisfied, and his body will the longer withstand fatigue. Where labour is used as a punishment, it is certainly an aggravation of the penalty to make the labour of a kind that manifests no result. The perpetual walking up the rounds of a treadmill, or perpetual turning a winch to drive unseen machinery,

is therefore a penal labour of a most severe character, although the actual muscular exertion which either demands is not great.

As rotatory motion is generally that kind of movement best applicable to machinery, being most easily modified in direction and velocity, labour at the winch is the kind of manual exertion chiefly used. Before steam-machinery became common, the number of operations effected by manual labour was very much greater than it is now. At present, except for special purposes—such as driving a printing-machine, or a turning-lathe, where the quantity of work to be done would not warrant the application of costly motive power—manual labour at the winch is little used. Engineers of the last generation, with whom human and animal labour was an important consideration as a source of motive power, devoted considerable care to experiments as to the best modes of applying it, and the quantity of power developed. The main result of their deductions was, that in moving machinery the power of a horse was on the average equivalent to a weight of 33,000 lbs. lifted 1 foot high in 1 minute; and this has accordingly been adopted by engineers as a standard by which all powers, whether of steam-engines or water or windmills, are measured.

The power of a man is about one-fifth of that of a horse; that is to say, on the average a man can exert a power equivalent to the lifting of a weight 33,000 lbs. one foot high in five minutes, or 6,600 lbs. one foot high in one minute. These estimates are not applicable to a single effort for a short period, but to labour, such as could be sustained for hours daily, and continued on successive days. When the power of a man is continuously applied to a winch, it should not be estimated at more than half that given above; that is to say, 3,300 lbs. raised one foot high in one minute. It is found in practice that a man working a winch can for a considerable period move his hand through 120 feet per minute, exerting an average of 30 lbs. At some parts of the revolution of the winch he exerts more than double of this pressure, at other parts less than half; but, on the whole, the mean pressure may be taken at 30 lbs. Now, 30×120 gives a power of 3,600 lbs. raised one foot per minute, which is rather above the estimate of 3,300, which we have given above. If the radius of the winch be 1 foot, the diameter of the circle in which the hand revolves is 2 feet, and its circumference $6\frac{2}{7}$ feet; therefore he makes about 19 revolutions per minute, for $19 \times 6\frac{2}{7} = 120$ nearly. For a short period he could turn the winch at the rate of 25 or 30 revolutions per minute, and exert an average pressure at the handle of 40 lbs. to 50 lbs. But this high estimate should only be used in calculating when a weight is to be lifted during some minutes, and not when the work is constant. For driving machinery affording a constant resistance during several hours, the lower estimate of the man's power, viz. 3,300 lbs. raised one foot high in one minute, should be taken.

In carrying a load on his back, over level ground, a man can develop a power of about 8,000 lbs. moved over one foot in one minute. In carrying a load up a staircase his power is equivalent to 2,000 lbs. lifted one foot high in one minute. In dragging a hand-cart on level ground his power is about 20,000 lbs. moved one foot in one minute. In rowing a boat the power developed is about 25,000 lbs. moved one foot in one minute.

For temporary efforts, the following estimates have been made:—A man can bear for a short time, standing still, about 300 lbs. weight. He can lift vertically about 280 lbs. He can grasp by his hand with a pressure of about 100 lbs.

The principal applications of manual labour are not those in which great force is required to be developed. Dexterity and skill of manipulation are the qualities desired in a workman. As more and more progress is made in mechanics, so does the necessity

for employing the bodily forces of men diminish. Indeed, a large proportion of the skilled workmen now employed are engaged in merely watching the operation of self-acting machines. Machines are more rigorously true in the work they perform than the hand; but they are limited to the constant repetition of similar operations. They cannot set their work, adjust their own movements, or vary their efforts, as man can do; but in all these matters they may be controlled by man. It is not our province to inquire into the moral effect produced on workmen by the extended use of self-acting machinery. It is true that much of their corporal labour is saved, and that a vast deal more work is done more economically; and necessaries and luxuries thus brought within the reach of millions who would else be deprived of them. At the same time it must not be forgotten that the workman is to a certain extent degraded into a mere machine, only a little elevated above that whose operations he watches and controls. He has no longer to acquire skill and dexterity in his craft—machinery does the work better than he could ever hope to do; he has not to think out the easiest and most effective methods of performing it—for the machine does a certain kind of work, the material is placed on it, the power is applied, and he has only to look on. Farther, the kind of work about which he is employed almost exclusively, is of the same unvarying character; and he is thus subjected to a monotony of occupation which leaves many intellectual faculties dormant. In former generations the millwright was a man conversant with machinery in general, and with the various operations required in its manufacture and application. He could make patterns, mould, forge, turn and fashion timber and metal to his uses, and generally—we may say necessarily—was a man of intelligence and ingenuity. Now we have pattern-makers, moulders, smiths, turners, fitters, and workmen for attending planing, screwing, slotting, punching, and other machines; all, in a manner, different trades, requiring no versatility of mechanical talent, and not even dexterity of hand.

No doubt the vastly increased quantity of work is now done in all the better style for this subdivision of labour; but we fear the moral and intellectual character of the workman is considerably lowered.

2. **Natural Movements of Air and Water.**—History affords us no information as to the period when, or the race by whom, the movements of the winds and tides were converted into useful mechanical forces. Many savage nations, who for ages could have had no intercourse with people more civilized than themselves, possess vessels that float on the surface of the waters, impelled by the wind or the tidal currents of the sea and rivers, as well as by oars; and it is probable that the idea of utilizing the motion of the wind or the current of the water has suggested itself to many separate individuals, without any communication among each other. The most simple mode of applying the force of wind is that which is of most general use, the propulsion of vessels on the water. The most casual observation of the natural effects of wind shows its greater power on large surfaces than on small ones; and a little reflection would offer a reason for this, and suggest the extension of surface to receive the pressure of aërial currents when greater force is sought to be derived from them. A savage floating in his canoe finds that, by holding up the blade of his oar to catch the favourable breeze, he makes progress without effort; by holding up two oars, the velocity of his progress is increased; and by stretching between the oars a mat or skin so as to increase largely the surface on which the wind can press, the speed of his canoe is proportionally augmented. From the first rude notion of a sail, the steps of transition to the complex rigging and arrangements of canvas in a first-rate man-of-war are easy and obvious; indeed, in making use of wind-power for propelling vessels, there are only two points

requiring consideration: those are, first, the quantity of the force developed or required; and, secondly, the direction in which it is to be made to act. The amount of wind-force depends upon the velocity of its movement, and the quantity of surface on which it acts. At first sight it might appear, that by doubling the velocity of the wind we should double its pressure on a certain area: but this is by no means the case, for on doubling the velocity, the pressure is quadrupled; on tripling the velocity the pressure is increased nine times; and so on, the pressure being always proportioned to the square of the velocity, or the velocity multiplied by itself. On a little consideration, this law becomes obvious, as we will endeavour to show. Let us suppose the air to consist of a number of particles or molecules; even if it do not consist of actual solid particles, we may at all events suppose it to consist of a number of separate masses, as small and light as we please; and though rare and attenuated as compared with solid bodies, yet certainly as much as they subject to the general laws which govern all matter. Now, if we double the speed with which any one of these masses moves, we double the force with which it strikes on any body placed in its way; if we triple its speed, we triple the force of its blow; and so on, the force being always exactly proportional to the velocity when any one mass or particle is taken into account. But when we extend our conception to the motion of an unlimited number of such particles, which constitutes a continuous fluid, we observe that, by doubling the speed of the fluid's motion, we double the number of masses striking any body opposed to it in a given time; by tripling the speed we triple the number striking; and so on, the number of particles striking, as well as the force of each, being proportional to the velocity of movement.

Upon the whole, then, the pressure on any surface, arising from a continuous series of blows from masses of air, must be as the force of each multiplied by the number in a given time; and as on doubling the velocity the number is doubled, and the force or pressure is 2 × 2, or 4 times, on tripling the velocity the pressure is 3 × 3, or 9 times, and so on, the pressures being always, as we have stated above, proportional to the velocities multiplied by themselves or the squares of the velocities. It is convenient to estimate the pressures of wind currents on some unit of surface, such as one square foot, while the velocities are generally stated as being at a certain number of miles per hour. The following table gives the pressures per square foot of surface produced by winds of different degrees of strength, as distinguished by their respective names, with the corresponding velocities in miles per hour.

Table of the Velocities of Winds in miles per hour, and their pressures in lbs. on a surface of one square foot.

	Miles per hour.	Pressure. lbs.
Hardly perceptible	1	0·005
Gentle breeze	5	0·123
Brisk gale	10	0·492
Very brisk gale	20	1·968
High wind	30	4·429
Very high	40	7·873
Storm	50	12·300
Great storm	60	17·715
Hurricane	80	31·490

The following is a simple rule for determining very nearly the pressure per square

foot, when the velocity in miles per hour is known:—Multiply the velocity by itself, halve the product, and point off two figures to the right.

Example 1.—Required the pressure of a wind blowing at 40 miles per hour. 40 × 40 = 1600; half is 800; pointing off 2 figures, we have 8 lbs., which nearly agrees with 7·873, the pressure in the table.

Example 2.—Required the pressure of a tempest at 70 miles per hour.

$$\begin{array}{r} 70 \\ 70 \\ \hline 2)4900 \\ \hline 24{\cdot}50 \text{ lbs.} \end{array}$$

The pressure of a wind on any surface, such as a square foot, being known, it is easy to estimate that on any other surface; for if, instead of 1 square foot, 2 square feet were opposed to the wind, we should find the pressure doubled; on 3 square feet it would be tripled; and so on in regular proportion. Knowing, then, the velocity of the wind, we compute its pressure on any known surface by multiplying the pressure on 1 square foot by the surface in square feet. Thus, if we wished to ascertain the pressure of a gale at 20 miles per hour on a ship's sail 20 feet wide by 30 feet high, since 20 × 30 = 600 square feet of surface, and the pressure of the gale on 1 square foot being nearly 2 lbs., we have 600 sq. ft. × 2 lbs. = 1200 lbs., the total pressure on the sail.

When the wind is made use of as a moving force, the surface on which it blows must move with it, for if it simply rested opposed to the air, it would sustain pressure but develop no moving force. The sail of a ship at anchor is certainly pressed on by the wind, and tends to force the vessel from its moorings; but until it does force it, no moving power is developed, for no motion is produced. But when a ship is in motion, the sail is, to a certain extent, going with the wind, and therefore receives a pressure due only to the excess of the velocity of the wind over that of the ship. Suppose the wind were blowing at the rate of ten miles per hour, and the ship sailing before it at the rate of seven miles per hour, the actual velocity with which the wind strikes the sail is only three miles per hour, the difference between its rate and that of the ship. The pressure on the sail is, therefore, only that due to three miles per hour. If we suppose, now, that the velocity of the wind increased to twenty miles per hour, the speed of the vessel propelled would also be increased probably to fourteen miles per hour, and the pressure on the sail would be that due to six miles per hour, the excess of twenty over fourteen.

Theoretically, as the resistance to the vessel's motion is that of the fluid in which it moves, the pressure due to increased velocity should follow the same law as that of the wind, and the example given above would be a probable case. In practice, however, the form of the vessel, the disturbance of the surface of the water, the bagging of the sails under increased pressure, and other circumstances, affect the regularity of the laws of water-resistance and wind-pressure so much, that we are not in a condition to compute accurately the ratio of the speed of the vessel to the velocity of the wind. Were the sail of a vessel fixed in a middle position, and the vessel itself capable of moving with equal ease in any direction, as it would be if it were made quite circular instead of being long and narrow, the vessel would be propelled exactly in the direction in which the wind might blow. But as this result would by no means suit the views of men desiring to sail in some particular direction, it is necessary to contrive the form of the

vessel, and the position of the sails, so that the pressure of the wind in one direction may be converted into a pressure in another direction. The vessel is therefore made long and narrow, so that it may be subject to much less resistance from the water when moving in the direction of its length, than if it were to move sideways; the sails are made capable of being turned about to different angles with the length of the vessel, so as to receive the wind obliquely; and, to direct the vessel and overcome any tendency of the wind to blow the front or back part of the vessel round out of its proper course, a rudder or helm is mounted at the stern, to balance, by its resistance in the water on one side or the other, the force tending to make the vessel deviate. When the wind strikes on a surface presented to it obliquely, its pressure is diminished. Suppose a certain surface five feet wide (Fig. 101) exposed directly to the wind, or at right angles to the direction of the wind, the pressure on it may be considered to be made up of five pressures each on one foot of the width. But if the same surface be turned obliquely to the wind, so that while the surface itself still remains five feet wide, the quantity of air intercepted is to be measured by three feet in width, the length of the line drawn at right angles to the direction of the wind intercepted between parallels to it from the extremities of the surface, then the pressure is only that on three feet of surface.

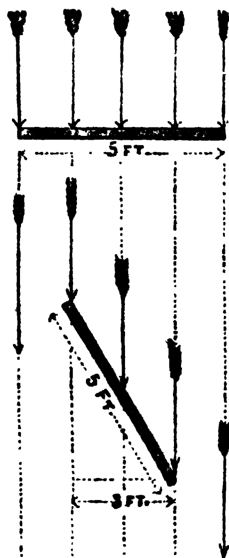


Fig. 101.

But besides the loss of surface from obliquity, there is also a loss in intensity of pressure. A column of air A D (Fig. 102) striking a surface exposed obliquely to it, would be reflected in D B, and press upon the surface at right angles. The quantity of pressure compared with what it would be if the surface were not oblique, is found by taking A D any length, to represent the direct pressure, and drawing A C and D C parallel and perpendicular to the oblique surface. Then while A D measures the direct impulse, C D measures its effective impulse to move the surface in the direction D E, and A C measures the loss from obliquity, or the amount of force expended parallel to the oblique surface, and therefore not effective upon it. But, farther, it is seldom the case that the effective pressure of the wind on the surface of a sail set obliquely to it is employed directly. If A B (Fig. 103) represent the surface of the sail of a vessel, C D the direction of the wind, D E the direction of the vessel's course, we may take C D any length, representing the force of the wind; then C F, perpendicular to the sail,

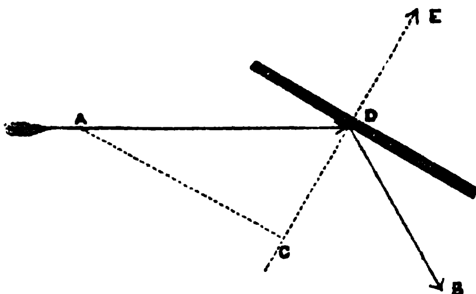


Fig. 102.

measures its effective force on the sail, and the length of C G, parallel to D E, and intercepted by F G, which is perpendicular to D E, measures the effective force in the direction of the ship's course. The line F G measures the effort of the wind to propel the vessel sideways, or to make what is technically called leeway.

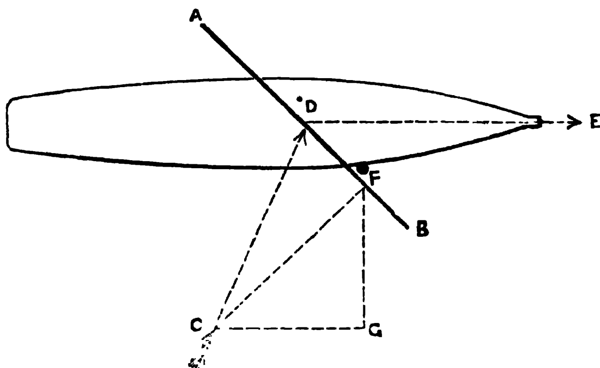


Fig. 103.

We may take a practical example as an illustration of how the effective impulse of the wind may be computed :—A sail 20 feet wide and 20 high is set with its edges directed to N.W. and S.E.; the wind blows at 30 miles an hour from the south, and the vessel sails due east with a speed of 10 miles per hour. Having drawn a plan of the

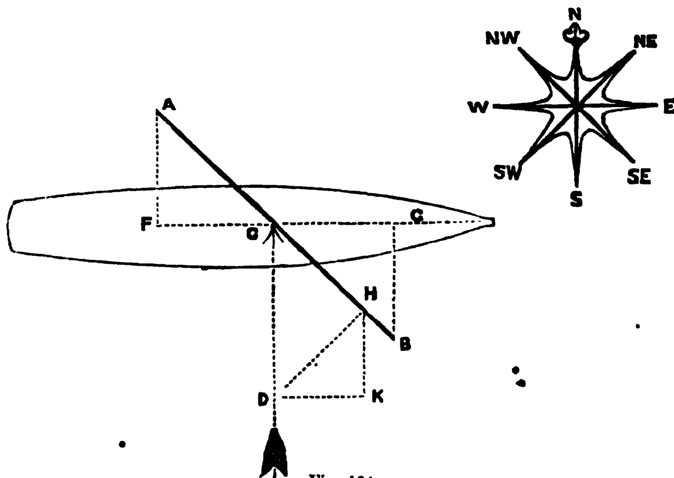


Fig. 104.

vessel (Fig. 104), showing D C the direction of the wind, and A B that of the sail, in the first place from A and B we draw parallels to D C, cutting off F G, the effective width of sail to receive the wind. A B measures 20 feet, and F G would be found to measure about 14 feet; therefore the effective surface of the sail is 14 width $\times$ 20 height $=$ 280 square feet. The velocity of the wind 30 miles per hour being represented by D C, its

velocity, resolved perpendicularly to the sail, is measured by D H, about 21 miles per hour. This velocity, resolved into D K, in the direction of the vessel's course, becomes 15 miles per hour, which is to the actual speed of the vessel, 10 miles, as 3 to 2. The pressure on the surface of the sail is not that due to 21 miles per hour, the measurement of D H, because the vessel and sail are continually escaping from the wind; but to the excess of D H over the speed of escape, or the same part of 21 as 5 (the excess of the wind's velocity in D K over 10, the velocity of the vessel) is of 15, the wind's velocity in D K. As 5 is $\frac{1}{3}$rd of 15, and 7 is $\frac{1}{3}$rd of 21, the effective pressure on the sail is that due to 7 miles per hour—about 0·245 lbs. per square foot. This multiplied by 280 square feet, the effective surface of the sail, gives 68$\frac{1}{2}$ lbs. for the pressure tending to blow off the sail from the mast while the vessel is moving at 10 miles per hour. Were the motion of the vessel entirely obstructed, the pressure on the sail would be that due to 21 miles per hour, about 2$\frac{1}{4}$ lbs. per square foot, or $280 \times 2\frac{1}{4} = 630$ lbs. The tendency of the wind to produce leeway, or to force the vessel sideways from its course, happens in this case to be the same as its direct force, for K H is equal to D K. If the vessel be 6 times as long as it is broad, the resistance to its movement sideways should be 6 times that to its direct movement, without taking into consideration the form of the part immersed. The shape, however, is made so as to offer as much resistance as possible sideways; so that, under equal impulses, the resistance to side-movement or leeway may probably be at least 10 times that to direct movement. It might be possible to compute the angle at which a sail should be set when the direction of wind and the course of the vessel are given, so as to obtain the greatest possible effect. But as in practice the sails of a vessel are numerous and various in position and direction of action, it would be difficult to apply such computation. A sailor, after a few trials of his vessel under sail, is able to estimate very correctly the angles of his sail and course with the direction of the wind, so as to get the best effect. If the wind blow almost directly

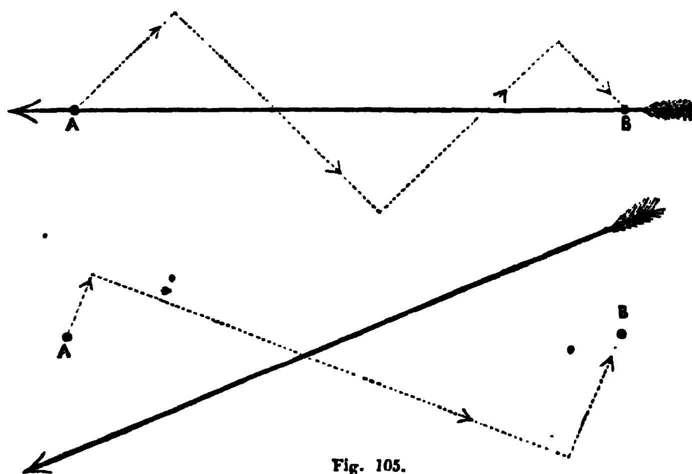


Fig. 105.

against the ship, it is necessary to tack. If, for instance, in sailing from A to B (Fig. 105), the wind blew from B towards A, or nearly so, the vessel would be steered in

various tacks or directions oblique to that of the wind, making progress on the whole. The choice of those tacks, so as to get over the given distance with as little deviation as possible, or, at all events, to do it with the least possible loss of time, is an important part of the sailor's art.

The force of wind has occasionally been applied to moving bodies on land, but not in a manner that can be generally used. Some of the Arctic voyagers having to travel long distances in sledges over ice, have taken advantage of a favourable wind to propel their vehicles by means of kites attached to them. Chariots have also been made with sails, so as to be propelled by wind along a road as ships are propelled at sea. But the uncertainty and variations of the wind render such applications of its power matters of curious experiment rather than of practical utility.

For giving motion to machinery, windmills have been and still are very extensively used. Engineers of the last generation devoted great attention to the construction of windmills, and brought them to great perfection. The introduction of steam-power—a power economical, manageable, and always to be depended on—has, in a great measure, superseded that of wind as a mover of machinery. It is true that after the first cost of a windmill, the power is comparatively inexpensive; but it is so variable in intensity—sometimes, when it is not required, exerting great force, and sometimes, when it may be most wanted, totally ineffective—that it is generally preferable to apply a force, perhaps considerably more expensive in its production, but constant, steady, and completely under control.

Windmills are of two kinds, horizontal and vertical. The former have been very little used, for it is found in practice that they are by no means so effective as the latter. The mode of constructing a horizontal windmill is like that represented on the plan (Fig. 106), or some modification of the same principle of construction. A wheel mounted on a vertical axis or shaft, and having flat vanes or boards fitted round its circumference, is inclosed in a circular casing, which is fitted with boards fixed obliquely, or in such lines as if produced inwards would touch the circumference of the wind-wheel. By this arrangement the wind, from whatever point it may blow, causes the wheel to revolve in the same direction. Part of the breeze passes between the oblique boards of the casing, and acts on the blades of the wheel; while part is intercepted by the boards, and either reflected inwards so as to propel the blades in the same direction, or reflected outwards so as not to act upon them in the opposite direction.

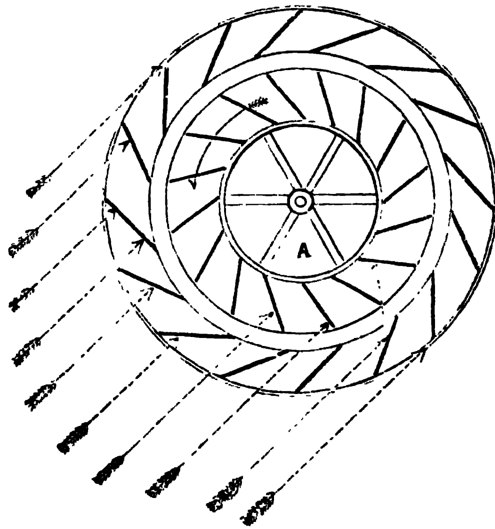


Fig. 106.

Sometimes horizontal windmills have been made with a casing partially surrounding the wind-wheel (Fig. 107), and capable of being turned round by means of a vane, so as to permit the wind only to act on one side of the wheel, while the other is completely sheltered.

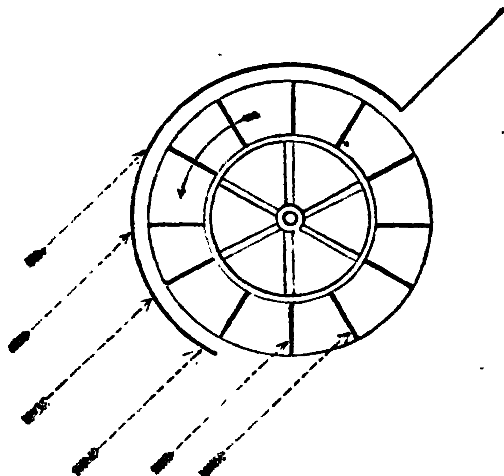


Fig. 107.

The vertical windmill, as is well known, consists of an axle or shaft, nearly horizontal, mounted in bearings at the summit of a tower, with four or more blades or sails attached to it. These sails are set at an angle with the axis, so that when the wind blows directly on the face of the mill, its oblique action on the sails is resolved into two forces—one in the direction of the axis, and the other perpendicular to it, the direction in which the sails revolve. Numerous experiments and computations were made to determine the most advantageous angles for setting the sails, and their most effective forms and proportions. If we suppose the radius of a sail divided into six equal parts (Fig. 108), and circles traced through the points of division, the velocity of each point in revolving is proportional to the part of its circle intercepted between two radii, or proportional to its own radius. If, then, we make a series of plans of the sail at these different parts, we see that as we approach the centre we should increase the obliquity of the sail to its plane of motion, so as to allow for its more slow escape sideways from the impulse of the wind. The sails accordingly are not made flat surfaces inclined equally to the plane of their revolution, but surfaces of varying inclination, somewhat like portions of screw

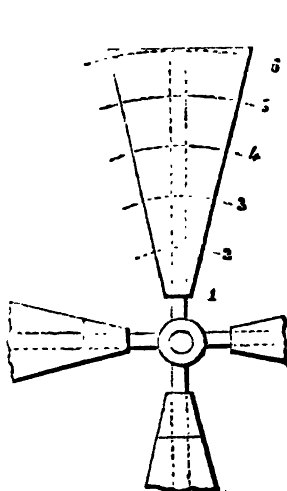


Fig. 108.

blades, twisting as it were from a certain obliquity at their extremes to a greater obliquity at the centre. The angles found most advantageous in practice are given by the celebrated engineer Smeaton as follow, as well as those used by some other engineers :—

Distance from centre	1	2	3	4	5	6
Inclination to plane of motion (Smeaton)	18°	19°	18°	16°	12½°	7°
,, ,, (otherwise)	24°	21°	18°	14°	9°	3°

In the angles given by Smeaton, an irregularity is observed in the first, which should by theoretical reasoning be greater than the second; whereas Smeaton makes it less. The following rule may be adopted as a very near approximation. To find the angle at which the sail should be inclined to the plane of revolution at any distance from the centre :—

Rule.—Multiply 18 twice by the distance from the centre, divide the product twice by the total radius, and subtract the quotient from 23; the remainder is the inclination in degrees.

Example.—In a windmill 60 feet in diameter, required the inclination of the sail 20 feet from the centre.

Here 30 feet is the total radius, and $\frac{18 \times 20 \times 20}{30 \times 30} = 8$, which, subtracted from 23, gives 15°, the angle of that point.

Were we to divide the radius 30 feet into 6 equal parts, and calculate the angles at each point, we should find them correspond nearly with the means of those given by Smeaton and others, as may be seen by the following table :—

Parts of radius	1	2	3	4	5	6
Distances from centre	5 feet	10 feet	15 feet	20 feet	25 feet	30 feet
Angles (Smeaton)	18°	19°	18°	16°	12½°	7°
Angles (others)	24°	21°	18°	14°	9°	3°
Means	21°	20°	18°	15°	10¾°	5°
Angles by the rule	22½°	21°	18½°	15°	10½°	5°
Difference from means	1½°	1°	½°	0	¼°	0°

Having determined the proper inclination of the sails at different distances from the centre, it next becomes important to inquire how much of the surface of the whole circle should be filled with sails. Mills are generally made with four strong wooden arms, or radii, fixed firmly in a central socket, and steadied and stiffened by tie-rods, connecting their extremities together, and with a projecting strut on the central boss. The width of each sail at the extreme should be about half of the radius, so that in a mill 60 feet diameter, or 30 feet radius, each sail would be 15 feet wide at the extreme. The part of the arm next the centre for about ⅙th of the radius, that is 5 feet in the case supposed, is not fitted with sails, because the surface there is so little effective, as well from its short leverage as from its obstructing the wind reflected from the head of the turret behind it. The width at the inner end should be ⅓rd of the radius, or 10 feet. The surface of each sail is therefore 312½ square feet, and the total of the four is 312½ × 4 = 1,250 square feet.

The total area of a circle 60 feet in diameter is somewhat above 2,800 square feet, so that not half the surface of the circle is clothed with sails. There would be no disadvantage in extending the surface by making the sails broader, or more numerous, until it became $\frac{3}{4}$ths of the whole surface. Beyond this, additional sail-surface is disadvantageous, for it appears to obstruct the free passage of the currents reflected from the sails, and thus clog their motions. It is found advantageous to arrange the surface of a sail somewhat in the proportions of the diagram (Fig. 109), which represents the front view of one sail.

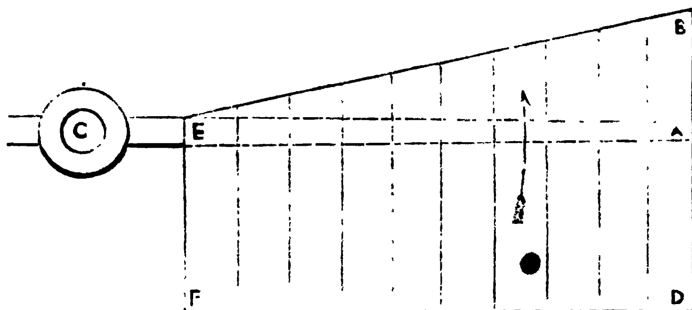


Fig. 109.

If A C be	. . .	30 feet.
Then A E	. . .	25 „
A D or E F	. . .	10 „
A B	. . . . .	5 „

The covering of the surface, so as to catch the impulse of wind, formerly consisted of canvas fixed on a roller at one side of the arm, on which it could be rolled like a window-blind; or from which it could be unrolled so as to cover the whole sail, which was filled in with wooden framing to support the canvas pressed against it by the wind. Sometimes the canvas, instead of being in one sheet, was subdivided into numerous separate sheets mounted on rollers; and apparatus was provided so that the canvas might be wound on the rollers or unwound at pleasure, while the mill was in motion. As the wind is exceedingly variable, and as the quantity of work required of the mill also might vary to a considerable extent, it was found necessary to provide some apparatus by which the mill might regulate itself, so that its velocity should not be excessive at one time and too small at another. One mode of effecting this object was to apply to the machinery of a mill a governor, like that of a steam-engine (which we shall have occasion to describe in detail hereafter). This governor consists of two heavy balls suspended from the summit of a vertical revolving spindle by jointed rods (Fig 110). The spindle being at rest, the balls hang close to it on each side; but on the spindle being caused to revolve rapidly, the balls, impelled by centrifugal force, fly away from the central axis, as marked by the dotted lines. A system of levers and rods connected this apparatus with the sail-rollers, so that when the balls flew outwards from increased velocity, the sails were furled; and when they fell inwards from diminished speed of revolution, the sails were unfurled. The quantity of surface thus presented to the wind was adjusted to its force, and a tolerably equable velocity of the machinery was attained. In some more recent mills an ingenious contrivance for regulating the surface of sail according

to the force of the wind has been successfully adopted. The sails consist of a framework filled in with louvre-boards hinged on pivot-pins near one of their edges, and all connected by levers and rods with a sliding-boss on the central axis of the windmill (Fig. 111). When the wind blows strongly against the louvre-boards, it forces them out of their vertical position, and passes freely through the openings between them. The surface of the sails is thus diminished by the pressure of the wind itself. To prevent its being too much diminished, the sliding-boss connected with the louvre-boards is pressed upon by a lever loaded by a certain weight sufficient to balance, as far as may be desirable, the pressure tending to force aside the louvres, and thus to keep them, to a certain extent, up to their work. When the load on the mill—that is to say, the quantity of work effected by it—is varied, the weight may be varied accordingly; and thus the effective amount of surface in the sails may be adjusted to the average force of the wind, and the work to be done by it. When the wind-force exceeds or falls short of its average, the greater or less inclination of the louvres very nearly compensates for the variation.

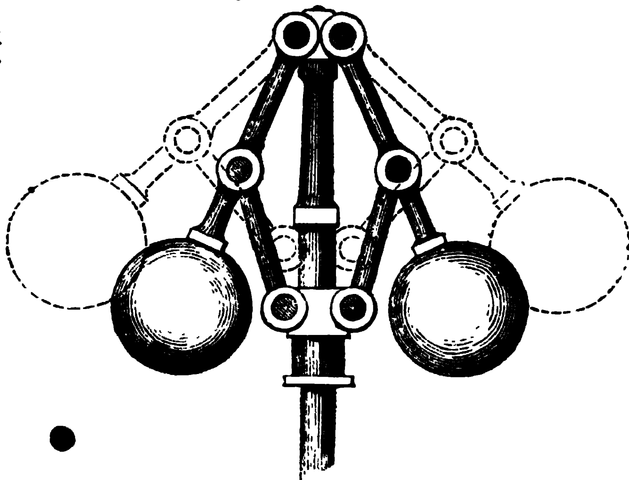


Fig. 110.

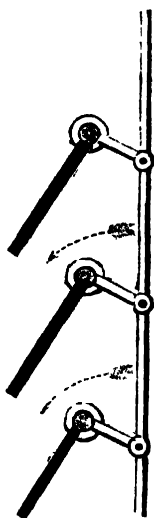


Fig. 111.

The sails of a windmill should directly face the wind in order to receive its most advantageous action; but, as the direction of the wind often changes, it is necessary to adopt some arrangement for varying that of the mill-shaft accordingly. The summit of the mill-tower, in which the mill-shaft is mounted, is therefore made to revolve, so that at any time the direction of the shaft may be varied, and the sails presented to the wind. In old mills, and indeed in many small mills still existing, this change of direction is effected by hand. A long lever is fixed to the moveable cap or summit of the tower, and extends obliquely to the ground. The miller watches the direction of the wind, and by moving this lever turns the cap round to its proper position. But in large mills this would require considerable power; and, moreover, constant attention would have to be paid to the changes of the wind. Were a single change neglected, the mill might be destroyed; for as the sails are made and strengthened by tie-rods to receive the wind's pressure on their

face, a change of the wind to the opposite direction might throw a great strain on their back, for meeting which no provision is made. A simple mode of making the change of direction self-acting, is to fit the back of the cap with a large vane, which, like that of a weather-cock, would cause the sails to be presented to the wind from whatever quarter it might blow. But when mills are of considerable size, the vane would require to be very large and cumbrous. The contrivance generally employed is neat and ingenious. Behind the cap (Fig. 112), on the side opposite that through which the wind-shaft passes, a framing is made to project outwards. On this framing there is mounted a small windmill, on an axis transverse to that of the main arms. The cap rests on rollers fitted in the circular top of the tower, so that it may move freely round; and a toothed circular rack is also fixed on the summit of the tower. A spindle, fitted with bevil-gearing so that it may be caused to revolve by the revolution of the small mill, conveys motion to a toothed pinion, which gears into the circular rack. When the main mill has its face presented to the wind, the small one stands edgeways to it, and therefore remains at rest; but as soon as the wind veers, it begins to act on one side or the other of the small mill, and thus causes it to revolve. The pinion is thus made to travel along the fixed rack, and turn the cap of the mill round until the main mill is again brought to face the wind in its new direction. This arrangement is found to be very effective; and when it is properly applied, the mill requires no attention in respect of direction to the wind.

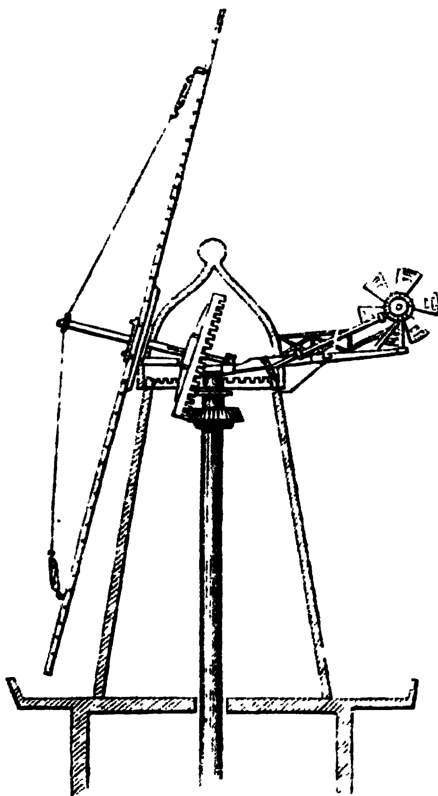


Fig. 112.

In estimating the velocity with which the sails of a windmill revolve, we have to consider not only the force of the wind upon them, but also the resistance to their motion occasioned by the work done by the mill. A B (Fig. 113) may represent the edge of a surface presented obliquely to the wind, and capable of moving in the direction C D at right angles to that of the wind. If the surface be free and unresisted in its motion, and the wind be considered to produce its full effect upon it, the proportion of its velocity to that of the wind would be estimated by that of the line B B′ to the line B A′; for it is clear that while the wind travels over the distance B A′, the surface moves to the position dotted, that is over B B′. But if the motion of the surface be resisted, its

velocity in relation to that of the wind is diminished. In the case of windmill sails, we may suppose such a load of work on the mill that the velocity of the sails is not more than half what it would be were there no resistance. We may, therefore,

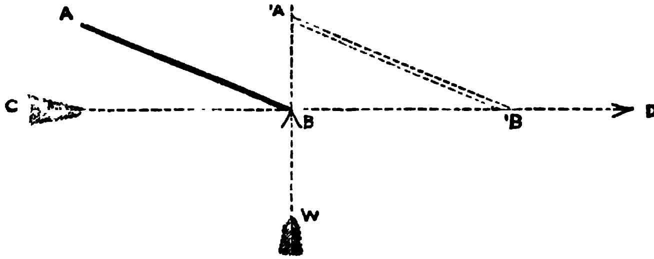


Fig. 113.

assume that the velocity of the sail relatively to the wind would be expressed by the ratio of half the length of the line B B' to the length of A B'. Taking the wind as a gentle breeze, the velocity of which in the table is about 5 miles per hour, and the inclination of the sail or angle A' B B' half-way from the centre 18°, we should find the half of B B' to be about $1\frac{1}{2}$ times A B, or the velocity of the sail $1\frac{1}{2} \times 5 = 7\frac{1}{2}$ miles per hour—about 660 feet per minute. If the windmill be about 60 feet in diameter, the diameter of the middle point of the arm is 30 feet; the circumference of the circle in which that point revolves is 94 feet, and the number of revolutions made per minute is therefore $\frac{660}{94}$—about 7.

If now we calculate the speed of the extremities of the arms, we find that it is 1320 feet per minute, or about 15 miles per hour; three times that of the wind, which we have assumed as 5 miles per hour. Did we assume a wind of greater velocity, we should have to take into account the self-regulating arrangement, which diminishes the amount of surface exposed, and therefore prevents the mill from attaining so much increase of speed as it would without regulation. Under ordinary circumstances the speed of the outer extremities of the arms ranges from 20 to 30 miles per hour. We may assume 30 miles per hour when the wind blows at 10 miles with a pressure of about $\frac{1}{2}$ lb. on the square foot. The total surface of the sails unfurled in a mill 60 feet diameter, is 1250 square feet; we may suppose half lost by furling, leaving 625 effective. As the surface is set obliquely to the wind, the pressure in the direction of motion would be reduced from $\frac{1}{2}$ lb. to about $\frac{1}{7}$ lb. as a mean over the whole of the arms, giving a total pressure in the direction of motion of about 90 lbs. The mean velocity of the arms is half that of the extreme, 15 miles per hour or 1320 feet per minute. We have, therefore, 90 lbs. moving at 1320 feet per minute, which is equivalent to a force of $90 \times 1330 = 118{,}800$ lbs. moving at 1 foot per minute. A horse-power is reckoned as equivalent to 33,000 lbs. moved 1 foot per minute; therefore, the power of the mill we have reckoned is about $3\frac{1}{2}$ horse-power.

When we double the diameter of a mill, we quadruple its power, for we quadruple its effective surface. The areas of circles are proportional to the squares of their diameters; and as the similar parts of the areas are occupied by sails, they are also as the squares of the diameters.

It is not at all an easy matter to estimate the powers of windmills. The proper guide as to power, velocity, and construction is experience. Some of the works of Smeaton contain much valuable information respecting this branch of Practical Mecha-

nics; and to these we must refer such of our readers as require a more full discussion of the subject than our limits permit us to offer.

As a force applied to the movement of machinery, wind has few advantages except its little cost after the first outlay for a windmill has been made. It is chiefly available in flat countries, where there is no opportunity of obtaining the preferable power of water, and where there is little interruption to the aërial currents. In hilly countries windmills are often subject to derangement from the excessive force of the gusts of wind that often occur in such regions. In tropical countries, particularly islands and places near the sea-shore, the daily occurrence of the land and sea-breezes occasioned by the action of the solar heat on the land, provides a certain amount of wind-power which may be almost always depended on. But in these countries, on the other hand, there often occur tornadoes or hurricanes of extreme violence, that sweep away almost everything that may oppose their progress; and thus frequently destroy windmills, and occasion renewed outlay in their re-construction. The principal use to which windmills are devoted in temperate climates is for grinding corn; in tropical climates, such as the West Indian Islands, they are employed for driving sugar-cane mills. In fenny and marshy countries, such as Holland or some of the eastern counties of England, they are used for drainage, either by working pumps or turning a wheel contrived for lifting the drainage water from the surface of the ground into canals at a higher level, by which it is carried off into the sea. In all situations, however, where the cost of fuel is not extravagantly great, steam-power is gradually superseding that of wind, because its certainty of action more than repays the cost of its production. Whole districts, the drainage of which is dependent on wind-power, may frequently remain many weeks under water from the prevalence of calm weather, and the agricultural operations of the season may be so seriously interfered with that whole crops are lost, or become immensely deteriorated. In sugar-growing countries again, the derangement of wind-machinery by a hurricane or tempest, may occur at the season when the sugar-cane have to be crushed; and the loss of a few days in crushing the canes may seriously damage the sugar in respect of quantity as well as quality. Upon the whole then, whenever the cost of fuel is not excessive, it is not advisable to incur the outlay of extensive works for securing wind-power. A very small steam-engine, kept constantly in operation, is far more effective than a windmill of much greater power, because the latter is so variable and uncertain in its action. The only operations suited to wind-power, are such as need not necessarily be completed at certain periods, but may be conducted occasionally as the wind may serve. Nor should the machinery driven by wind require very nice regularity in its action; for, notwithstanding all the ingenious arrangements for equalizing the wind-force, it is still unsteady at the best.

Every part exposed to the wind should be greatly in excess of the strength required to resist the average strain to which it may be exposed. The tempest of an hour—nay, a momentary gust—may frequently destroy a windmill that has stood under ordinary winds for years; as a safeguard against too much strain, the windmill should always be left free to revolve, even if the machinery which it drives be thrown out of gear. The shaft or axis of the mill generally carries a large wheel, to which is fitted a strap of iron loaded so as to press on its circumference, and act as a friction-break either to hold the mill fast for purposes of repair during light winds, or to check its velocity when the winds are too strong for the work required.

Water Power.—The movements of water are much more serviceable for the purposes of power, and steady in their operation, than those of air. In level countries, where

the streams are slow and languid in their flow, this power is not attainable; but in hilly countries, where the rivers and streams fall frequently from a high level to a lower, water-power is easily obtained, and is most advantageous as a steady inexpensive prime mover. The most common way of employing water-power is to cause the current to act on the circumference of a large wheel, so as to give it a rotatory motion, which is communicated by means of shafts and wheelwork to the machinery required to be driven. Such watermills are generally used for grinding or thrashing corn, crushing bones for manure, raising water to irrigate land, in mining districts for crushing or otherwise operating on the ores, and in manufacturing districts for working cotton, woollen, or flax machinery. Water-wheels are of three kinds, named according to the mode in which the water current is made to act upon them:—

1. Undershot, when the wheel is fixed over a stream with inconsiderable fall, but considerable velocity.

2. Overshot, when the fall of the water is so great that the stream may be directed upon the upper part of the wheel.

3. Breast-wheels, when the stream can be directed on or near the middle or breast of the wheel.

1. *The Undershot-wheel* may be best understood by conceiving the action of the paddles of a steam-vessel reversed; that is to say, while in a steam-vessel the paddles are caused to revolve, and were the vessel fixed would produce a current in the water by their revolution, in the case of the undershot-wheel the natural current of the water pressing on the floats immersed in it causes the wheel to revolve. It is sufficiently clear that the power derived from this arrangement depends upon the intensity of the pressure which the water exerts on the floats, and the amount of surface pressed upon. If we suppose the wheel at rest, and its float standing vertically in the water, we may easily compute the pressure on every square foot of its surface by ascertaining the speed at which the water flows against it. This pressure, like that of the wind, is proportional to the square of the velocity of current; for by doubling the velocity we bring double the number of particles in contact with the float, and we also double the force of each particle in striking it, so that upon the whole we quadruple the pressure. So also, by taking 3 times the velocity, we have 9 times the pressure; and, generally, if we know the pressure due to one velocity, such as 1 foot per second, we can compute that due to another velocity, such as 10 feet per second, by multiplying the velocity by itself, and taking the pressures in those proportions.

By an investigation of the mechanical laws which govern the motion of fluids, we can ascertain that the velocity with which a fluid flows from an orifice in any vessel is the same as that which a heavy body would acquire by falling through a height equal to that of the fluid column above the orifice. This is found by the following rule:—Multiply the square root of the height (in feet) by 8; the product is the velocity (in feet per second). Thus, if a stone were dropped from a precipice 100 feet high (neglecting the resistance of the air to its fall), its velocity when it strikes the bottom would be about 80 feet per second; for 10 is the square root of 100, and $8 \times 10 = 80$ feet per second. If, on the other hand, we knew the velocity, we should be enabled to calculate the height of fall by reversing the rule; that is to say, divide the velocity (in feet per second) by 8, and square the result for the height (in feet). Thus, if we ascertained that a stone had in its fall acquired a velocity of 80 feet per second, we should reckon that it had fallen 100 feet, for $\frac{80}{8} = 10$, and 10 squared or $10 \times 10 = 100$.

These rules do not take into account the resistance offered by the air to the motion of the body falling through it. In falling through great heights, this resistance has very great effect on the velocity; but for small falls, it may be neglected without material error. If, now, we apply these computations to the flow of currents of water, we can calculate the height due to a certain velocity of current; that is, the head of water whose pressure on its lower particles gives them the velocity in question. Thus, if on measuring the velocity of a stream at a rapid we found it to be 16 feet per second, since $\frac{16}{8} = 2$, and $2 \times 2 = 4$, we should reckon that, in order to have this velocity, it must be pressed on by a column or head of water 4 feet high. It must, therefore, press upon any body immersed in it with the same force as it is itself subjected to; for it is the peculiar property of fluids to convey pressures equally in every direction through them. It does not necessarily follow that, at the point where the velocity is measured, there is an actual fall of 4 feet; it is sufficient that the water has somewhere fallen enough to acquire the velocity measured, or that any forces whatever have combined to give it that velocity, or to reduce its velocity to that degree. The velocity is, in fact, the expression of the result of all the forces and resistances that have acted on the water up to the point where it is measured. Having computed the head of water pressing, it is easy to compute the amount of pressure exerted on a square foot. A cubic foot of water weighs $62\frac{1}{2}$ lbs.; therefore the bottom of a cubical box, measuring 1 foot every way, is pressed on by a force of $62\frac{1}{2}$ lbs. when the box is filled with water; in other words, the pressure of a column of water 1 foot high is $62\frac{1}{2}$ lbs. on every square foot of bottom surface. Were the height of the column increased, the pressure would be increased in like proportion; every additional foot of height would throw an additional pressure of $62\frac{1}{2}$ lbs. on every square foot. The intensity of pressure (in lbs. per square foot), then, is $62\frac{1}{2}$ times the height in feet. Thus, the pressure due to a head of 4 feet is $62\frac{1}{2} \times 4 = 250$ lbs. per square foot. We may now combine the two computations—that for head due to given velocity, and that for pressure due to given head—into one rule, for determining the pressure per square foot due to a given velocity, as follows:—As the square of 8 is 64, which does not much differ from $62\frac{1}{2}$, we may, without material inaccuracy, avoid dividing the velocity by 8 before squaring the quotient, and again multiplying by $62\frac{1}{2}$, and thus have the very simple rule.

Square the velocity (in feet per second), and the result is nearly the pressure (in lbs. per square foot). Thus, when the stream is moving with a velocity of 16 feet per second, its pressure per square foot is $16 \times 16 = 256$ lbs. nearly. By the former computation, we found 250 lbs., less by 6; that is, less than $\frac{1}{41}$st part of the whole. So far we have found the means of computing the pressure on a float-board *at rest*, in a stream flowing with a given speed; but in the case of an undershot water-wheel, the float is in motion in the same direction with the current, and therefore the relative velocity of the latter in acting upon it is so much diminished. The relative velocities of the float and of the current may, of course, be varied by applying more or less resistance to the motion of the wheel. It is necessary to know the relation of the two velocities, so as to derive the best possible effect from the current. If we take a particular case, and try various relations, we may find the most advantageous—bearing in mind that the result to be obtained is the maximum of power or useful effect; that is to say, the pressure on any float multiplied by the velocity of its motion, which product gives the power of that float as a mover of machinery. If we take the velocity of a current as 6 feet per second, and form a table of velocities of float from 0 feet per second up to 6 feet per

second, of pressures due to the excess of the stream's velocity over that of the float, and of the powers or products of those pressures multiplied by the corresponding velocities of float, we shall find the power greatest when the velocity of the float is exactly 2 feet per second, or ⅓rd of that of the stream. Thus—

Velocity of stream.	Velocity of float.	Excess of stream over float.	Pressure or excess squared.	Power or press. × veloc.
6	0	6	36	0
„	1	5	25	25
„	2	4	16	32
„	3	3	9	27
„	4	2	4	16
„	5	1	1	5
„	6	0	0	0

Were we to take any other velocity of stream we should find the same result, that the velocity of the float should be ⅓rd of that of the stream, in order to attain the maximum effect. If this rule be adhered to, the excess of the stream's velocity over that of the float is ⅔rds of itself; and the pressure per square foot would be the square of ⅔rds of the stream's velocity, or $\frac{4}{9}$ths of the square of the velocity, since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. The power would depend on the number of square feet pressed upon, and the velocity of the float, and would therefore be found by multiplying the surface of the float by $\frac{4}{9}$ths of the square of the stream's velocity, and the product by the float's velocity, or ⅓rd of the stream's velocity. As there are generally several floats immersed in the water, it may appear that the surface acted on is considerably larger than that of one float; but when it is remembered that the volume of water contained between any two of the floats, if it press the one forward by its direct action, must equally press the other backward by its reaction, we cannot safely estimate more than the surface of one float as really effective. The velocities of currents are generally reckoned in feet per second, while the velocities of moving parts of machinery, in estimating power, are reckoned in feet per minute. Taking a horse-power as 33,000 lbs. lifted one foot high in one minute, and assuming the circumferential speed of an undershot-wheel as one-third of that of the current, we may estimate its power as a mover of machinery by the following rule:—

Multiply the surface of the float (in square feet) three times by the velocity of the stream (in feet per second), and divide the product by 3,800; the quotient expresses the horse-power.

Note.—If V = velocity of stream, v = velocity of float, p = pressure per square foot $= (V - v)^2$ and $P = \text{power} = pv = \overline{V - v}^2 v$, to find v, so that P may be a maximum.

$$P = \overline{V - v}^2 v = V^2 v - 2Vv^2 + v^3.$$

$$\frac{dP}{dv} = V^2 - 4Vv + 3v^2 = 0.$$

$$\therefore\ v^2 - \frac{4}{3}Vv = -\frac{V^2}{3} \text{ and } v^2 - \frac{4}{3}Vv + \frac{4}{9}V^2 = \frac{1}{9}V^2.$$

$$\text{Hence } v - \tfrac{2}{3}V = \pm \tfrac{1}{3}V, \text{ or } v = V, \text{ or } \tfrac{1}{3}V.$$

$$\frac{d^2P}{dv^2} = -4V + 6v, \text{ and substituting for } v \text{ each of its values.}$$

$$-4V + 6V = 2V \text{ positive, gives } P \text{ a minimum when } v = V.$$

$$-4V + 2V = -2V \text{ negative, gives } P \text{ a maximum when } v = \tfrac{1}{3}V.$$

Example.—An undershot-wheel, having floats 2 feet deep and 10 feet wide, is moved by a stream running at $7\frac{1}{2}$ miles per hour: required its power.

Here the surface of a float is $10 \times 2 = 20$ square feet.

The velocity is $7\frac{1}{2}$ miles per hour: or (as a mile is 5,280 feet, and an hour 3,600 seconds) the velocity is $\frac{5280 \times 7\frac{1}{2}}{3600} = 11$ feet per second.

The power therefore is $\frac{20 \times 11 \times 11 \times 11}{3800} = 7$ horse-power.

The speed of the floats being $\frac{1}{3}$rd of that of the current, is $\frac{11}{3}$ feet per second, or $\frac{11 \times 60}{3} = 220$ feet per minute. We may take this as the speed of the middle part of the float; and if the wheel be 23 feet in extreme diameter, its diameter at the middle of the floats would be 21 feet, and circumference there 66 feet; if this moves at the rate of 220 feet per minute, the wheel must make $\frac{220}{66} = 3\frac{1}{3}$ revolutions per minute.

The speed of a stream may be generally estimated by throwing on it a body that floats, but is immersed some depth—as a bar of wood loaded at one end to float vertically—and watching the time occupied by its passage over a certain known distance. Care should be taken that the body fairly attain the speed of the current before its motion is reckoned in the time. It is advantageous to make the inner edges of the floats stand somewhat above the general level of the water, which becomes heaped up behind them, and would otherwise pour over the edges (Fig. 114). Indeed, the difference of level caused by this heaping up of the water behind, and its hollowing in front of a float, almost measures the head of water pressing on it. When practicable, the stream should be narrowed to the width of the wheel (Fig. 115), as by this means not only is its velocity augmented by the necessity of a certain body of water passing more quickly through a diminished channel; but also the water acting on the floats is prevented from escaping sideways without giving its full effect.

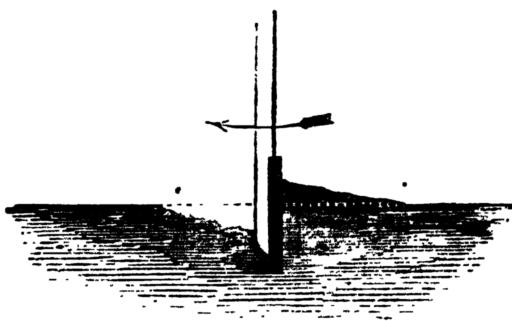


Fig. 114.

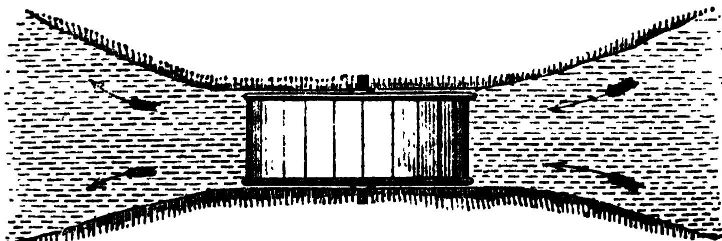
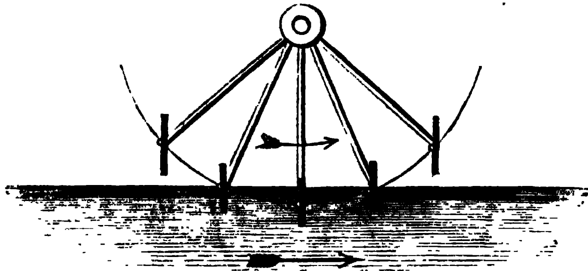


Fig. 115.

As the velocity of streams to which undershot-wheels are applicable is never very great, and as the velocity of the floats should not much exceed one-third of that of the stream, such wheels are necessarily slow in their revolution, and can therefore be applied with most advantage in driving machinery where quick speeds are not required. When it is necessary to convert the slow revolution of the wheel to rapid motions in the machinery, there are considerable losses from the friction of the gearing. For such purposes as that of working pumps or fulling-mills, and generally for slow, heavy work, these wheels are very serviceable. The principal objection to their use arises from the circumstance, that with a stream of average rapidity, very little power is obtained without a very large and cumbrous wheel, involving considerable outlay, and extending over a great breadth of the stream. By making the diameter of the wheel large, no greater power is obtained, except what may be attributable to the more direct action of the water on the floats, which enter and leave the water more vertically when the wheel is large. The circumference of a large wheel should move with the same speed as that of a small one; and, therefore, the greater the wheel, the smaller number of revolutions does it make in a given time. The only way of increasing the power is to extend the surface of the floats. This may be done by making them deeper or wider. Additional depth of the float, even where the depth of the stream permits it, is by no means so effective as additional width; for a wide shallow float enters and leaves the water with ease, while a deeper one presses the surface of the water down in entering, and lifts it up in leaving, and thereby encounters considerable resistance to its motion.

Occasionally undershot wheels have been made like the feathering paddles of steam-vessels, where the floats are capable of being turned on pivots (Fig. 116) so as to maintain a vertical position while immersed in the water, and thus receive its most direct impulse, while they enter and leave it with the least possible resistance.


Fig. 116.

When it is considered that twice in every day a great tidal stream flows and ebbs along our coast and in our estuaries, it is surprising that advantage has not more frequently been taken of this enormous power by the erection of undershot-wheels along the course of the tidal currents. In this country tide-mills are rare; and neither their number nor magnitude render them important as sources of power. Occasionally, however, they have been employed with advantage. Where there is a great tidal stream, and consequently a considerable difference between the levels of high and low-water, fixed wheels would be almost useless, for at high-water they would be too much immersed and at low-water too little. It is in such cases necessary to mount them on a floating stage or barge, so that the whole mill may rise and fall with the tide, the amount of immersion remaining constant. Again, as the tide flows alternately in opposite directions, when it is required that the machinery move only in one direction, it is necessary that tide-mills, in such cases, should be fitted with apparatus by means of which the direction of movement may be reversed; or, where the mill is floated on a

barge, the swinging of the barge by the tide effects the required change of position to suit the change in direction of the current.

If it were practicable to make use of the tidal stream in such a river as the Thames, without interfering with its navigation, the power derived from it would be enormous. If we suppose the breadth, 1200 feet, occupied by tidal mills side by side, with floats immersed 3 feet deep, the total float-surface would be 3600 square feet in one section of the river. The velocity of current we may take, on the average, as 3 miles per hour, nearly $4\frac{1}{2}$ feet per second; and the power, according to our rule, would be

$$\frac{3600 \times 4\frac{1}{2} \times 4\frac{1}{2} \times 4\frac{1}{2}}{3800} = 86 \text{ horse-power.}$$

Were such mills repeated at intervals of 220 feet along a mile of the river, there would be 24 of them, and the total power would be $86 \times 24 = 2064$ horse-power in a mile of the river's length. It is not, of course, presumed that such an arrangement is feasible: it is only offered as an illustration of the great mechanical power that might be derived from the natural movements of the water in tidal estuaries. In some rivers, such as the Rhine and the Seine, barges are moored carrying tidal mills of this kind. In such streams the level does not greatly vary, and the current sets continuously in one direction, so that the power is applied with constancy and facility.

2. *The Overshot Water-wheel* (Fig. 117) has its circumference divided by partitions into numerous compartments, or buckets, capable of containing water. The spout conveying the water to the wheel either passes over its summit, or has a check at its end (Fig. 118), so as to discharge the water into the buckets a little beyond the summit of the wheel. As the wheel revolves, each successive bucket is brought under the spout and becomes filled with water, the weight of which, acting on one side of the wheel, is the moving force. The buckets, as they descend, become gradually emptied, and return up the unloaded side of the wheel, to be again filled and descend.

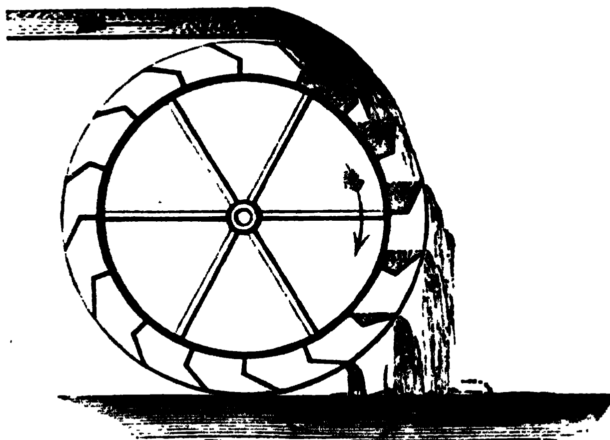


Fig. 117.

Such wheels are only applicable where there is a considerable fall of water; for the height of the head above the stream as it flows away from the wheel, technically called the tail-water, must be equal to the diameter of the wheel, or nearly so. In overshot wheels the velocity of the water is no element of power, except in so far as the quantity conveyed by the spout to the wheel depends upon the velocity with which it flows. If the velocity of discharge be considerable, a positive disadvantage results from the too rapid dash of water into the buckets, causing it to overflow, while the bucket remains

only partially filled. It is easy to see that the quantity of water issuing from the spout during the time which a bucket occupies in passing under it, should barely exceed that which the bucket will hold; if it falls short of that quantity, the bucket is only partially filled in its passage; and if it much exceeds that quantity, the force of its flow causes it to dash over and become wasted without effectually filling the bucket. The diameter of the wheel being limited by the height of the fall, when it is desirable to take advantage of a large quantity of water discharged from the spout, the breadth of the wheel must be increased, and the water in the spout caused to spread itself out to a wide sheet, so as nearly to cover the whole breadth of the wheel. The sheet of water should always be a few inches narrower than the face of the wheel, to save the water from dashing ineffectively over the edges (Fig. 119). Another point of great importance in the construction of the buckets is to leave a passage for air at the inner upper angle of each bucket, otherwise the bucket can become only partially filled, in consequence of the elasticity of the air confined in it compelling the supply-water to dash over the edges instead of filling the bucket.

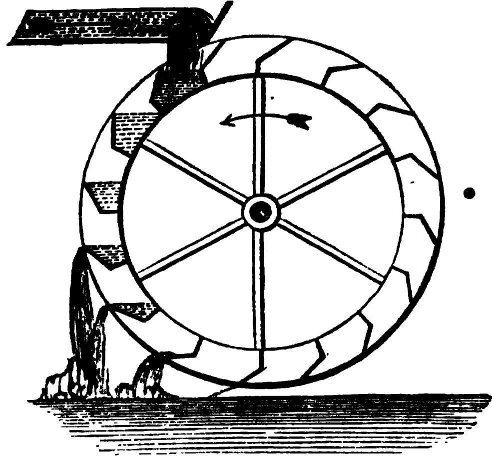


Fig. 118.

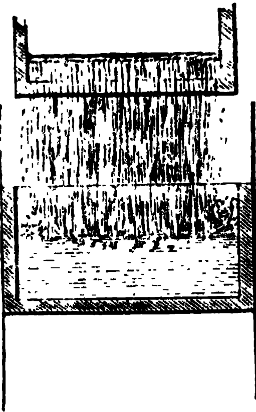


Fig. 119.

In constructing an overshot-wheel it is necessary to give consideration to the following points:—

1. The point of the circumference at which the spout should discharge so as best to fill the buckets.

2. The best form of bucket for receiving the water, and for retaining it, through a considerable part of its descent.

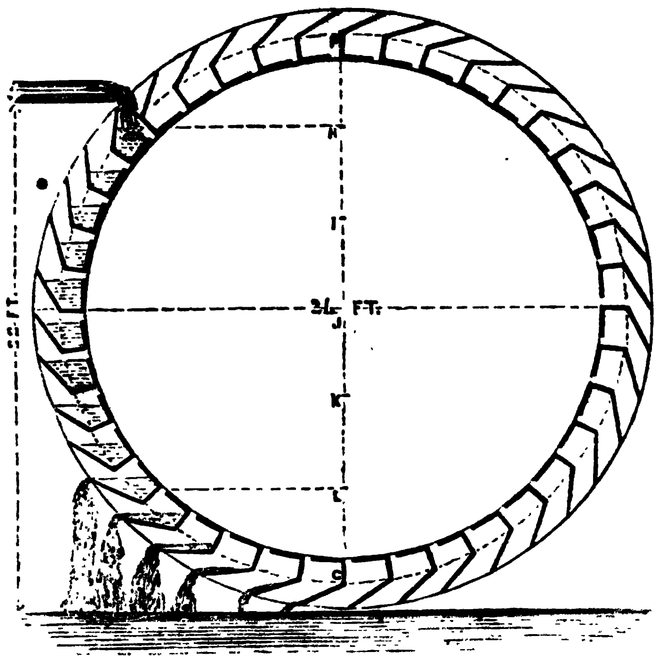


Fig. 120.

3. The best speed at which the circumference of the wheel should travel so as to obtain the greatest effect from the moving load of water which its buckets contain.

If we suppose that a fall of water about twenty-two feet in height is to act upon an overshot-wheel (Fig. 120), we may make the wheel about twenty-four feet in diameter, and the depth of the buckets, measured towards the centre, two feet. Dividing this depth into two equal parts, each one foot, marked by the dotted circle (Fig. 121), dividing the circumference of this circle into a convenient number of equal parts B B equal to the number of buckets (40 in the diagram), and drawing lines A B A towards the centre of the wheel through the points of division, we are enabled to determine the form of the buckets. The casing extending between A A is called the sole of a bucket, and is left with a narrow slit open at A for the escape of air from the bucket

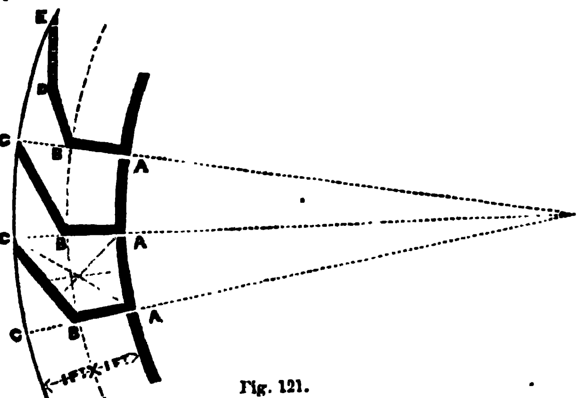


Fig. 121.

when the water pours into it. The board A B is called the start, and the inclined part B C the arm of the bucket. Sometimes this inclined part is made in two parts at different degrees of obliquity, like the dotted lines B D, D E; of which B D would be called the arm, and D E the wrist, the start A B being called the shoulder. These names are doubtless given from the resemblance of the section to the form of a bent arm. The whole circumference of buckets and soles is called the shrouding.

On dividing the vertical diameter F G of the mean circle of the shrouding into six equal parts at the points H, I, J, K, L, and drawing horizontal lines through H and L to meet the circumference, we observe that at the upper line the bucket is filled, and therefore the weight of its contents begins to act in causing the wheel to revolve, while at the lower line it begins to empty itself, and its action may there be considered to cease; or whatever effect the water may have beyond this point is so small that it may be neglected as an element of power. The total effect of the water, then, in causing the wheel to revolve, may be reckoned to be that of the weight of water contained in ten buckets descending through a height equal to two-thirds of the diameter of the mean circle, viz. $22 \times \frac{2}{3} = 15$ feet nearly. The diameter of the mean circle is equal to the height of fall; and we may, therefore, by taking a wheel of just proportions, generally obtain a descending weight acting through a vertical height two-thirds of the height of fall; and the weight itself consisting of the contents of one-fourth of the total number of buckets. The capacity of those buckets for containing water depends manifestly on the breadth of the wheel or the length of the buckets, as well as their sectional area. The area of those in the diagram, reckoning up to the level line bounded by the air-slit at their filling-point, and by the lip of the bucket where the discharge begins, may be taken at little above $1\frac{1}{8}$ square foot; and we may suppose, for facility of calculation, that their length or the breadth of the wheel is 1 foot, giving each bucket a capacity for containing $1\frac{1}{8}$ cubic foot of water, weighing about 70 lbs. The contents of the 10 buckets, therefore, weigh 700 lbs.; and this weight is constantly moving with the velocity of the wheel.

In determining the absolute power to drive machinery, we must ascertain the velocity in relation to that with which the water flows from the spout. The circumference of the wheel must move at such a rate that no bucket shall pass the spout without being filled from it. The total circumference of the wheel being $75\frac{1}{2}$ feet, divided into 40 equal parts, we have for the distance from lip to lip of each bucket $\frac{75\frac{1}{2}}{40} = 1{\cdot}888$ nearly, or about $1\frac{7}{8}$ foot. If the wheel makes 1 revolution per minute, each bucket passes any fixed point in $\frac{1}{40}$th of a minute, or $1\frac{1}{2}$ second; and the velocity of any point in the circumference is $\frac{75\frac{1}{2}}{60}$, about $1\frac{1}{4}$ foot per second. It has been stated that the most advantageous circumferential velocity of an overshot-wheel is at the rate of 2 to 3 feet per second. Taking $2\frac{1}{2}$ feet per second for the case we are discussing, the wheel would make 2 revolutions per minute, and each bucket would pass a fixed point in $\frac{3}{4}$ths of a second. As the water issuing from the spout has a certain depth or thickness, some part of the time of the bucket's passage must be deducted in order to ascertain the time allowed for influx of the water. Deducting $\frac{1}{3}$rd of the time, that is $\frac{1}{4}''$ from $\frac{3}{4}''$, we have $\frac{1}{2}''$ as the time during which the bucket remains under the spout to be filled; and in this time $1\frac{1}{8}$ cubic foot, the contents of the bucket, must flow from the spout—that is, $2 \times 1\frac{1}{8} = 2\frac{1}{4}$ cubic feet in 1 second. As the spout is 1 foot broad, and we must not reckon the depth of water in it above 6 inches or $\frac{1}{2}$ a foot, the sectional area of the water-channel

is ½ a square foot, through which 2¼ cubic feet must flow per second. The velocity of the water must therefore be $2 \times 2\frac{1}{4} = 4\frac{1}{2}$ feet per second. Should the velocity of the stream be less than this, either the wheel must move more slowly or the spout must be inclined to meet it at a lower level, so that the water may attain greater velocity from additional fall. Should the velocity of the stream exceed this, either the wheel must be permitted to move more quickly, or it and the spout must be widened, so as to present greater capacity of bucket and diminish the speed of influx. Recurring to the power of the wheel which we suppose to revolve at the speed of 2½ feet per second, or 150 fee per minute, with a force of 700 lbs. at its circumference, we find the effect to be equivalent to $700 \times 150 = 105000$ lbs. moved 1 foot per minute, $\frac{105000}{33000} =$ about 3⅙ horse-power.

Had we estimated it in another way by taking the quantity of water issuing from the spout, viz. 2¼ cubic feet per second, or 135 cubic feet per minute, weighing about 8840 lbs., and reckoning its fall or effective movement 15 feet, the distance descended while it remains in the buckets, we should have found the power to be $\frac{8440 \times 15}{33000} =$ about 3¾ horse-power. But it must be remembered that all the buckets do not act with their full leverage to turn the wheel in descending through 15 feet, being nearer the centre of the wheel above and below the middle level than when they pass that point. The former estimate, therefore, of 3⅙ horse-power is to be taken as the more correct one.

In order to ascertain what fraction it is of the power actually developed by the descent of the water—that is, of the force necessary to raise the water up again to the level whence it flowed—we have to consider that 2¼ cubic feet of water issue from the spout every second, and descend 22 feet, or that a weight of $2\frac{1}{4} \times 62\frac{1}{2} =$ to about 140 lbs. moves 22 feet per second, or $22 \times 60 = 1320$ feet per minute; or that $140 \times 1320 = 184,800$ lbs. moved through 1 foot per minute. This is equivalent to $\frac{184800}{3300} =$ rather more than 5½ horse-power. Of this the mill has been found to render 3⅙ horse-power, or about 57 per cent. available for driving machinery.

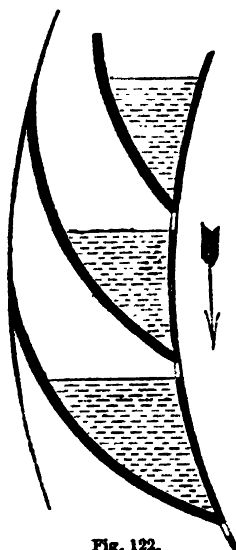


Fig. 122.

It has been stated by some engineers, that as much as 70 to 80 per cent. of the power expended by the fall of water has been made available by means of overshot-wheels; but we are inclined to think that, with the best known construction and proportions, the useful effect does not certainly exceed 70 per cent. of the water-power.

Of late years, many of these wheels have been made of iron: the partitions of the buckets are constructed of three iron plates bent to a curved form, and the obliquity is made considerably more than in the wooden shrouding of former times (Fig. 122). The diameter of such wheels is made somewhat greater than the height of fall, so that the water enters the buckets some distance below the summit, when the inclination of the bucket is suited for the reception of the stream. Even if the buckets were filled at the summit

of a wheel, and did not empty themselves till they reached the lowest point, the additional effect of their contents would be of little advantage, as it would act more to press the wheel down on its bearings than to turn it round. It will be found advantageous in practice to reckon the diameter of the wheel as $\frac{1}{8}$th more than the fall. Thus, for a fall of 24 feet, we should make the wheel 24 and $\frac{1}{8}$th of 24, 3; altogether, 27 feet in diameter.

It has been recommended that the velocity of the wheel should be made dependent on the height of the fall; that is to say, that it should be $\frac{1}{4}$th of the velocity which the water would acquire in reaching the bottom by free descent. We can see no reason why such a rule should be observed; for, as we have formerly stated, the velocity of the circumference should be so proportioned to that of the water flowing from the spout, that the buckets may be properly filled during their passage. It is true that, by inclining the spout, we may increase the speed of the stream flowing from it, and thus render a greater velocity of wheel practicable; but, being limited to a certain fall, whatever inclination we give to the spout, we take so much from the height after the water is delivered on the wheel, and consequently reduce the moving weight on the descending side of the wheel. We are, therefore, inclined to adhere to the maxim formerly received among millwrights, that the velocity of the wheel's circumference should not exceed 3 or 4 feet per second, and that, perhaps, it would most advantageously be fixed at 2 to 3 feet per second.

The number of buckets may be determined by making it double the number of feet in the wheel's diameter: thus, in a wheel 24 feet in diameter, the number of buckets would be 48. According to this rule, the space from lip to lip of buckets would always be about $1\frac{1}{2}$ feet. Where the stream in the spout is wide and shallow, it may be made less; and where the stream is deep, it should be greater. But, practically, its size within a few inches is of no great importance; and we should recommend that a division of the circumference by 6, 8, 4, or such numbers and their multipliers, should be made, so as to bring each division nearly to 18 inches.

In order to provide for the escape of air from the buckets, it is better to make their width exceed, by several inches at each side, that of the stream, than to provide air-slits in the sole; for, by this arrangement, each of the buckets may be made to hold a considerably greater quantity of water than when the air-slits limit its depth.

Every precaution should be taken to secure a free flow for the tail-water, as the resistance arising from the immersion of the lower part of the wheel in a languid stream takes considerably from its effective force. It is better to sacrifice a few inches of head by inclining the tail-course, so as to give the water some velocity (at least that of the wheel) in its escape, than to let it act as a drag on the wheel, by making the tail-course too nearly level.

3. *Breast-wheel and Chain of Buckets.*—All descriptions of wheels where the water is received on their circumferences, fall under the denomination of overshot-wheels, even if the water be not shot over their summits; indeed, according to the systems now pursued in rendering water-power available, there is no case where a really *overshot*-wheel should be adopted. Instead of making the diameter of the wheel less than the height of fall, so that the spout could be carried over it, the diameter should always be greater, as we have described, so that the water may be delivered at some point below the summit. Instead of an overshot-wheel, in some cases an endless chain of buckets (Fig. 123) has been employed for obtaining power from a fall of water. In theory, this arrangement appears one likely to prove more effective than that of the wheel, for the weight

of water is retained in the buckets, and acts with constant force throughout the whole descent. Practically, however, the apparatus is not of so substantial and permanent a character as the wheel; the chain has numerous joints, all subject to wear and decay from rust; and when they become deranged, the increased friction and inequality of action considerably diminish the efficiency of the apparatus.

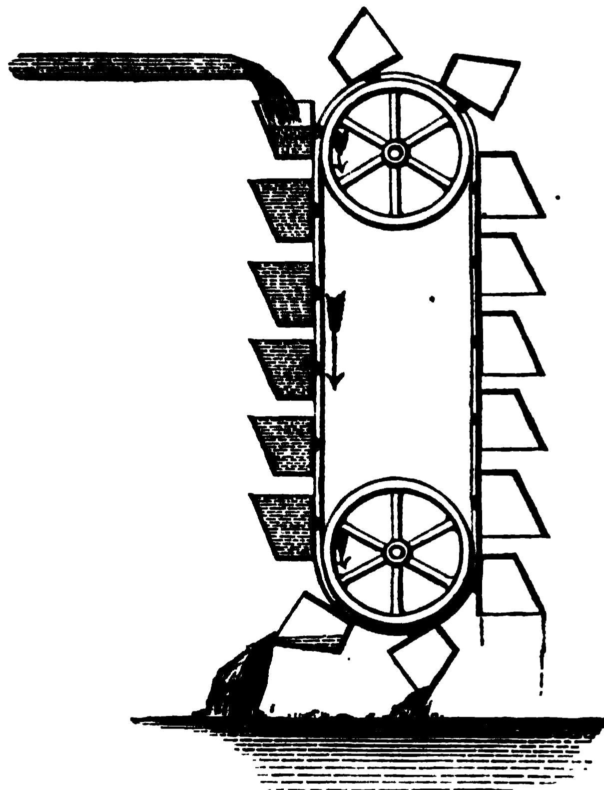


Fig. 1:3.

The breast-wheel is an arrangement intermediate between the undershot and overshot-wheels. It consists of a wheel fitted with floats or paddle-boards round its circumference, revolving with its lower part in a channel which nearly fits it. Each float has a back-plate or sole, so that the wheel is somewhat like an overshot-wheel with buckets open on their outer sides. When a considerable stream of water falls over a height not sufficient to render an overshot-wheel applicable, and yet greater than would be required for an undershot-wheel, the breast-wheel is applied with great advantage. The floats fit as nearly as possible, without rubbing-friction, to the bottom and sides of the channel, or sweep, in which they revolve; and thus, after passing the point where the water is delivered upon them, they act almost as close buckets, containing a load of water which urges them onwards.

At the point where the floats receive the water, some force arises from the impulse or velocity with which the water strikes them, as well as from the mere weight of their contents. Some millwrights have thought this impulse a most essential element of power, and have therefore contrived the spout so as to throw the water on the floats with great velocity. Others, and among them Smeaton, whose opinions on such matters are always to be received with reverence, have arranged the spouts so as to deliver on the wheel at as high a level as possible. By this arrangement the impulse from velocity

is lessened, but the height through which the water afterwards acts by weight is increased.

If we suppose that a certain stream, flowing with a velocity of 8 feet per second and having a fall of 8 feet, is applied to driving a breast-wheel 20 feet in diameter, having 40 floats (Fig. 124), we may inquire whether it be more advantageous to deliver the stream at once on the wheel, or to slope its course downwards 3 feet before it meets the floats. In the one case we have the impulse due to a speed of 8 feet per second on one float marked 9, and the weight of the water contained in eight others, marked 1 to 8 inclusive. In the other case we have the impulse due to the increased velocity of stream upon one float marked 7, and the weight of water acting on six others marked 1 to 6 inclusive. Farther, as in the second case the velocity of the delivered water is greater, its stream must be shallower, and therefore it must strike on a less area of float; and if the wheels move at rates respectively proportional to those of their streams, the same quantity of water being supposed to be delivered in each case, each of the buckets in the second wheel must contain less water than each of those in the first. Let us assume that in each case the wheel revolves at a rate which makes its circumference travel at one-third of the velocity of the stream, which we found to be the most advantageous speed for receiving impulse in the case of undershot-wheels. In the first case the velocity would be $\frac{8}{3} = 2\frac{2}{3}$ feet per second. In the second case we must calculate the velocity of stream due to increased fall. The fall to produce 8 feet per second is 1 foot; and adding to this the 3 feet of additional fall, we have a fall of 4 feet; the velocity due to which is 8 times its square root, or 16 feet per second. The circumference of the second wheel, then, travels at the rate of $\frac{16}{3} = 5\frac{1}{3}$ feet per second, twice the velocity of the first; and if in the first the buckets be exactly filled, in the second they can only be half filled, or need have only half the capacity. If we take the area of float in the first case 1 square foot, and in the second ½ square foot, the float marked 9 in the first sustains a pressure due to 8 feet per second the velocity of the water less by $2\frac{2}{3}$ feet per

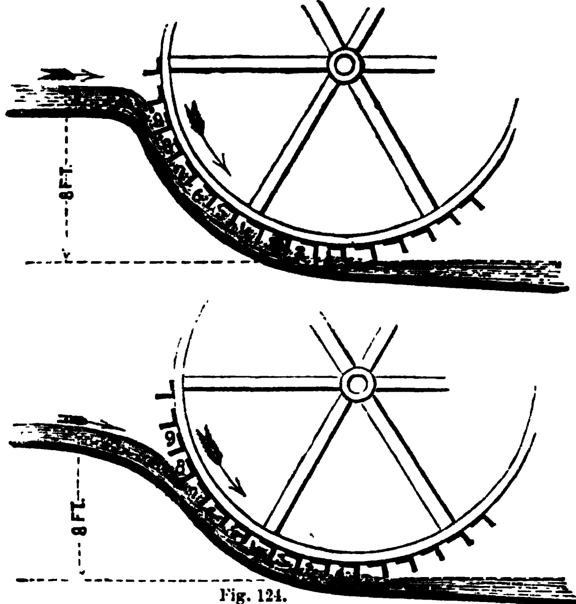


Fig. 124.

second its own velocity; that is to $5\frac{1}{3}$ feet per second, equivalent to a column $\frac{4}{9}$ths of a foot high on 1 square foot of area, about $\frac{4}{9}$ths $\times 62\frac{1}{2} = 28$ lbs. moving at the rate of $2\frac{2}{3}$ feet per second, or $2\frac{2}{3} \times 60 = 160$ feet per minute, which gives a power of $28 \times 160 = 4{,}480$ lbs. moving at 1 foot per minute. In the second case the float 7 is pressed on by a column sufficient to give 16 less by $5\frac{1}{3}$, that is $10\frac{2}{3}$ feet per second, which implies a height of $1\frac{7}{9}$ foot; and this pressing on $\frac{1}{2}$ square foot gives 56 lbs. moving at $5\frac{1}{3}$ feet per second, equivalent to 17,920 lbs. moving at 1 foot per minute, 4 times the effect of float 9 in the first case, as might have been surmised, because the velocity is doubled.

It remains now to compute the effect of the remaining floats in producing power. The total quantity of water issuing is 8 cubic feet per second, or $8 \times 62\frac{1}{2} \times 60 = 30{,}000$ lbs. per minute. In the first case this keeps 8 buckets continually full, and moves them at $2\frac{2}{3}$ feet per second, or 160 feet per minute; in the second case it keeps 6 buckets half filled, or 3 buckets quite full, and moves them at $5\frac{1}{3}$ feet per second, or 320 feet per minute. As each bucket holds 1 cubic foot, or $62\frac{1}{2}$ lbs., the power of those in the first case is $8 \times 160 \times 62\frac{1}{2} = 80{,}000$ lbs. moving 1 foot per minute; and of those in the second, $3 \times 320 \times 62\frac{1}{2} = 60{,}000$ lbs. Adding to each of these results the power derived from the impulse of the water, we have in the first case 84,480 lbs. moved through 1 foot per minute $= 2{\cdot}54$ horse-power; in the second case 77,920 lbs., equivalent to $2{\cdot}36$ horse-power. The result is, therefore, in favour of the first case; and thus Smeaton's view of the circumstances is borne out.

If the floats be tolerably well fitted to the sweep, so that there is little loss of water by escape past their edges, the circumferential speed of the wheel should be considerably more than one-third of that of the stream. A rate as high as two-thirds or three-fourths is practically attained with advantage. When this is the case, the impulse from excess of the stream's velocity over that of the float is much diminished, and the principal element of power is the load of the water contained in the buckets. If, then, the fall of the spout be made just sufficient to deliver the water supplied by the stream or reservoir, all the rest of the fall is most advantageously applied in the sweep, care being taken that sufficient fall is left to carry off the tail-water with full velocity, so that it do not become heaped up and retard the ascending floats.

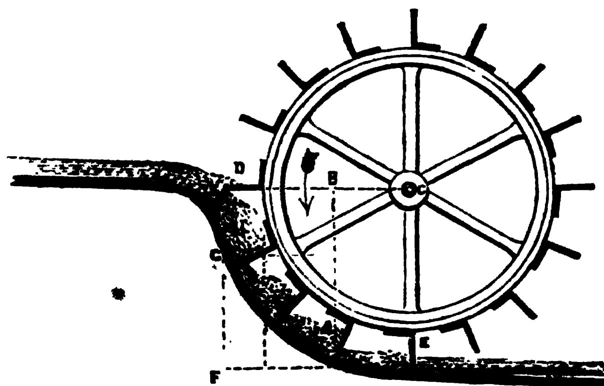


Fig. 125.

In estimating the power of a breast-wheel, we may suppose, for the sake of simplicity, that the water is delivered on the horizontal line of the centre, and keeps all the buckets, from that line to the bottom, full (Fig. 125). Now the effect of the weight of any bucket, such as A, to turn the wheel, depends upon the leverage with which it acts, which would be measured by the length C B of the horizontal line intercepted between

the centre and the middle of that bucket. Were we to divide the circumference from D to E into a great number of equal parts, and calculate their combined effect as dependent on the leverage with which they respectively act, we should find it to be the same as if one weight—bearing the same proportion to the total weight in the circumference D E as the length of C D, the radius, bears to the length of the circumference D E—acted at D. In other words, the effect of all the weight of water in D E to turn the wheel is the same as that of a column D F of the same width and thickness hanging at D. The same principle is true if the water do not deliver at the level of the centre; for if it delivered at G, the effect of the weight of water in G E would be the same as that of a column of equal area and of the height G F acting at D.

If, now, we take the particular case of a wheel 25 feet in diameter, with buckets 1 foot broad and 1 foot deep, receiving the water at the level of the centre, and making 3 revolutions per minute, we may compute its power, and the proportion which its useful effect bears to the expended power of the water. The buckets being 1 foot deep, the circle passing through their middle points would have a diameter of 24 feet, and therefore a radius of 12 feet and a circumference of $75\frac{1}{2}$ feet, making 3 revolutions per minute. The water in the buckets, therefore, moves at the rate of $75\frac{1}{2} \times 3 = 226\frac{1}{2}$ feet per minute; and the weight of the column, having an area of 1 square foot, and being 12 feet high, is $12 \times 62\frac{1}{2} = 750$ lbs. The power, then, is $\frac{750 \times 226\frac{1}{2}}{33000} =$ about $5\frac{1}{8}$ horse-power.

The quantity of water required to fill the buckets is $226\frac{1}{2}$ cubic feet per minute, for it must 3 times fill the whole circumference every minute; and as there must be considerable waste from the inaccuracy with which the floats fit the bottom and sides of the sweep in which they revolve, we may reckon 20 per cent. more, or altogether 270 cubic feet per minute, to cover this waste; that is, $4\frac{1}{2}$ cubic feet per second. If we take the stream at the spout 1 foot wide, and 9 inches or $\frac{3}{4}$ths of a foot deep, its area is $\frac{3}{4}$ths of a square foot, through which $4\frac{1}{2}$ cubic feet have to flow per second. The velocity of the water must, therefore, be 6 feet per second, or that due to a fall of nearly 7 inches. The water in working the wheel has to descend 12 feet, and we must allow at least 5 inches more of depth at the bottom of the wheel to clear the floats of back-water, and the total descent is therefore 13 feet: in other words, in order to raise the water up to the proper level to work the wheel, we should have to lift 270 cubic feet 13 feet high every minute. The power required for this would be $\frac{270 \times 62\frac{1}{2} \times 13}{33000}$ = about $6\frac{5}{8}$ horse-power. We found the effective power of the wheel about $5\frac{1}{8}$ horse-power; that is, 77 per cent. of the power expended. We believe that, practically, this estimate would be found too high, and that we could not depend on obtaining in useful effect more than 60 to 70 per cent. of the water-power expended.

Terminology.—The terms undershot, overshot, and breast-wheels have been applied in a somewhat different way from that in which we have used them. The term *undershot* has been used when the water is delivered on the wheel anywhere below the level of its centre, and thus the wheels which we have called *breast*-wheels would be among the *undershot;* the term *overshot* has been used in those cases only where the spout is actually carried over the summit of the wheel; and the term *breast* has been applied to wheels where the water is delivered somewhere above the central level. We think, however, that the classification we have adopted here is more distinct, as it refers

not only to the different points where the water is delivered, but also to differences in the construction of the wheels. Thus, the *undershot*-wheel is that which receives the water-pressure on simple paddles or floats immersed in the current, and is acted on by its force only; the *overshot*-wheel receives the water at a high level in buckets formed in its circumference, and is moved simply by the weight of water contained in them; the *breast*-wheel receives the water on paddles or floats nearly fitting a sweep in which they revolve, and is thus put in motion partly by the weight of water lodged on and between the floats, and partly by the pressure on the floats arising from the velocity of the current. The peculiar construction of each kind of wheel is adapted to different conditions of the fall of water. The undershot-wheel is to be used when there is a volume of water moving with considerable velocity, but with very little local fall, as in the case of river streams and tidal currents. The velocity of the current of a river arises from numerous little falls, or from a continuous inclination, without any considerable difference of level within a limited space. The velocity of a tidal current, again, arises from the pressure of the tidal wave, or body of ocean water, elevated above its average level by the gravitating influence of the moon; but this wave appears only as a gentle and almost imperceptible inclination of the water surface, except in some estuaries, such as the Solway Frith, where it presents itself as an elevated body of water rushing with considerable velocity towards the land.

The overshot-wheel is applicable when the water has a considerable local fall, nearly equal to the diameter of the wheel; and the breast-wheel when the local fall is not great—less, for instance, than half the diameter of the wheel—but when it is of considerable volume and moves with considerable velocity. In order to apply either an overshot or a breast-wheel, it is generally necessary to make extensive arrangements for conducting the water from an elevated level to the wheel, instead of permitting it to follow its natural channel. When a stream has a considerable fall—such as 40 or 50 feet in each mile of its length—a dam or weir is built across it at some convenient position, so as to check its progress there, and a new channel is formed for conveying its waters to the mill, and thence back to the bed of the stream at some point below the dam. As the artificial channel is made with only sufficient declivity to secure the flow of the water in such quantities as may be required, it is thus possible to obtain at the wheel nearly the total fall which the channel of stream has, estimated from the point where the dam is built to that where the tail-water of the mill re-enters. If, for instance, the stream in its natural channel is found to have a fall of 60 feet in a mile—this difference of level being made up either of numerous small local falls or of a continuous declivity, or both—an artificial channel is formed by its side, or as near it as the levels of the ground permit, having a constant declivity for half a mile amounting to 5 feet of difference of level; the water acts on a wheel with a fall of 20 feet, and a declivity of 5 feet is allowed in the length of the tail-course. The difference of level in the channel for half a mile—that is to say, 30 feet—is thus made up, and the power due to $\frac{2}{3}$rds of that fall is thus secured for driving machinery. The current of the stream itself would probably not have so great a velocity at any place as to make it practically available for an undershot-wheel, on account of the irregularities of its channel, and the numerous resistances opposed to its progress.

Reservoirs.—Where there is no stream of sufficient magnitude to give the necessary power, and the power is not required to be constantly in operation, it is usual to form a large dam or reservoir for collecting the constant small tribute of the stream, so that the volume of water thus accumulated may be employed to drive the mill as occa-

sion requires. For moving agricultural machinery, such as thrashing-machines, this plan is very commonly resorted to. The corn of a farm is generally thrashed in the winter season, when there is the best supply of water in the streams and from the drainage of the soil. A small mill-dam or reservoir collects during the night sufficient water to drive the mill through the following day. And it is thus possible, even in localities where apparently no water-power can be obtained, to secure enough for the work to be done, by executing a properly contrived dam, and turning into it the drainage of the surrounding fields.

When water is applied to manufacturing purposes, requiring the constant supply of large volumes of water, reservoirs or dams are sometimes executed on a gigantic scale, in order to store up against a season of drought the superfluous supply of rainy weather. In some of the hilly districts of England and Scotland, these works are of a most important character, and the interests of large local populations are dependent on their efficiency and permanence. When the differences of level in the district are considerable, numerous mills are worked successively by the same water, that which has driven the higher flowing along an artificial channel till it arrives at the next lower; and so on in constant succession for great distances. In such cases the mill-owners frequently combine to execute works for the benefit of all, and of much greater magnitude than a single capitalist could undertake. By such arrangements they are enabled to throw immense dams or retaining-walls across some valley, and can thus collect in a vast reservoir the drainage of an extensive range of hills, which would otherwise flow along its natural course to the sea without being turned to useful account as a mover of machinery. While the rains or melting of the snows contribute much more water than is required, the reservoir is filled; and when the water in it attains the highest level required, it is permitted to overflow into its natural channel. When the season of drought arrives, the mills, that would otherwise be at a stand-still, derive an ample supply from the reservoir, extending, in some cases, over many square miles of valley. When the depth of this reservoir is considerable, great strength is required in the dam; and, notwithstanding the ingenuity and labour expended on some of these structures, they sometimes give way; and the enormous volume of water thus suddenly set free rushes impetuously onward to the sea, devastating whole districts in its course, destroying crops and buildings, and too frequently causing a great sacrifice of life.

The sluices or valves for opening and closing the water-channel of a mill are generally of very simple construction; they consist of a plate of wood or metal made to fit against a framework fixed in the channel, and pressed against it by the water. This plate is made to slide upwards in grooves fitting it at each side; and when it is of large dimensions, it is raised or depressed by racks and pinions, or screws fitted with appropriate gearing. By opening or shutting the sluice, the wheel is put in motion or stopped at pleasure. A channel is always provided to carry off the surplus water to the tail-course of the wheel, when it rises in the spout or lead above the sill of the waste channel.

Regulators.—Various contrivances have been applied to regulate the speed of water-wheels. The most effectual is the steam-engine governor, or conical pendulum (Fig. 126). A throttle-valve, or plate moving on an axis or pivots at its middle, is fitted into the lead. When it presents its edge to the current, it offers very little obstacle to its course; but when it is turned into an oblique position across the current, it arrests all the water except what can pass through the openings left between its edges and the sides of the lead. As it is poised on pivots in its middle, the pressure of water on each limb is the

same; and the only force required to move it in either direction is that required for overcoming the friction of its pivots, and the resistance of the water to its movement through it, a force very inconsiderable. The governor is connected by rods and levers to a valve of this description in such a manner, that when from too great velocity of the water-wheel the governor-balls fly out from their axis, the valve is closed, or partly so, and the supply of water to the wheel diminished. When the wheel moves too slowly, the balls fall down to the axis, and cause the valve to open for the passage of a greater volume of water to the wheel. By this arrangement the movement of the wheel is regulated with great nicety, and the quantity of water supplied to the wheel is suited exactly to the work which it has to do. The consequence is a great saving of tear and wear to the machinery, and a regularity of movement better suited to almost all mechanical operations than variations in velocity.





Fig. 126.

The power which can be derived from a given stream of water may be computed with tolerable accuracy. When, by levelling the ground, it is ascertained how much fall may be secured, making ample allowance for the declivity of channel to and from the intended wheel, the volume of water delivered in a certain time is to be computed by measuring the area of the existing channel, and the velocity with which the water flows through it. The area of channel may be found by dropping a plumb-line, at numerous equal-measured intervals, across some part of the channel where the water moves with tolerably equable velocity, and tracing out the section according to the measurements so taken. The area can then be calculated by the ordinary rule for mensuration of superficies: for example, if the total width of the surface of the stream be 9 feet, and the soundings taken at every foot be those marked in Fig. 127 (in feet and fractions of a foot), the area is the sum of all those depths, viz. 8 square feet. The velocity of the current may then be ascertained by throwing into it a floating body at some distance above a marked length of channel, so that before it floats within the range of the marked distance it may have attained the speed of the current. The time of its passage over the marked distance may be then observed by a stop-watch. We may assume, for instance, that the marked distance is 20 feet, and that the floating body occupies 5 seconds in passing this; we conclude that its velocity is $\frac{20}{5} = 4$ feet per second. We must not, however, assume this to be the velocity of the whole stream, for at the bottom, and particularly at the shallow sides of the channel, the friction of the water on the rough surface considerably retards it. The effect of this retardation may be often observed upon a streak of foam spreading across the channel, the middle part

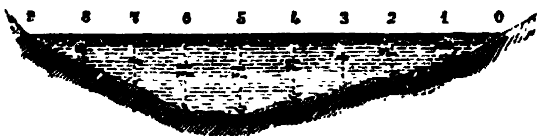


Fig. 127.

advancing rapidly and breaking away from the side portions, which sometimes are even caused to move backwards by the eddying currents. In a shallow stream, like that in the case we have supposed, we should not, perhaps, be safe in assuming more than half the middle surface speed as the average of the whole stream, say 2 feet per second. Multiplying this by the area, we find that 8 × 2 = 16 cubic feet of water per second is delivered by the stream. Having levelled the ground in the neighbourhood of the stream, we find that by building a dam 4 feet high at A (Fig. 128), and excavating a channel with a gentle declivity to B, a convenient site for a mill,

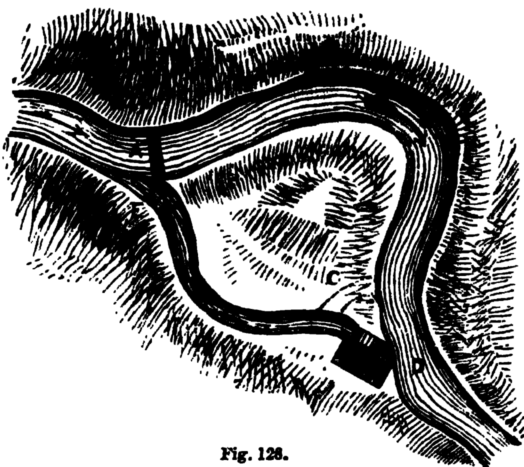


Fig. 128.

and thence into the bed of the stream, we can get a fall of 15 feet at the mill-wheel, allowing for declivity of channel from A to B, and a proper fall for the tail-water from B to the stream at D. We should, therefore, have 16 cubic feet of water per second falling down 15 feet of vertical height as our moving power. To lift this quantity up the height of its fall, we should require $\frac{16 \times 60 \times 62\frac{1}{2} \times 15}{33000} = 27$ horse-power. By a proper arrangement of wheel we may expect to secure about $\frac{2}{3}$rds of this for effective working power, which we should therefore estimate at $27 \times \frac{2}{3} = 18$ horse-power. A sluice would be fitted across the lead at E, so that the water might be excluded from it for purposes of cleaning or repairing the channel; and the proper sluice and regulating valve would be fitted at the mill with a sluice and waste water-course C for emptying the lead when the entering sluice at E is closed.

The Turbine-wheel.—When the volume of water is small, but the fall considerable, an apparatus called a turbine is frequently applied with great advantage. The principle of its action is similar to that of the well-known firework called the Catherine-wheel, or of the revolving jet sometimes applied to fountains. For a considerable period it has been known as a philosophical toy called Barker's mill. This consists of a vertical tube, with two horizontal branches closed at the end, mounted on a vertical axis on which it can freely revolve (Fig. 129). Near the extremity of the horizontal arms, holes A A are made on opposite sides; and when water is poured into the upper part of the tube, it flows through these holes, and makes the arms revolve in the opposite direction. The cause of their motion may be very simply explained. If we suppose the apparatus at rest, and the holes closed, when the tube and arms are filled with water, every square inch of the inner surface of those arms is equally pressed on by the column of water in the vertical tube; for it is the property of fluids to communicate pressure equally in all directions. Under these circumstances there is no tendency to produce motion in

any direction; but if the holes A A be opened, then, while the pressure on one side of the tube B remains the same as before, that on the other is lessened by as much as its surface is diminished. If we suppose each hole to have an area of one square inch, then each side of the tube B sustains a pressure on one square inch more than the other side; in other words, there is a pressure on B exceeding that on A by that due to the area of the hole in A. This excess of pressure causes motion in the direction in which it acts—that is, opposite to the flow of the water issuing from the holes; and the force of the movement depends upon the amount of unbalanced area in each arm, and the intensity of pressure upon it.

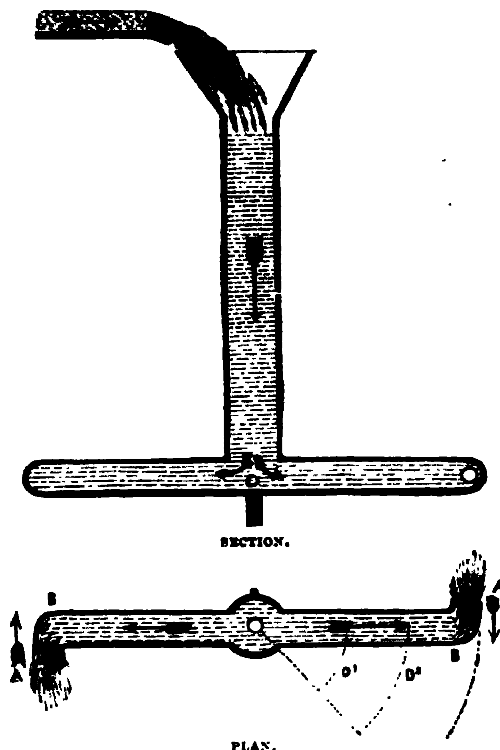
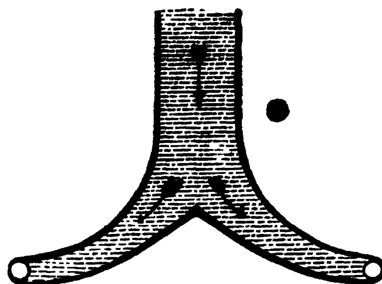


Fig. 129.

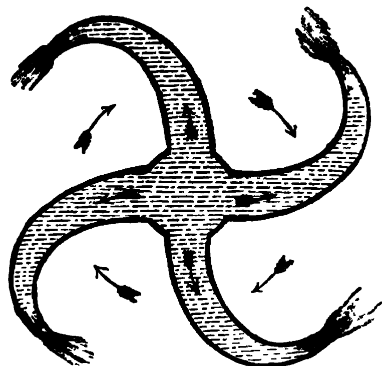
In the simple Barker's mill there is considerable loss of power from impediments to the flow of the water. The water descending the tube with considerable speed is suddenly arrested at O, and spread out laterally; losing by this angular bend a considerable part of its velocity. For the same reason it again loses speed in issuing from the holes A A; and, farther, a considerable part of the power is expended in giving the water a circular motion as it passes along the arms. A certain weight of water, as for instance 2 lbs., having descended the tube to O, has merely vertical motion; half of it (1 lb.) has suddenly to be turned at right angles along each horizontal branch, and is immediately put into circular motion with the arm in its revolution, as well as direct motion along the arm. The farther it flows along the tube, the more rapid is its circular motion; for if we take any points D_1 D_2 along the tube, and trace circles through them, we observe that the circumferences of these circles increase as their radii; and as each of the circumferences is passed over in the same period, the time of a revolution, the circular velocity of the water at each point increases in like proportion. In order to obviate these defects, the form of the tube and its arms is modified. Thus, the arms turn from the vertical to the horizontal direction by a gentle curvature, as seen in the section (Fig. 130), gradually changing the vertical movement of the water into a horizontal movement with little loss of force. And again, the arms, of which there may be any convenient number, bend also horizontally, as seen on the plan; so that while they revolve, the water contained

in them is really moving almost in a straight line, instead of being swept round in a circle.

As the mouths of the arms must be made of such an area as to permit the issue of the water at the proper velocity due to its fall, if they are made too small, less water passes them than can be supplied, and the machine is not so powerful as it might be with the given supply of water. If, on the other hand, the mouths are made too large, the velocity of the issuing water is diminished; and the pressure on the opposite sides of the arms tending to drive them round, is diminished with it. The area of the mouths being decided according to the quantity of water and its velocity from vertical fall, the arms, are made to taper gradually to that area, so that the velocity of the water may gradually increase to suit the gradually diminished area of its channel, as it would naturally do during its vertical descent. Due consideration having been given to these points, as well as to the best mechanical arrangements for strength, durability, and economy of execution, the machine becomes a turbine, practically applicable in many cases with great advantage. M. Fourneyron in France, and Messrs. Whitelaw and Stirrat in Scotland, have executed many of these machines, and made interesting experiments on their power and the best modes of constructing them. Their simplicity and efficiency, and the small space they occupy, give them an advantage over water-wheels; and it is said that they are capable of deriving from a fall of water quite as much effective power as wheels of the best construction, even if the volume of water be large. Experiments conducted by Morin in France lead to the conclusion that turbines are actually more effective than wheels under similar circumstances, the useful effect averaging from 70 to 78 per cent. of the power of the water. It has been found that even the immersion of the arms to a depth of several feet in water does not materially affect their action; so that even greater height than that of the fall measured to the level of the tail-water can be taken advantage of.



SECTION.



PLAN.

Fig. 130.

In estimating the power that may be derived from a given fall by means of a turbine, $\frac{3}{4}$ths of the power required to raise the water up again may be reckoned as the useful effect. Thus if the volume of water be 16 cubic feet per second and the fall 15 feet, the power required to raise it would be $\frac{16 \times 60 \times 62\frac{1}{2} \times 15}{33000} = 27$ horse-power; and the power of the turbine to drive machinery may be taken at $\frac{3}{4}$ths of 27, about 20 horse-power.

It is probable that turbines will in many cases take the place of water-wheels. They are as yet comparatively novel, and not widely known, or looked on with prejudice; but as improvements are gradually made in their construction and adaptation, and as they become more common in their application, these prejudices will doubtless give way, and no longer interfere with the extended use of a simple and elegant apparatus, instead of the large and cumbrous wheels now generally used. For regulating the motion of turbines, arrangements may be made similar to those used for governing water-wheels; the quantity of water supplied to the turbine being regulated according to the speed required, and the work to which it is applied. The power of the turbine is found to be very nearly proportional to the quantity of water passing through it, so that having found its maximum power, or the greatest quantity of water that it will use, we can employ $\frac{1}{2}$ or $\frac{3}{4}$ths of that power by reducing the supply of water to $\frac{1}{2}$ or $\frac{3}{4}$ths of the maximum.

Contrivances for Raising Water.—Before quitting the subject of water-power, we may notice some contrivances by which a volume of water is made to raise a smaller volume to a greater height for purposes of irrigation or the like. The simplest of these is the Persian wheel. A breast or undershot-wheel of the ordinary kind has a number of buckets hung on pivots to its circumference (Fig. 131). These buckets dip into the water at their lowest level, and being filled, are carried up one side till they come in contact with the side of a spout fitted near the summit of the wheel. They are canted over by this spout, and discharge their contents into it, to be conveyed away for their required purposes. The empty buckets descend on the other side to be again filled and lifted as before. Thus the force of a stream having inconsiderable fall is made to lift a certain quantity of water nearly the whole height of the wheel. The power of the wheel is expended on lifting a continuous weight of filled buckets up one side; and the quantity of water contained in them, as well as the height to which it is lifted, depend upon the size of the floats of the wheel and the pressure of the stream upon them.

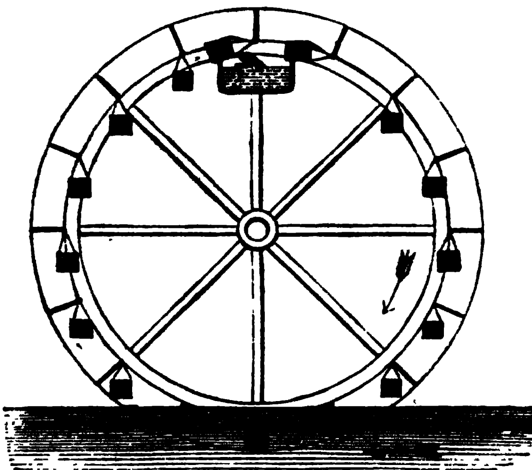


Fig. 131.

We may suppose a wheel 20 feet in diameter, with floats 8 feet broad and 1 ft. 6 ins. deep, worked at the circumferential velocity of 4 feet per second by a stream flowing at the rate of 12 feet per second. Subtracting 4 from 12, we get 8 feet per second as the excess of velocity of the stream over that of the float of the wheel; and the pressure due to that excess is ($\frac{8}{8}$ or 1 squared) that of a column of water 1 foot high. The area of the float is $8 \times 1\frac{1}{2} = 12$ square feet; therefore the total pressure on the float is equivalent to 12 cubic feet of water, and it moves at the velocity of 4 feet per second, or

240 feet per minute, giving the same force as $240 \times 12 = 2880$ cubic feet of water moved 1 foot per minute. We may deduct $\frac{1}{3}$rd of this to allow for various losses, thus leaving an effective power equivalent to 1920 cubic feet of water lifted 1 foot high per minute, or $\frac{1920}{20} = 96$ cubic feet lifted 20 feet (the height of the wheel) per minute. As the wheel's circumference may be taken at 60 feet moving at the rate of 120 feet per minute, the wheel makes 2 revolutions per minute, and therefore twice in every minute lifts the contents of all its buckets, amounting to 96 cubic feet, as we found above. The buckets may therefore altogether contain 48 cubic feet; and if their number be 24, each may have a capacity of 2 cubic feet. If we estimate the quantity of water acting on the wheel as the area of the float multiplied by the velocity of the stream, it appears that 8 ft. $\times$ 1$\frac{1}{2}$ ft. $\times$ 12 ft. $= 144$ cubic feet passes per second, or $144 \times 60 = 8640$ cubic feet per minute, of which 96 or $\frac{1}{90}$th part is lifted 20 feet high by the wheel which it moves.

The hydraulic ram is an ingenious contrivance, by which a small fall of a considerable body of water is made to raise a much smaller volume of water to a considerable height. From a reservoir or dam A at the height of a few feet above the lower level of the stream at B, a large pipe conducts the water; this pipe has an aperture D on its upper side near to its lower end, and the aperture is closed by a valve opening upwards

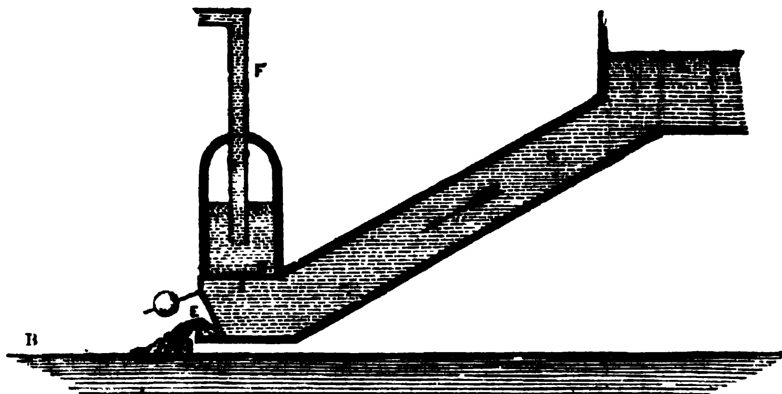


Fig. 132.

into an air-vessel, from which a small pipe F leads to a cistern at a level considerably above that of A. At the lower end of the inclined pipe there is a hinged valve E, opening inwards, and kept open by a weight fixed on a lever projecting from the valve. This weight is adjusted nicely, so as to counterbalance the pressure of the water on the surface of the valve E, but not greatly to exceed it. When the weight opens the valve, the whole of the water in the inclined pipe C begins to flow downwards, and issue at the opening made by the valve at E. Having acquired a certain velocity, it presses with greater force on the valve, and closes it in opposition to the weight, thereby completely arresting its own flow; but the momentum of the large body of water flowing along the pipe C cannot be suddenly destroyed, but must expend itself somewhere. It therefore lifts the small valve D with considerable force, and part of it flows into the air-vessel and up the pipe F. The momentum being thus absorbed, and the water in

the pipe C having become still, the valve E again is opened by the weight, and the operation is repeated. Thus, by the alternate opening and closing of the valve E under the quiescent and moving pressures of the water, a certain portion of the water is forced up the pipe F, and is prevented from returning by the closing of the valve D. The object of the air-vessel is to provide an elastic spring for the water propelled upwards: every time that the water is injected into it, the air in its upper part is compressed into a smaller space; and being perfectly elastic, tends to resume its former volume. It therefore exerts a pressure on the water, and continues its flow along the pipe F during the intervals that elapse between the successive discharges through the valve D. In estimating the power of this apparatus to raise water, we may suppose it arranged with the flow-pipe vertical instead of inclined, as it is usually made for convenience, the principle not being altered, but the details of calculation simplified by the vertical arrangement (Fig. 133). We may suppose that the height from the valve to the surface of the water in A is 4 feet, and that the area of the valve E is 1 square foot. The pressure on the valve is, therefore, the weight of 4 cubic feet of water, namely, $4 \times 62\frac{1}{2} = 250$ lbs. The effect of the weight to lift the valve by means of its lever must be somewhat greater than this. If the valve could be suddenly lifted so as to leave an opening to the full extent of its area, 1 square foot, the water would descend in C at the rate of 16 feet per second, the velocity due to 4 feet head; but as the valve opens gradually, and is only open for a short time, and that not to its full extent, we may take the velocity of the descending column at not more than $\frac{1}{4}$th of this rate; that is to say, at 4 feet per second. When this movement ensues, the valve is pressed on not only by the weight of the column above it, as before, but also by the weight of such a column as is due to a velocity of 4 feet per second, which will be found by calculation to be $\frac{1}{4}$th of a foot high, adding $62\frac{1}{2} \times \frac{1}{4}$, about $15\frac{5}{8}$ lbs. to the load on the valve. This additional load overcomes the leverage of the weight, and closes the valve; but the column of water, 1 foot in area and 4 feet high, contained in C, amounting to 4 cubic feet, is thus arrested whilst moving at the rate of 4 feet per second; and its momentum, which is equivalent to that of $4 \times 4 = 16$ cubic feet, moving at 1 foot per second, or 1 cubic foot at 16 feet per second, must be given out as a force propelling the water along the small pipe at the side. If we suppose that this pipe communicates with a cistern 36 feet high, the velocity due to that height is 48 feet per second; and the momentum of 1 cubic foot, moving at 16 feet per second, being equivalent to $\frac{1}{3}$rd of a cubic foot moving at 48 feet per hour, we should expect that this quantity—$\frac{1}{3}$rd of a cubic foot—would be propelled upwards to the high cistern at each closing of the valve. We must, however, recollect that the momentum of the larger volume has not only to balance that of the smaller,—it must considerably exceed it, because it has to lift the valve, to give the motion upwards to the column, to overcome friction in the pipes and other impediments, and upon the whole may be reckoned effective to the extent of not more than $\frac{1}{4}$th of its estimated force. We may estimate, then, that $\frac{1}{12}$th of a cubic foot is propelled

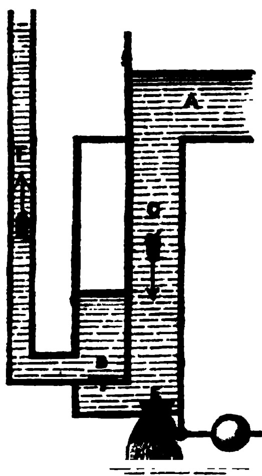


Fig. 133.

up to the high cistern by the descent of 4 cubic feet through 4 feet from the lower cistern. Should the action not last so long as a second, a smaller volume will descend, and a proportionally smaller quantity will be sent upwards, and conversely; but the number of times the descent and ascent take place in a given period will be greater or less accordingly.

We have presented this calculation only as a rough approximation to the practical results. We are not aware that any carefully-conducted experiments with hydraulic rams have been given to the world, and we therefore do not venture to offer any estimate as a guide for practice; but have merely discussed a case with the view of opening the questions that have to be considered in dealing with such apparatus. In many situations where it may be desirable to raise water for the purpose of ornamental fountains, or of domestic supply, the hydraulic ram is applicable with great advantage. A neighbouring stream may be dammed so as to provide a fall of a few feet, if it has not sufficient local fall naturally; and the apparatus once fixed and properly adjusted will continue effective for a long period. It is exceedingly simple, entirely self-acting, and seldom liable to derangement, if care be taken to fix gratings on the pipe so that dirt or extraneous matter of any kind may be prevented from interfering with the action of the valves.

3. **Weight and Elasticity of Bodies.**—The gravitating attraction which the earth exerts on bodies near its surface, and the force with which elastic bodies tend to resume the condition from which they have been withdrawn, are frequently employed for giving motion to machinery, chiefly when the power required is small, but exerted for considerable periods. Weight and elasticity are not really sources of power; they rather afford the means of storing up efforts of short duration for subsequent use during longer periods. Before a body can act by its weight, it must be lifted to the height whence it has to descend; and, in the same way, before a body can act by its elasticity, its condition must be changed to the extent through which it has afterwards to return. Thus, in winding up a clock or watch, as much power is exerted as is afterwards given out by the weight or spring in moving the machinery with which either is connected. The weight of the clock and the spring of the watch are merely instruments for absorbing at the moment a certain amount of power, and giving it out by degrees afterwards. The simplest, and, it may be said, the only general mode of employing the weight of a solid body to give impulse to machinery, is to attach it by a chain or string to a cylindrical barrel mounted on bearings at some height from the ground, and connected by gearing with the machinery which it is intended to move. The barrel being caused to revolve by hand, or any other convenient power applied to it, the string is wound round it and the weight raised from the ground. When left to itself, the weight, attracted again towards the earth, descends, unwinding the string from the barrel, causing it to revolve, and thus giving the necessary impulse to the machinery. Did the machinery offer no resistance, the weight would always descend with a speed accelerated at every moment of its descent by the continued action of gravity, which exerts as much influence on a body in motion as on one at rest. In dropping a stone from a considerable height, whatever be its weight, we find that during the first second of its descent it acquires a velocity of 32 feet per second. Its velocity at the commencement was nothing, for it began to move from a state of rest; at every one of the instants into which we may conceive a second of time divided, it acquired more and more velocity, until it attained the final velocity of 32 feet per second. All these acquisitions in speed are equal in equal times, because the force of gravity is constant, and therefore exerts equal influences in equal times.

Had the body descended during the whole second at the final velocity of 32 feet per second, it would of course have passed through 32 feet of space; had its velocity remained the initial velocity, which was nothing, it would have descended through 0 feet; but as the velocity began with 0 and ended with 32, its average throughout the second was 16 feet per second; and therefore the body descends in the first second through 16 feet. During the next second, the body, starting with a velocity of 32, acquires an additional velocity of 32, and therefore ends with a velocity of 64 feet per second; the average being 48 feet per second, and therefore the descent being 48 feet of height. Adding this to the space descended during the first second, 16 feet, we find that in the first 2 seconds the total descent is 64. Were we to pursue the investigation farther, we should find the velocity at the end of the third second 96 feet per second, and the total descent 144 feet, and so on according to the following law:—The velocity (in feet per second) acquired by a falling body is 32 times the time (in seconds).

The space (in feet) passed through by a falling body is 16 times the square of the time, or the square of 4 times the time. Hence it follows that the time (in seconds) occupied by the descent of a falling body through a given height (in feet) is $\frac{1}{4}$th of the square root of the height; that the velocity (in feet per second) is 8 times the square root of the height; and that the height is the square of $\frac{1}{8}$th of the velocity.

Example 1.—Required the velocity acquired by a falling body in 5 seconds.

$$32 \times 5 = 160 \text{ feet per second.}$$

Example 2.—Required the height fallen by a body in 5 seconds.

$$16 \times 5 \times 5 = 400 \text{ feet, or } (4 \times 5 =)\ 20 \times 20 = 400.$$

Example 3.—Required the time occupied by a body falling through 400 feet.

Sq. root of 400 is 20, and $\frac{1}{4}$th of 20 is 5 seconds.

Example 4.—Required the velocity acquired in falling 400 feet.

Sq. root of 400 is 20, and 8 times 20 is 160 feet per second.

Example 5.—Required the distance fallen by a body when it has acquired a velocity of 160 feet per second.

$$\frac{160}{8} = 20, \text{ and } 20 \times 20 = 400 \text{ feet.}$$

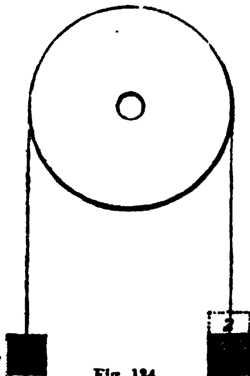


Fig. 134.

These rules only apply correctly when a body falls in vacuo, for the resistance of the air materially retards the velocities, especially when they become considerable, and when the body has considerable bulk in relation to its weight. Were it not for this resistance, every rain-drop, descending as it does from a height of many hundred feet, would strike with a force as fatal as that of a rifle-bullet.

When part of the weight of a falling body is balanced by the ascent of some weight connected with it, its velocity at each movement is materially diminished. When two equal weights, each 3 lbs., are hung by a string over a pulley (Fig. 134), they exactly balance each other—that is to say, the gravitating attraction exerted on one is equal to that exerted on the other; and as the one cannot descend, in accordance with this force, without causing the other to ascend in opposition to it, no motion takes place; but if we add to the one 2 lbs., we apply to that side the gravitating force of 2 lbs., and the motion takes place in obedience to it. The force of 2 lbs. has, however, not only to put

in motion its own mass, but also that of the two weights, in all 8 lbs., or 4 times its own weight; and therefore the velocity which they all acquire in 1 second can only be the $\frac{1}{4}$th of that which the 2 lbs. alone would acquire—that is to say, 8 feet per second instead of 32. The distance passed over must therefore be only $\frac{1}{4}$th of what it would be were the moving weight left to itself without having to put the others in motion. In all such cases, then, of weights partly balanced, if we know the total of the weights put in motion, and the excess of the one over the other, we find the velocity or the space in a given time by the following rule:—

Find the space or velocity as before, multiply by the excess of one weight, and divide by the sum of all.

Example 6.—Required the space passed through in 4 seconds by a weight of 19 lbs. connected by a string over a pulley with one of 13 lbs.

The excess of the one is 6 lbs.; the sum of both is 32 lbs. The space would be for a single weight, $16 \times 4 \times 4 = 256$ feet, and

$\frac{256 \times 6}{32} = 48$ feet is the space descended in 4 seconds under the circumstances given.

It does not often happen that in practice two different weights are hung over a pulley as we have just described. The effect, however, is just the same if there be one weight acting on a barrel, and if the movement of the barrel be resisted by some known force. If, for instance, we had a weight of 19 lbs. attached to a barrel connected by machinery with some other load, and we found that 13 lbs. hung to the barrel instead of 19 lbs would exactly balance the load upon the machinery, so that no movement of the barrel took place, we should estimate, as we have done above, that 19 lbs. had to set in motion itself and 13 lbs., the drag of the machinery, and that the space passed through in four seconds would be 48 feet, as we have computed. All such estimates, however, are made without regard to the resistance caused by friction. The rubbing surfaces of machinery are so irregular, and subject to such changes of condition by wear, temperature, deficiency of lubrication, and other causes, that friction cannot be estimated as a regular resistance. No machinery moved by a weight, without some special regulating apparatus, can be expected to move with uniform velocity, or with velocity varying according to any uniform law, on account of these irregularities of resistance The law of motion which a falling body obeys is, therefore, scarcely ever practically applicable In all machinery moved by weights, some contrivances are introduced for providing a resistance so much greater than that of the mere friction, that the weights upon the whole may be made to descend uniformly, and not with the accelerated velocity due to gravitating force alone. Thus, in the time-keeping part of a clock, the weight which puts the whole in motion is arrested at every moment of its descent by the pendulum and escapement. The motion of the weight downwards becomes in this case a series of extremely short descents, recurring at equal intervals, whenever the pallets of the escapement permit the train of wheels to move. The weight employed is so much in excess of the resistance from friction, that the irregularity in the time of each of its partial descents is exceedingly small. The regularity of the clock's motion, therefore, depends upon the uniformity of the times which the pendulum occupies in each of its beats, the interval during which the weight descends and the train moves being too small for any irregularity to manifest itself. In the striking part of a clock, there is no necessity for extreme regularity. The whole striking train is kept at rest until the time-keeping part comes to such a point, that it removes the obstacle to the motion of the striking-weight. This, being relieved, begins to descend,

and would go on descending with accelerated velocity, moving its train of wheels, and causing the hammer to strike the bell faster and faster, were not a resistance provided sufficient to prevent the acceleration from becoming too great. The apparatus generally used for giving this resistance consists of a small revolving fan or wheel, with flat blades, caused to rotate very rapidly by the striking train. The resistance which the air offers to the quick passage of the fan-blades through it increases as their velocity increases, but in a much higher ratio; and gradually as the weight becomes accelerated in its descent, and the fan consequently rendered more rapid in its rotation, the resistance of air to the fan becomes as great as the effort of gravitating attraction on the weight. When this velocity has been attained, the weight continues to descend with nearly uniform speed, and the strokes of the hammer on the bell are made to succeed each other at nearly equal intervals. These arrangements are sufficiently complete for their purpose; for, though a perfectly uniform motion is not attained, the speed is so nearly regular that the ear does not appreciate any marked difference in the intervals between the strokes. When a perfectly uniform motion is required, as it is sometimes for measuring intervals of time accurately, as in astronomical observations, it is necessary to have arrangements more delicate, so that all irregularities may be properly compensated. A very ingenious apparatus was contrived some years ago by Mr. Froude, of Dartington, Devon, for giving perfectly uniform velocity of rotation to a cylinder. This motion was necessary for enabling the inventor to obtain diagrams exhibiting the flow of air into a vacuum, the propelling power of a screw on a boat fitted with it, and other interesting mechanical phenomena. In one of his apparatus he employed a flat disc B (Fig. 135) at the end of an arm suspended from a vertical axis A, which was caused to rotate by a train of wheels connected with a weight and with the cylinder, which was required to be uniformly moved. The disc revolving through the air offered a certain resistance, depending on its velocity; and if at any instant the velocity increased, the disc immediately moved outwards from the vertical line, owing to its increased centrifugal force, to some such position as that indicated by the dotted lines. When at this distance, the radius of its orbit D E was greater than the former radius B C; and therefore the speed with which the disc was driven through the air was proportionately greater, and the resistance of the air to its motion increased accordingly. The momentary increase of velocity, therefore, produced a considerable increase of resistance, and thus reduced itself to its average rate; and, conversely, a decrease of velocity diminished the resistance, and thus restored itself. By a proper adjustment of the disc as to area, the length of the arm by which it swung, and a weight on that arm, the apparatus served to move the cylinder for many minutes with a velocity in which there was not the slightest appreciable irregularity. A piece of paper was fixed on the cylinder, and a pencil, connected with a spring-balance acted on by the pressure to be measured, traced on the paper as it revolved a diagram, indicating the intensity of the pressure at every instant of the motion.

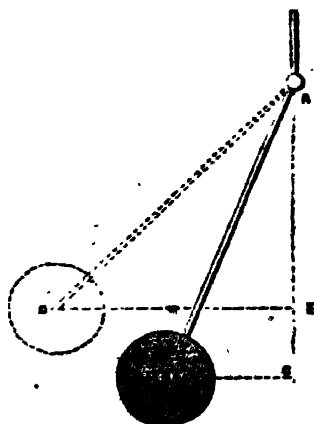


Fig. 135.

When the elasticity of a solid body is employed as a force for moving machinery, the body is called a spring, and generally consists of a strip of steel wound into a spiral form round an axis A, as shown in Fig. 136; one end of the strip being fixed to the axis, and the other at B to the side of the cylindrical box in which the spring is contained. When the box is fixed, as in a Geneva watch, the axis is turned round by a key, and the spring is thus drawn into closer convolutions. A wheel is fixed on the axis, and is connected by gearing with the escapement and hands of the watch. The effort of the spring to unwind itself causes the axis to revolve in the direction opposite to that in which it was wound, and thus puts the train of wheels in motion. It is the property of all elastic bodies to exert a force which is nearly proportional to the amount of strain to which the body has been subjected. As we wind up the spiral spring of a watch, we apply greater and greater strain the farther we wind; and so when the spring unwinds itself it exerts the greatest force at first, and gradually decreases in power the farther it is unwound. This decrease of force is an irregularity which no modification of the form of spring can obviate, but which may be considerably diminished by making the spring with a great number of convolutions. Thus, we may suppose that a spring of 20 convolutions, or turns, is wound up from its neutral condition to the full power required by 10 revolutions of the axis. In unwinding itself it would lose by the first revolution $\frac{1}{10}$th of its force, by the second revolution another $\frac{1}{10}$th, and so on, until by five revolutions backwards it would lose $\frac{5}{10}$ths or $\frac{1}{2}$ of its full force. But if we suppose the spring had forty convolutions, and that we wound it up by 20 revolutions of the axis to the same force as the former spring, in unwinding itself it would lose by the first revolution $\frac{1}{20}$th, by the second another $\frac{1}{20}$th, and so on till by the fifth it would lose $\frac{5}{20}$ths or $\frac{1}{4}$th of its full force. The decrease of power in the second instance is only half that in the first, the axis performing in each case an equal number of revolutions. It is therefore important to give all springs which are required to exert a tolerably equal force throughout their recoil as great a number of convolutions as possible, and to employ as few revolutions of the axis as possible for moving the machinery.

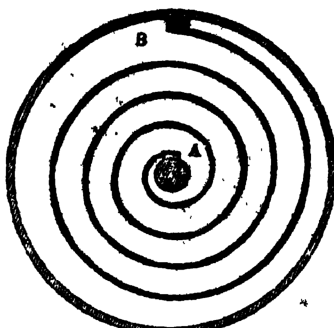


Fig. 136.

In other watches, where smallness of bulk is not so much studied, and in clocks moved by springs, the axis of the spring is fixed, and the box *c* (Fig. 137) containing it is caused to revolve by the recoil of the spring, and to wind upon its outer surface a small chain *d* attached to a fusee *a*. This fusee is a conical barrel, with a screw-groove cut in its surface to receive the coils of the chain. When the spring is beginning to recoil, and therefore acting with its greatest force, the chain pulls the fusee round at is upper part where the diameter is smallest, and therefore acts with least leverage on the train. As the spring loses force, the chain is pulled from a larger diameter of the fusee, and acts with

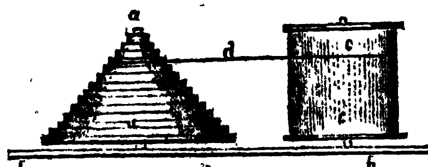


Fig. 137.

more and more leverage on the train as its tension becomes less. By properly proportioning the increase in the diameter of the fusee to the diminution in the force of the spring, the power applied to the train can thus be made perfectly regular throughout the whole range of the spring's recoil. The fusee is not a regular cone, the outline of its section being part of a hyperbolic curve, that being the form which furnishes the proper proportion of increase of diameter to diminution of spring force.

In machinery moved by springs, modes of regulating the speed may be employed as in machinery moved by weights. Thus, in spring-watches and clocks the balance-wheel and pendulum are used for governing the time-keeping train, and the fan for governing the striking part. In musical-boxes the fan is also employed, and serves to give the movement of the barrel sufficient regularity to suit the regular time of the music.

Mr. Froude, to whom we have already referred, contrived a very ingenious regulating apparatus for machinery moved by a spring, employing the resistance of the air as a retarding force in a manner somewhat similar to that we have described, but with a difference in the details, made with a view to render the apparatus portable and capable of acting justly without regard to level. The regulating part of the apparatus consisted of an axis with a longitudinal slot cut through it, mounted in bearings at A A (Fig. 138), and connected with the train of machinery. Within the slot there was fitted a short transverse spindle B, to which were fixed two thin blades, kept in the position indicated on the figure by means of a small spring made to act on their spindle. When this was made to rotate with considerable velocity in the bearings A A, the blades tended, by their centrifugal force, to fly outwards to some such position as that marked by the dotted lines, and thus encountered more resistance from the air. By the proper adjustment of the area and weight of these blades and of the spring which acted on their spindle in opposition to their centrifugal force, extreme regularity in the motion of the train was maintained.

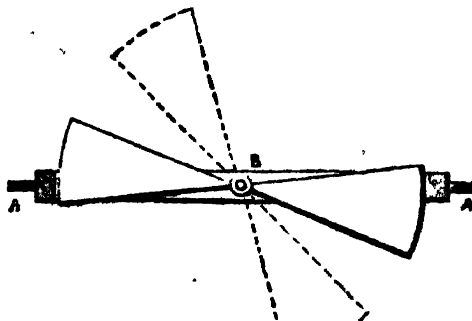


Fig. 138.

In general, for moving machinery with regular speed, weights are preferable to springs, because the force of a weight is constant, while that of a spring varies according to its tension. But weights act only vertically, while springs act in any direction. For all portable apparatus which cannot be maintained in a constant level so as to make the direction in which the weight acts constant with relation to that on which it acts, springs must be employed. And so for the regulation of portable apparatus, escapements in which the elasticity of a spring is used for recoil, instead of the weight of a pendulum, must be likewise employed. The details of all such apparatus, and the many ingenious contrivances applied to them, form the subject of a distinct art, that of Horology, which will be treated of hereafter.

Besides contrivances for employing the weight and elasticity of solid bodies for

giving motion to machinery, there are others by which the weight or pressure of liquids and the elasticity of gases are also applied for such purposes.

Among these the hydraulic lift and the hydraulic crane may be particularly noticed. The hydraulic lift, in its most simple form, consists of a cylinder closed at bottom, and fitted with a plunger or ram, which passes through the top and supports a stage (Fig. 139). A leather collar, fitting round the plunger, is placed in a recess provided in the neck of the cylinder, so as to prevent the escape of water around the plunger. A pipe from a high cistern conducts water into the cylinder, and another pipe permits the water to issue from it. Each of these pipes is provided with a stop-cock, so that the water may be admitted to the cylinder, or allowed to flow from it at pleasure. Since liquids communicate pressure equally in all directions, every part of the cylinder and the plunger is pressed upon by a force proportioned to the height of the cistern which supplies the apparatus. Every 27 inches of height produces a pressure of 1 lb. on every square inch of surface exposed to it; and by making the area of the bottom of the plunger sufficiently large, a considerable weight can be raised on the stage which it carries. Thus if the cistern at the top of a house be 54 feet above the ground-floor, the pressure on the internal surface of any vessel on the ground-floor connected with it by a pipe is 24 lbs. per square inch. If this vessel be a cylinder, such as we have described, fitted

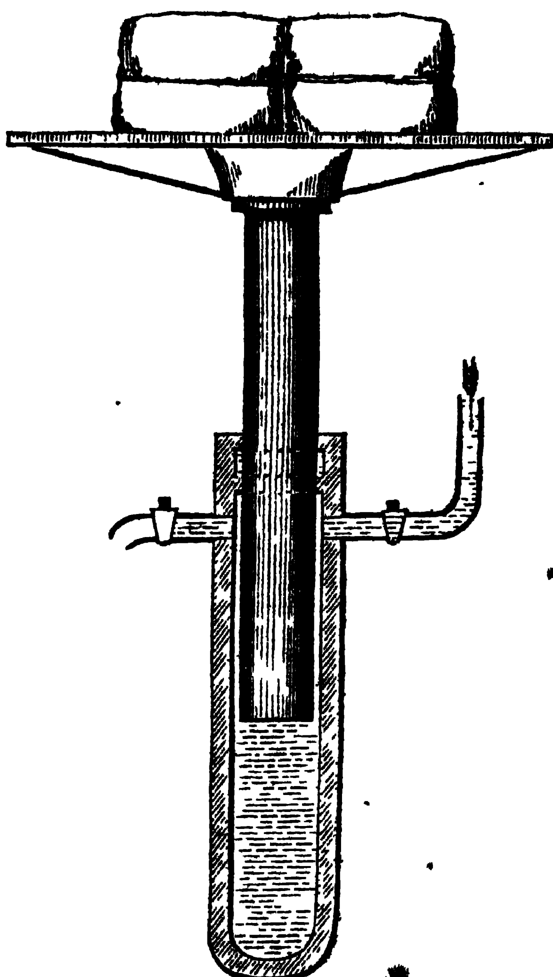


Fig. 139.

with a plunger 10 inches diameter, having therefore a sectional area of 78½ square inches, the pressure forcing this plunger upwards amounts to 78½ × 24 = 1884 lbs. and if from this we deduct 764 lbs. as a weight exceeding that of the plunger and platform, there remains a pressure of 1120 lbs. forcing the plunger upwards. A load then

of this amount, half a ton, may be placed on the platform when it is in its lowest position, and the cock being opened to the cistern, the whole will be raised to a height equal to that of the plunger, which may be made sufficient for lifting goods from one floor to another. By closing the cistern-cock and opening the other, the water is permitted to leave the cylinder; and the plunger, no longer subjected to upward pressure, descends to its former position.

The water-pressure may be made to act in a way somewhat different in detail, but similar in principle, by fitting the cylinder with a piston, connected by a rod passing tightly through the cylinder cover with a rope or chain, which may be led by pulleys in any convenient direction for lifting weights attached to it. The water-pressure acting on the piston, forces it down the cylinder, and thus pulls the chain and lifts the weight. Such an arrangement constitutes the hydraulic crane. The advantage of employing water for lifting weights in this manner consists in the circumstance that a small steam-engine, or other power, can be constantly employed in raising the water to a high cistern, and that great lifting power can be obtained for a short time by the expenditure of a portion of the water thus raised. The water lifted to a height becomes, in fact, a reservoir of power, ready to be used when required, and accumulates the efforts of a small power acting through a considerable period, ready to be expended as a great power acting through a short time. Farther, this power is completely under control, for by the mere turning of a stop-cock, or the opening or closing of a valve, it can be put in action or arrested at pleasure, and the speed with which a weight is moved, and the height to which it is raised, can be regulated with the greatest nicety.

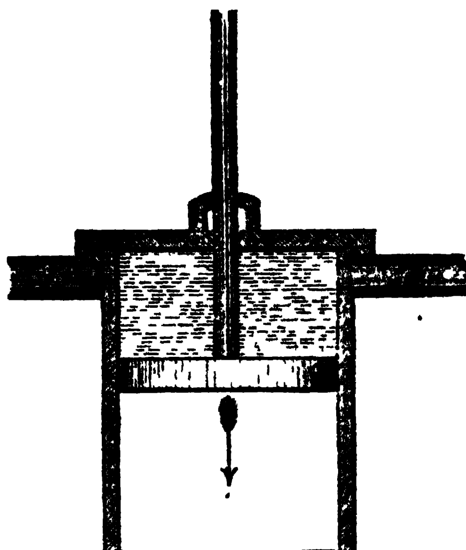


Fig. 140.

Apparatus have sometimes been applied in which water flowing from a high level, or compressed air, may be made to act as steam in a steam-engine, for giving motion to machinery. The construction of such apparatus is very similar to that of the steam-engine, and may be easily understood by one who has studied the arrangement of the latter.

The pressure of the atmosphere has also been employed as a moving-force in the case of the atmospheric railway. We must, however, defer any consideration of this subject until we shall have discussed the steam-engine.

4. **Heat and Electricity.**—In treating of heat, electricity, magnetism, and chemical action, as sources of power, we embrace a very wide range of matter, which may with advantage be subdivided. We may here merely notice the branches which this division of the subject embraces.

Heat is the power which is applied in the steam-engine, the most advantageous and widely-employed of all apparatus for rendering natural powers serviceable to man. This branch of practical mechanics will therefore demand considerable space for its special discussion.

Electricity, magnetism, and chemical action are the powers applied in the electric-telegraph; and in some apparatus that have been contrived for employing them in moving machinery. The electric-telegraph has attained an importance, and presents so many points for consideration, that we must give it also special consideration in another place. The attempts that have been made to render electricity, magnetism, and chemical action available as prime movers of machinery, have not been crowned with much success We do not think it necessary, therefore, to devote space in a work on practical mechanics to the discussion of contrivances which have not been made practically available. It is not to be denied that the phenomena attending electricity, magnetism, and chemical action, present forces which could be and have been applied as prime movers. It has been found, however, that the great cost hitherto attending their production, without any compensating benefits from their application, must place contrivances for applying them in the list of the ingenious and interesting rather than the useful. Among these we may notice an engine contrived to act by the alternate decomposition of water into its elements—the elastic gases, oxygen and hydrogen—and its re-composition by their combustion. The one operation, decomposition, produced a pressure of the elastic gases liberated from their condensed liquid condition; the other operation condensed the gases again into water, and thus removed the pressure produced by the former. The cost of decomposing water, as it was proposed, by electricity, was so great as to render the scheme practically abortive.

The apparatus intended to utilize electricity and magnetism as prime movers, have been devised on two distinct principles. In the one set, the attractive and repulsive force of a magnet has been employed as the element of power, the magnet being rendered alternately active and neutral by the movement of a current of voltaic electricity around the iron constituting it; or its cessation. However great may be the attractive force of an electro-magnet on iron very near it, it is found that on an increase of distance, the force diminishes very much. This limitation of the magnet's power to short distances renders the problem of employing it as a moving force by no means an easy one.

The other set of electro-magnetic engines were contrived to take advantage of the singular properties by which magnetism and voltaic electricity appear to be connected to one another. It is found that when a stream of electricity is made to pass along numerous wires in the neighbourhood of a magnetic-needle, the needle is deflected so as to stand transversely to the current. And, conversely, it is found that the mere position of a magnetic-needle within a coil of wire encircling it induces a current of electricity along the wire. Motion is caused by this reciprocal action of magnetic bodies and electrical currents; and it has been endeavoured to render this motion available in driving machinery. In this, as in all electro-magnetic machines, however, the cost of producing the power, by the solution of zinc or some other metal in the galvanic battery, has been found too great; and none of these contrivances have ever come into competition with the steam-engine, in which the cost of producing the power by the combustion of fuel is far more moderate.

THE STEAM-ENGINE.

It is usual for writers on the steam-engine to give an outline of its history, tracing up from the earliest times successive contrivances and suggestions for rendering the elasticity of vapours applicable as a motive power. This mode of introducing the student to the consideration of the mechanical arrangements, which now constitute a complete steam-engine, is not without its advantages. It is calculated to open the student's mind to the defects of successive inventions, and the modes in which they have been gradually removed, until what was at one time looked on as an ingenious and interesting philosophical toy, has become an apparatus that has changed the whole face of civilized society, and almost brought the elements within the control of man. By watching the steps of inventors in this progress, the student becomes well prepared to comprehend the details of the perfected apparatus, and the reasons why certain modifications have been introduced as improvements, while others have been found practically inapplicable. But it happens with the steam-engine as with almost all other mechanical contrivances, that its earliest forms are by no means the most simple: and that improvements have often consisted more in discarding useless complications than in adding new parts. We think, upon the whole, that the steam-engine may be best studied in its most simple forms, regardless of their historical sequence, especially when a practical knowledge of its construction and applications is desired, rather than a popular theoretical acquaintance with its leading features. We propose, therefore, to discuss, in the first place, the general properties of steam, and the modes adopted for generating it in a manner suitable for rendering it available as a prime mover. And we will, secondly, consider the special forms of apparatus, or engines, through which the steam is made to act for producing the movement of machinery. By this course we shall hope to give the reader a clear view of the whole subject; and we shall take such opportunities as offer themselves of referring to the various inventions that have, from time to time, been brought to bear.

To the advantages arising from the use of steam as a motive power, to the changes that have been effected by its introduction, and the still greater changes that may be confidently expected from its more extended application, we need scarcely refer. These are patent to all; for in the present condition of society few operations are conducted in which the steam-engine does not play the most prominent part.

Expansive Power of Liquids.—When a liquid is exposed to heat, it expands in bulk; and when the heat is carried to a certain point, part of the liquid rises from the rest in the form of vapour, occupying a much greater volume than it did in its liquid condition. It is probable that all substances, solid as well as liquid, are capable of being changed into the vaporous form by heat. Some bodies, such as ether and alcohol among liquids, and iodine among solids, rise in vapour at comparatively low temperatures. Others, again, such as many of the metals, even after they have become liquid under the influence of heat, require very considerable accessions of temperature before they pass into vapour. Water, one of the substances most widely diffused in nature, and therefore most cheaply obtained, holds a middle course in this respect between these extremes, and attains by no great accessions of temperature properties which render it especially serviceable for human use. It is, indeed, among the most obvious of the beneficent arrangements of Providence that this substance, almost everywhere accessible; possessed of no corroding power like acids or alkalies; of no intoxicating qualities like spirits; not of inconvenient gravity like metals,—should be at the same time capable of absorbing vast quantities of heat, and of giving it out in the shape of active motive power. It has been

proposed to employ the vapours of ether, alcohol, sulphuret of carbon, or mercury, for moving engines; but no advantages from their use in this way have presented themselves, such as could bring them into competition with water. We will, therefore, confine ourselves to the latter substance, which is the only one turned practically to account.

The atmosphere which envelops the earth, though invisible, is nevertheless possessed of weight, and presses on the surface of all bodies in it exactly as the water contained in a cistern presses on bodies immersed in it. The pressure of the atmosphere at the surface of the earth is about 15 lbs. on every square inch. This pressure varies with changes in the density of the atmosphere, but not to a great extent. The average is generally stated at somewhat less than 15 lbs. per square inch; but for all practical calculations 15 lbs. may be assumed without involving material error. Water contained in any vessel exposed to the air is thus pressed upon, and retains its liquid form at ordinary temperatures. But if the temperature of the water be raised to 212°, as measured by Fahrenheit's thermometer, its vapour at that temperature has an elastic force which exactly balances the atmospheric pressure, and part of the water consequently rises in the form of vapour and mingles with the air, displacing as much of the latter as is equivalent to its own bulk. Were the pressure of the air removed, vapour would rise from the water at much lower temperatures than 212°, and occupy the vacant space in the vessel containing it. Thus, when a saucer containing water is placed under the receiver of an air-pump, as the air is extracted from the receiver, vapour rises from the water; and however low may be the temperature of the water, vapour continues to rise from it as the pressure of air on its surface is diminished. Even under the full atmospheric pressure, water is constantly evaporating at ordinary temperatures, for the air seems to have a power of dissolving water or taking it up in a vaporous form, just as water dissolves salt and retains it diffused through it in a liquid form.

At every different temperature at which vapour may exist, its elastic force is different, being greater the greater the temperature, but not proportionally so. When we inquire what is meant by the elastic force of a vapour, we can only say that it is a property common to all aëriform bodies, by which they react on any surface compressing them, or tend to expand into a larger volume than that in which they are confined. There seems, indeed, to exist a repulsive force among the particles which constitute a gaseous body, pushing them asunder with equal power in every direction, and requiring some contrary compressing force to retain them in their places. We have no example of a gaseous body existing without being contained in an envelope of some kind, or being pressed upon by some surrounding medium. The air at the surface of the earth is pressed upon by the weight of air above it, and exerts an elastic force exactly balancing the pressure of the superincumbent fluid. If its elastic force were either greater or less than the force compressing it, it would expand and lift the air above it; or it would collapse under a pressure greater than it could resist. As we ascend higher, the weight pressing on the air is diminished, because there being a certain amount of the atmosphere left below, there remains a less amount above. We find accordingly that at any height above the earth's surface the elastic force of the air is diminished in proportion as the pressure upon it is lessened. At the top of a high mountain, like

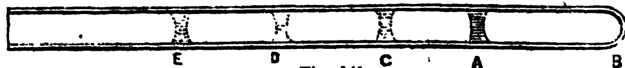


Fig. 141.

Mont Blanc, the elastic force of the air is not more than half that of the air at the surface. If we were to introduce a drop of oil into a small glass tube closed at one end (Fig. 141),

so that the oil should form a film, or diaphragm, across the tube at some point A, we should enclose between A and B a portion of air in the same condition as that surrounding us. If, now, we ascended a mountain we should find the air-film at points C, D, E successively, as we attained greater height. Before our ascent commenced the elastic force of the air in A B pressed the oil-film outwards with exactly the same force as that of the weight of superincumbent air pressing it inwards, and accordingly the film remained at rest. As we ascended we subjected the film to less pressure inwards, because we had less weight of air above us; and accordingly the elasticity of the air in A B overbalanced the pressure, and pushed the film to some such point as E; not by fits and starts, but gradually as we ascended.

We have now to inquire why there should be any point such as C where the film could rest; in other words, why the elastic force of air in the tube, having once overbalanced the pressure of the external air, should again become equal to it, and no longer force the film outwards. The reply to this question involves the consideration of a most important law to which all elastic fluids are equally subjected. It is called Marriott's law, after the name of the person who gave it to the world, and is to this effect:—*The elasticity of a gas is proportional to its density*—that is to say, the greater the number of particles of a gas we force into a certain space, or the smaller we make the space containing a certain weight of gas, the greater we make its elastic force, and conversely. Referring now to the air in the tube, we find that when the film has arrived at C, the space containing the air is extended from A B to C B, and the elastic force is diminished in like proportion. In fact, the density and elasticity of the air within the film is exactly the same as that of the air without, and the film remains motionless at C as long as this equality subsists. Again, as we ascend, the external pressure diminishes; the elasticity of the air within pushes the film outwards, and thus extends its space until its elasticity is reduced to an equality with the pressure of the external air; and so on without limit. In descending we should find the film moving inwards, according as it became subjected to a greater external pressure; to positions such that the air confined within it became of a density sufficient to balance the pressure. Notwithstanding the extreme simplicity of Marriott's law, it is one of the greatest importance, and should be thoroughly understood by any one desirous of an acquaintance with the steam-engine, for it applies to steam as closely as to any gas. From this law it follows, that if we forced air or steam occupying a volume of 2 cubic feet into a vessel of the capacity of 1 cubic foot, we should find the elasticity doubled, because the density would be doubled, or the volume reduced to half. Conversely, if we permitted 1 cubic foot of air or steam to occupy a volume of 2 cubic feet, we should reduce its elastic force to half of what it was before expansion, because we have doubled its volume, or reduced its density to half of what it was. In order to illustrate these facts, let us suppose that a cylindrical vessel is fitted with a piston that can slide in it (Fig. 142), having an area of 1 square inch, and that a load of 20 lbs. placed on the piston forced it down to a position C D, an inch from A B, compressing below it the air or steam occupying the part A B D C of the cylinder. We should then say that the elasticity or pressure of the air or steam was 20 lbs. per square inch, because it

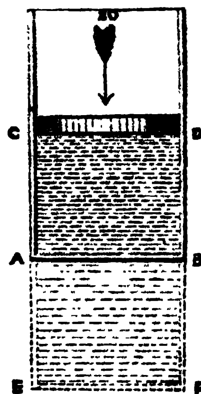


Fig. 142.

balances or supports a load of 20 lbs. placed on a piston exposed to its elastic force. If we suppose another inch added to the length of the cylinder, the added space A E F B being totally empty, and the former bottom A B suddenly withdrawn, so that the air or steam had its space or volume doubled, we should find 20 lbs. on the piston twice as much as it should be to retain the piston in its place, and would have to replace it by a load of 10 lbs., because the elasticity of the air or steam becomes half of what it was before the increase of its volume to the double of what it was. We have already stated that the pressure or elasticity of the air near the surface of the earth is about 15 lbs. on every square inch. If then a cylinder like that which we have just described were filled with air under the piston and placed in a vacuum, there would be required on the piston a load of 15 lbs. to retain it in its place. Were the load less than 15 lbs., the air under the piston would expand in volume and raise it; or were the load greater, the piston would be pressed downwards, increasing the density of the air below it, and proportionally its elasticity, until the load became exactly balanced. But a cylinder of the kind described, filled with air and not placed in a vacuum, requires no actual weight on the piston, because the surrounding atmosphere affords a load exactly equivalent to the weight that would be required in a vacuum. If the cylinder were filled with steam instead of air, and with no load on the piston, the steam would be said to be at atmospheric pressure, because its elasticity tending to force the piston upwards is exactly balanced by the pressure of the atmosphere tending to force it downwards. If the piston required a load of 15 lbs. upon it, the steam would be said to exert a pressure of two atmospheres, or of 15 lbs. above atmospheric pressure. So, if the steam sustained loads of 30 lbs., 45 lbs., 60 lbs., &c., placed on the piston, its pressure would be called that of 3, 4, 5, &c., atmospheres; or of 30 lbs. 45 lbs., 60 lbs., &c., above atmospheric pressure.

The pressure or elasticity of fluids, such as air or steam, is often expressed in terms of inches of mercury, or of the height of column of mercury which they can sustain. It happens that 2 cubic inches of mercury weigh very nearly 1 lb., and that 30 cubic inches weigh, consequently, about 15 lbs. Now, if we suppose a tube, having 1 square inch of sectional area, bent as in Fig. 143, and closed at both ends A and D, were filled with mercury to a height of 30 inches in one limb above the level in the other, the part A B being a perfect vacuum, and the part D C filled with air, since the weight of the column 30 inches high is 15 lbs., this pressure of 15 lbs. is communicated through the mercury in the bend to the air in C D, which consequently reacts with an elastic force equivalent to 15 lbs. on the surface of the mercury exposed to it. That is to say, the elasticity or pressure of the air in C D is 15 lbs., or 1 atmosphere; and the tube might be opened at D to the ordinary pressure of the atmosphere without effecting any change in the equilibrium

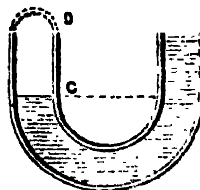


Fig. 143.

of the mercurial column. The instrument in this form would become the ordinary barometer, which measures the pressure or density of the atmosphere by the height of a mercurial column sustained in a tube, every 2 inches of height of mercury corresponding to 1 lb. of pressure per square inch.

If a bent tube (Fig. 144) were connected with a vessel A containing water, on heat being applied to the water, steam would be generated in A, and press, by its elastic force, the mercury downwards in one limb of the tube and upwards in the other, until it attained such a position that the excess of weight of mercury in the one limb and the pressure of the atmosphere on its upper surface C should exactly balance the elastic force of the steam in A. If the height of C above B were 60 inches, the steam would be said to exert a pressure of 60 inches of mercury, or 30 lbs. per square inch above atmospheric pressure, or to have a total elasticity of 3 atmospheres.

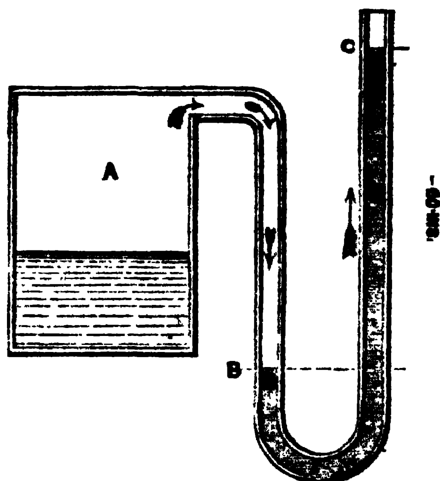


Fig. 144.

We have already said that the elasticity of steam is greater the greater its temperature. There is no simple rule for calculating the pressure due to any given temperature, as the law which governs its variations is of rather an abstruse character. The following table of the pressures and corresponding temperatures of steam, or the vapour of water, is compiled from the results of numerous experiments made with a view to establish some law.

In a treatise like this we must abstain from the discussion of this law, on account of the advanced analysis required for its investigation. Nor need we offer a formula for calculating the pressure corresponding to a given temperature, as the table contains results sufficiently accurate for all practical purposes. The table only applies to the case of steam in a boiler, or vessel, in contact with the water from which it is generated. Were we to remove any portion of steam into another vessel and then subject it to heat without water being in contact with it, we would simply expand its volume as we should air or any other gas by an increase of temperature; or confining its volume, elevate its pressure according to a totally different law.

In the first column of the table the temperatures are marked in degrees of Fahrenheit's thermometer.

The second contains the pressures in atmospheres corresponding to the temperatures.

The third gives the pressures in inches of mercury, or the heights in inches of mercurial columns capable of balancing the elasticities.

The fourth column gives the pressures in pounds per square inch above that of the atmosphere; or the loads in pounds required, in addition to that of the atmosphere, to keep down a piston having an area of 1 square inch pressed upwards by the steam.

TABLE I.

Temperature, Fahrenheit.	Pressure in Atmospheres.	Pressure in inches of mercury.	Pressure in lbs. above atmospheric.
60°	0·017	0·5	14¾ below.
120°	0·120	3·6	13¼ „
180°	0·500	15	7½ „
212°	1·000	30	0 above.
250°	2·000	60	15 „
275°	3·000	90	30 „
290°	4·000	120	45 „
305°	5·000	150	60 „
320°	6·000	180	75 „
344°	8·000	240	105 „
372°	12·000	360	165 „
432°	20·000	600	285 „

We would remark that for temperatures below 180° and above 344° the results are somewhat uncertain. It is within these limits, however, that any practical application of the table can be required, for we seldom have to do with steam pressing with elastic force of less than half an atmosphere on the one hand, or with pressures above eight atmospheres on the other.

The great secret of the power derivable from steam lies in the fact that a small volume of water by heat is expanded into a large volume of elastic vapour, tending to occupy a greatly increased space, and forcing any obstacle presented to its expansion with the pressure due to its elasticity. The volume of steam produced from a given quantity of water has been variously stated; but we believe it may be very correctly estimated at 1600 times when the pressure is equivalent to 1 atmosphere. That is to say, a cubic inch of water contained in a vessel open to the air, when boiled off into steam, occupies 1600 cubic inches of bulk, and forces the air contained in the vessel away to the extent of that expanded volume; or expands with a force of 15 lbs. on every square inch, which is the measure of the atmospheric pressure, and therefore the force resisting the expansion of the steam. But had the vessel containing the steam a volume of only 800 cubic inches, or half that to which the steam would expand at atmospheric pressure, then the density of the steam being doubled—or, in other words, the number of particles crowded into any part of the space being doubled—the pressure on every part of the vessel would be doubled also. If the cover of the vessel were a moveable piston having 1 square inch area, it would in this case require a load of 15 lbs. in addition to the atmospheric pressure upon it to keep down the elastic steam within.

Looking at the question generally, we see that the volume occupied by steam from a certain quantity of water is inversely as the pressure, because the pressure is as the density, and the density is inversely as the volume. The following is the rule for calculating the volume of steam at any pressure produced from a given volume of water.

Rule.—Multiply the volume of water by 1600, and divide by the pressure in atmospheres.

Example 1.—Required the volume of steam at 4 atmospheres (or having a pressure of 45 lbs. per square inch above atmospheric pressure) generated from 3 cubic feet of water.

$$\frac{3 \times 1600}{4} = 1200 \text{ cubic feet of steam.}$$

Conversely, to find the quantity of water necessary to generate a given volume of steam at a given pressure.

Rule.—Multiply the volume of steam by the pressure in atmospheres, and divide by 1600.

Example 2.—Required the water necessary to generate 1200 cubic feet of steam at 4 atmospheres.

$$\frac{1200 \times 4}{1600} = 3 \text{ cubic feet of water.}$$

Example 3.—A cylinder 12 inches diameter and 20 inches long is filled 120 times per minute with steam, having a pressure of 30 lbs. above that of the atmosphere; required the quantity of water necessary to generate the steam.

The area of a circle 12 inches diameter is 113 square inches, and the capacity of the cylinder is $113 \times 20 = 2260$ cubic inches. This capacity filled 120 times gives a volume of steam $= 2260 \times 120 = 271200$ cubic inches. As the steam presses with 30 lbs. above that of the atmosphere, or altogether with 45 lbs., that is 3 atmospheres, the volume of water is $\frac{271200 \times 3}{1600} = 508\frac{1}{2}$ cubic inches.

From these examples it is evident that a great force can be obtained by subjecting water to the action of heat. Just as a few grains of gunpowder on being ignited become suddenly transformed into a large volume of elastic gases, which by their expansive force propel a ball with great velocity, or burst asunder the solid rock; so a small quantity of water heated above 212° is changed into a perfectly elastic vapour, pressing upon the envelope containing it, and forcing any body opposing its expansion through a space sufficiently great to permit its enormous increase of bulk. But not only to the expansion of its volume does vaporized water owe its excellence as a moving force, for its increased volume can be suddenly reduced to a small bulk by the application of cold, or the removal of the heat which is necessary to its vaporous condition; and whatever force the steam exerted in expansion, is returned again by its condensation.

Thus if a vessel containing 1 cubic inch of water, and 799 cubic inches of air, were heated so as to turn the water into steam at twice the atmospheric pressure, the steam would force out the air and occupy its place; acting as it expanded with a pressure of 15 lbs. on every square inch above that of the atmosphere. Were the vessel now cooled so as to reduce the 800 cubic inches of steam to 1 cubic inch of water, the vacuum left by the condensed steam would be immediately filled by air pressed into it by the surrounding atmosphere with a force of 15 lbs. on every square inch. Were the vessel fitted with a piston or partition capable of sliding upwards or downwards in it without permitting the passage of fluid round its edges, the effect would be the same; for if the piston at A (Fig. 145) were in contact with the water before boiling, and raised to B by its expansion into steam on heat being applied, it must, in rising, have been subjected to a pressure of 30 lbs. on every square inch of its under surface, so as to overbalance the atmospheric pressure on its upper surface by 15 lbs. On cold being applied so as to reduce the steam to its original volume of water, the pressure on the under surface of the piston being removed, that of the air on its upper surface again forces it downwards from B to A, its



Fig. 145.

original position. Thus, both in the ascent and in the descent there is developed a force applicable to the movement of machinery properly connected with the moving piston. In order that we may attain some notion of the amount of this force, let us suppose that the piston has an area of 1 square foot, and can rise and fall 2 feet, and that we have the means of generating and condensing the steam in the vessel 50 times in every minute.

Since 1 square foot = ... 144 square inches,
And on every square inch the pressure is . 15 lbs.

The total pressure on the piston is . . . 2160 lbs.

This force is moved 2 feet up and 2 feet down 50 times per minute, or through a space of 200 feet per minute, and is therefore equivalent to $2160 \times 200 = 432000$ lbs. moved through 1 foot per minute. A horse-power being reckoned at 33000 lbs. moved 1 foot per minute, the force of the piston, as we have estimated it, is equivalent to $\frac{432000}{33000} = 13$ horse-power. From this example it will appear that by increasing the size of the piston, the distance through which it is moved, the rapidity of its alternations, and consequently the means of generating and condensing the steam, almost unlimited power can be attained.

In many steam-engines advantage is not taken of the power derivable from the condensation of the steam—its mere expansive power is employed; and, after having done its work, the expanded steam is allowed to escape into the atmosphere. This system is adopted for the sake of economy, lightness, and simplicity in the construction of the engine; and such engines are called *high-pressure* or *non-condensing: high-pressure*, because the steam must exert a pressure considerably higher than that of the atmosphere against which it has to act; or *non-condensing*, because the steam is not condensed after having done its work. In other steam-engines, called *low-pressure* or *condensing* engines, although greater power is derived from the steam, yet the machinery is rather more complex and heavy and more liable to derangement, and a large supply of cold water is necessary to effect the condensation of the steam. Of late years many engines have been advantageously employed where the steam is first caused to act as it does in a non-condensing engine; but instead of being blown out into the air, it is afterwards made to do duty as in a condensing engine. Such are called *combined* engines, because the principles of expansion and condensation are combined in their action to a greater xtent than in most others.

Before entering upon questions connected with the practical construction of the steam-engine, it will be advisable, in the first place, to discuss theoretically some of the principal facts connected with the expansion and pressure of steam, or generally of elastic fluids.

If we suppose a cylindrical vessel fitted with an air-tight piston, and containing within it a certain volume of air, steam, or any gaseous fluid perfectly elastic, we may represent the amount of pressure which the fluid exerts on the piston at B (Fig. 146) by the length of a vertical line B C. For example, if the area of the piston be 1 square inch, and the pressure on it at B be 15 lbs., we may draw a line or ordinate B C 15 inches long, taking 1 inch for each pound, or 15 half inches, or 15 tenths of an inch, or any other proportion that we may find convenient. If now we applied a force to the piston so as to push it along the cylinder to some place B_1, and there drew an ordinate B_1 C_1 having a length bearing to the pressure on the piston at B_1 the same proportion as the length

of B C bears to the pressure at B_1; and if, farther, we drew an ordinate at B_2, and others at any number of intermediate points, we might, by tracing a curve C_2 C_1 C through all the points so determined, exhibit graphically the rate of variation of pressure according

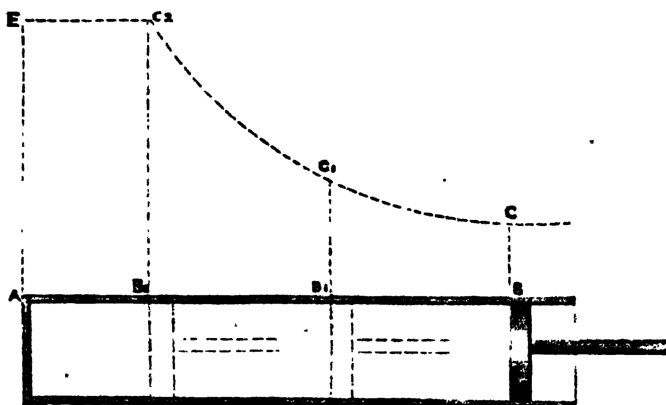


Fig. 146.

to that of volume or density. The law, to which we have already alluded, called Marriott's law, is very simple, viz. that the volume multiplied by the pressure is always constant—that is to say, the length of A B (which represents the volume of gas when the piston is at B), multiplied by the height of B C (which represents the pressure at B), gives the same product as that of the length of A B_1 multiplied by B_1 C_1, or of A B_2 multiplied by B_2 C_2.

Note.—The curve C_2 C_1 C (Fig. 146) is called the *hyperbola*, and it can readily be traced geometrically thus: If A B (Fig 147) be the given volume or length of the cylinder when the pressure is B C, take any other point B_2 and draw an ordinate B_2 G cutting in F a line C E parallel to A B; join A F, and prolong it to meet B C in D, and through D draw D C_2 parallel to A B, cutting B_2 G in C_2; then C_2 is the point of the curve corresponding to B_2, for A B_2 × B_2 C_2, or the area of the rectangle A B_2 C_2 H, is equal to A B × B C, or the area of A B C E, as might readily be proved. Any number of points, such as C_2, being found, the curve can be traced through them.

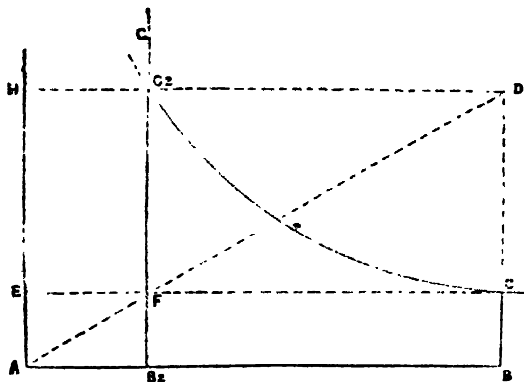


Fig. 147.

If, instead of filling the length A B of the cylinder with fluid, and then forcing in the piston so as to compress it, increasing the pressure in proportion to the increase of density, we supposed the piston to be at B_2, and the part A B_2 of the cylinder filled with elastic fluid, it would force the piston towards B with pressure gradually diminishing with the density according to the same law, and the curve C_2 C_1 C would terminate the ordinates representing pressures along the stroke or distance passed over by the piston. This is precisely the condition under which the piston of a steam-engine is ordinarily worked; and by taking a practical example we may ascertain what power is developed by the expansion of the steam. We shall suppose that in a steam cylinder of 1 square inch area the piston is close to the end A, and that by an opening there steam, having a pressure of 4 atmospheres, 60 lbs. per square inch, is admitted so as to force the piston away from A to the position B_2, 1 inch from A. The steam-opening then being closed, the cylinder contains 1 cubic inch of steam 4 times the density of steam at atmospheric pressure, and pressing on the piston with a force of 60 lbs. When the piston has reached B_1, 2 inches from A, the steam has expanded to 2 cubic inches, and is therefore half its former density, or twice that of steam at atmospheric pressure, acting on the piston with a force of 30 lbs. At B, 4 inches from A, the density is one-fourth of that at B_2, and the pressure on the piston is 15 lbs. By reckoning the pressures at a number of intermediate points, we might ascertain an average pressure of nearly 35 lbs. throughout the stroke, and thence calculate the power developed by the movement of the piston to be 35 lbs. moved through 4 inches, or 140 lbs. moved through 1 inch by the action of 1 cubic inch of steam at a pressure of 4 atmospheres—a quantity that would be generated from $\frac{1}{400}$th cubic inch of water.

Now, if we take another case, using the same quantity or weight of steam, but at a different pressure, we shall find a marked difference in the power developed. Let us suppose that steam at a pressure of 2 atmospheres, or 30 lbs. per square inch, is admitted so as to force the piston through 2 inches of its stroke; we have 2 cubic inches of steam at 2 atmospheres, which are equivalent to 1 cubic inch at 4 atmospheres, and would be generated from the same quantity of water, $\frac{1}{400}$th cubic inch, at nearly the same expenditure of heating power. We should in this case find the average pressure on the piston, throughout the stroke of 4 inches, to be about 24 lbs., or equivalent to a weight of 96 lbs. moved through 1 inch. At first sight it appears startling, that by an expenditure of no more heating power in the first case, we should have obtained 45 per cent. more power than in the second; but such is the fact, nevertheless: and that it is so may be clearly seen by the diagram (Fig. 148).

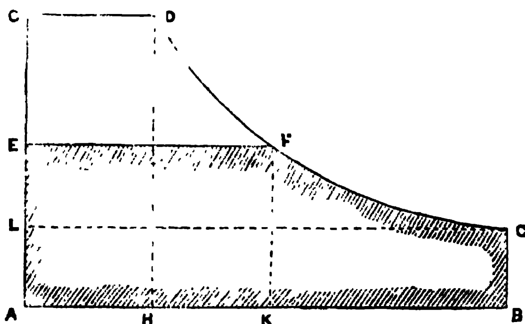


Fig. 148.

Let A B represent the stroke 4 inches long, A C the pressure, 4 atmospheres (represented by 4 inches in height), continued through 1 inch of the stroke, as shown by the line C D; then from D let the pressure-curve be drawn till it terminates at

G, B G being 1 inch high, representing the terminal pressure 1 atmosphere. Then the area of the compound figure A C D F G B represents the total power developed in the first case by the expenditure of a quantity of steam represented by the area of A H D C. Again, let the pressure in the second case of 2 atmospheres, represented by A E, 2 inches high, be continued through 2 inches of the stroke to F; a portion of the same pressure-curve F G will enclose the pressures during the rest of the stroke. In this case the power is represented by the area of the shaded figure A B G F E, the quantity of steam expended being as the area of A K F E. Now the area of A K F E, or $2 \times 2 = 4$, is precisely the same as that of A H D C, or $1 \times 4 = 4$; while the area of A B G D C exceeds that of the shaded figure by the portion E F D C—so much clear gain of power by the higher initial pressure, and greater range of subsequent expansion. We have here supposed that no resistance is offered to the movement of the piston. If it be conceived to move in opposition to the pressure of the atmosphere, the gain by expansion of the steam is even more marked. A line G L represents the constant atmospheric resistance, and cuts off from both figures an area A B G L, which leaves the effective overplus of force impressed on the piston so as to move machinery, represented in the one case by the area of L G F E, and in the other by L G D C. Numerically, since we found the total force in the two cases to be equivalent to 96 lbs. and 140 lbs. respectively moved through 1 inch, and as the atmospheric resistance is 15 lbs. through 4 inches, or 60 lbs. through 1 inch in both cases, the effective forces are as $140 - 60$, or 80, to $96 - 60$, or 36; that is to say, the one is more than double the other.

This mode of graphically representing forces is not only of an interesting, it is also of a most useful character. Watt invented an instrument which, being applied to a steam-engine, could draw upon a card a figure such as we have employed, representing the power developed by the action of the steam in the engine. He, however, seemed to consider it only as an interesting toy; but of late years this instrument has been found to be of the greatest utility. It is called the *indicator*, and is capable of not only representing with great accuracy the power developed in the engine to which it is applied, but also of exhibiting defects in design and construction, and of suggesting improvements. We shall have occasion hereafter to enter more particularly into the details of its construction and application.

The Boiler.—In all steam-engines, the boiler or apparatus for generating the steam is a part of prime importance. In devising a good boiler, the problem is to obtain the greatest quantity of steam, or to boil off the greatest weight of water with the least weight of fuel consistently with due simplicity, durability, strength, and economy of material and labour in its construction. It must be a vessel capable of containing water, and affording space for steam generated from it; every part of it being exposed to the pressure of the steam within, it must be capable of resisting this bursting force, and in its construction precautions must be taken for safety in case of the pressure tending to exceed the strength provided to resist it. A certain portion of its surface must be exposed to the action of the fire; and as the materials which we have to use in its construction suffer when exposed to excessive heat, and as we cannot, consistently with economy, afford to apply any portion of our fuel ineffectually, we must make provision for having the interior of the fire-surface covered with water to receive the heat communicated through it. The most simple kind of boiler is one of cylindrical form, (Fig. 149), placed horizontally, with a fire arranged under it so that the direct heat of the fire, and of the heated products of combustion in their passage to the chimney, act through the metallic casing on the water within. The water only partially fills the

boiler, leaving a space above it for steam, which is conducted from the boiler to the engine by a pipe leading from its upper side.

It is found by experiment, and indeed it seems to be a reasonable conclusion, that, within proper limits, having a certain quantity of fuel to dispose of, the larger the sur-

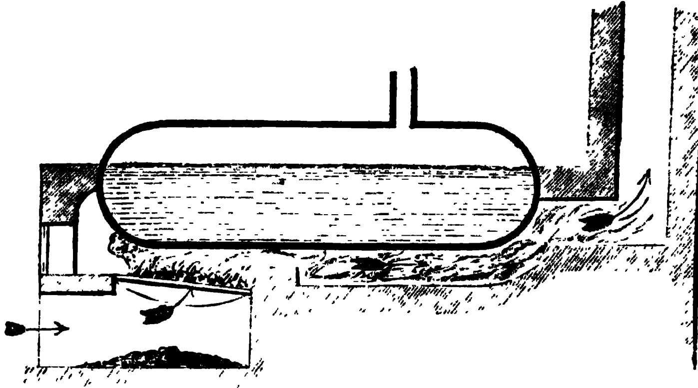


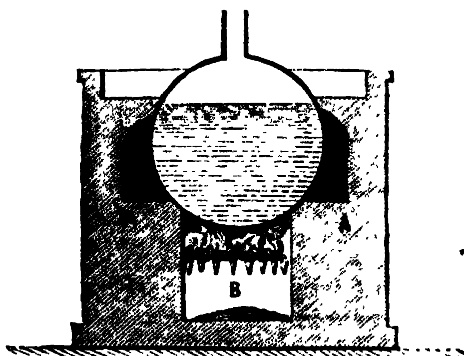
Fig. 149.

face of water over which the heat developed from combustion is spread, the greater will be the heating effect produced, or the larger will be the quantity of water turned into steam. In fact, it becomes the object of the engineer to allow as little as possible of the heat to escape by the chimney, and consequently to arrange his boiler in such a manner as to make the water absorb the greatest possible quantity of heat, by exposing the largest possible surface to the combustion, disposing that surface in the best manner.

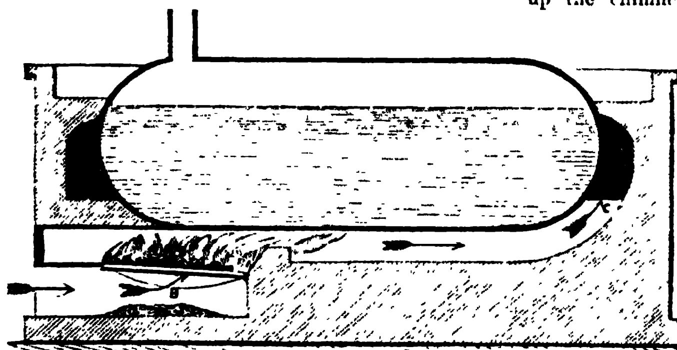
The strongest form in which a vessel can be made when it is intended that it shall resist internal pressure, is that of a spherical shell. If then in the construction of steam-boilers strength alone were studied, the spherical form would be generally adopted. But of all forms of vessels, the spherical is that which has the smallest surface in proportion to its capacity, and it is consequently ill-adapted for the purpose of a boiler where the amount of heating surface is important. Next to the spherical in point of strength, and superior to it in respect of superficial area, is the cylindrical form, with hemispherical or rounded ends (Fig. 149); and accordingly this form is very generally adopted for steam-boilers.

In order to render available as much as possible of the surface for receiving the heat, the heated products of combustion are not permitted to escape directly from the fire into the chimney, but are carried round the boiler by flues generally in the manner indicated in Fig. 150. The boiler is placed on two banks of brick-work A A, between which are fixed the fire-bars B, so that the flame may play on the bottom of the boiler. The products of combustion, heated to a high temperature, pass along under the bottom of the boiler, upwards at the far end C, thence along the side flues D D formed by brick-work, till they finally proceed by any convenient flue or channel E to the chimney. By this arrangement a large portion of the heat contained in the products of combustion is absorbed during their passage along the bottom and side surfaces of the boiler, and given to the water contained in it; while the brick-work, being a very imperfect con-

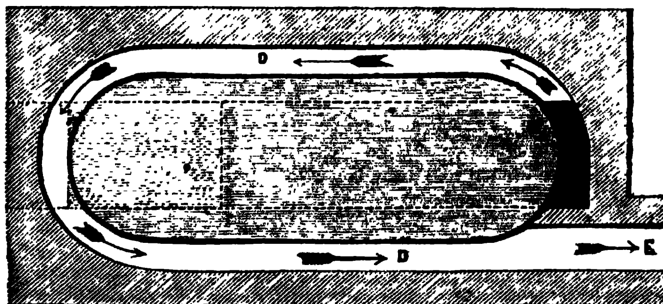
ductor of heat, permits very little to escape ineffectively. The power of a boiler arranged in this manner depends upon the extent of fire-grate, or quantity of combustible matter consumed in a given time, and the superficial area of boiler on which the products of combustion play in their course towards the chimney. That there should be some relation between these quantities will be evident from the following considerations. If the quantity of fuel consumed be very great while the flue-surface is small, the products of combustion will not have sufficient opportunity for parting with their heat, and will, therefore, carry up the chimney and waste a large amount of heating power, which, by better arrangements, might be made to tell upon the water. If, on the other hand, the fire-grate be too small, while the flue-surface is large, the products of combustion will have parted with their heat before reaching the chimney—a large portion of the flue-surface will thus be rendered useless, and the draught (caused by the ascent of heated air in the chimney) will be sluggish, and the combustion slow. We believe that practically it will be found



TRANSVERSE SECTION.



LONGITUDINAL SECTION.



PLAN.

Fig. 150.

advantageous to adopt the following rules as to size of fire-grate and quantity of flue-surface in a boiler, such as we have described, called the cylindrical egg-ended boiler.

To the diameter of the boiler multiplied by its length add one-half: the result may be taken as the flue-surface; and this product (in square feet) should be 10 times the horse-power. Thus, in a boiler 4 feet 6 inches diameter, and 15 feet long:—

Since 15 feet $\times$ $4\frac{1}{2}$ feet $=$	$67\frac{1}{2}$ square feet.
Add one-half	$33\frac{3}{4}$,, ,,
The flue-surface may be taken at . . .	$101\frac{1}{4}$,, ,,

Or equivalent to 10 horse-power. Again, for every horse-power there should be $\frac{3}{4}$ths of a square foot of fire-grate. For 10 horse-power there should therefore be $7\frac{1}{2}$ square feet of fire-grate, a surface that might be made up by taking the length of the fire 3 feet 9 inches, and the width 2 feet, since 3 feet 9 inches $\times$ 2 feet $= 7\frac{1}{2}$ square feet.

The converse rule for finding the dimensions of a boiler suitable to a given power is the following:—From 10 times the horse-power subtract its $\frac{1}{3}$rd part, and the result will be the product of the diameter by the length. Thus, to make a boiler of 10 horse-power:—

Since 10 $\times$ 10 $=$	100
Subtract $\frac{1}{3}$rd $=$	$33\frac{1}{3}$
The product of diameter $\times$ length $=$	$66\frac{2}{3}$

We are at liberty to take any convenient diameter and length that might make up this product within proper limits. Thus, making the

	ft.		ft. in.			
Diameter	3	and the length must be	22 3	product	$66\frac{3}{4}$	square feet.
,,	4	,, ,,	16 8	,,	$66\frac{2}{3}$	,,
,,	5	,, ,,	13 4	,,	$66\frac{2}{3}$	,,
,,	6	,, ,,	11 2	,,	67	,,
,,	7	,, ,,	9 6	,,	$66\frac{1}{2}$	,,

Any of these dimensions may be chosen according to the particular circumstances of the case. Were we to take a diameter smaller than 3 feet, with a greater length than 22 feet, or a diameter greater than 7 feet, with a length less than 9 feet 6 inches, we should find practical difficulties in fixing and working, and lose useful effect from the extreme lengthening or shortening of the flues.

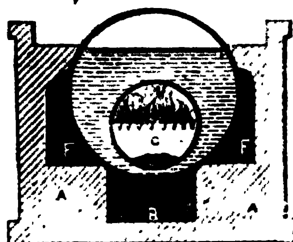
When it is desired to obtain greater heating surface within a smaller space, it is found very advantageous to construct the boiler of cylindrical form, with a cylindrical flue or tube passing through the water; so that not only may the exterior surface exposed to the flues receive heat from the products of combustion, but also the interior surface of the tube. When this tube is made of sufficient size to admit the fire within it, as in Fig. 151, the boiler is called a Cornish boiler, from the circumstance of its being first extensively applied with excellent effect in Cornwall. The arrangement of flues for a Cornish boiler is generally similar to that represented in the figure. The boiler rests on two banks of brick-work A A, with a space for the bottom flue B left between them. The fire-grate is fixed at C in the front portion of the tube; and the products of combustion pass along the tube, spread at the end E into the two side flues F F, descend at G G to the bottom flue B, and pass thence to the chimney. The quantity of flue-

surface in a boiler of this kind exceeds that in the simple cylindrical boiler by nearly the internal surface of the tube. But as the hot products of combustion pass chiefly along the upper side of the tube, and leave its lower side generally covered with a coat of non-conducting soot and ashes, we cannot safely reckon much more than half the surface of the tube as effective heating surface; that is, $1\frac{1}{2}$ time the tube's diameter multiplied by its length.

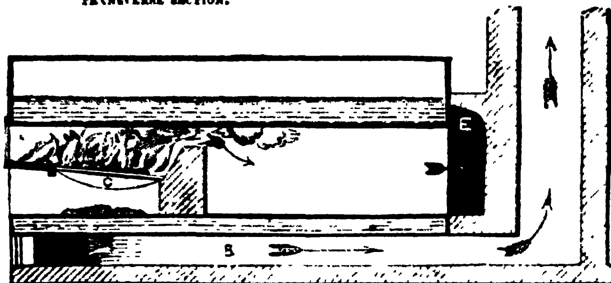
In order, then, to estimate the power of a Cornish boiler, we should calculate the external surface as before, and add to it the product of $1\frac{1}{2}$ time the diameter of the tube by the length for the total effective surface; allowing 10 square feet for every horse-power.

Or, more simply:—To the external diameter add the diameter of the tube (in feet), multiply by the length, add one-half to the product, and divide by 10 for the horse-power of the boiler.

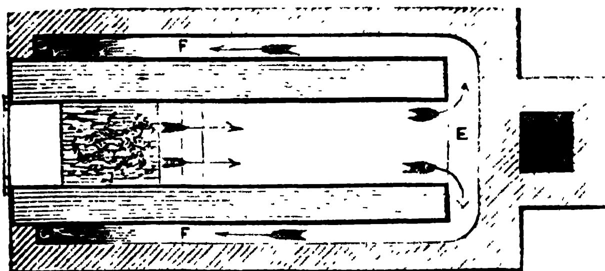
Example. — A Cornish boiler, 5 feet diameter, and 12 feet long, has a flue-tube 3 feet diameter. required its power.



TRANSVERSE SECTION.



LONGITUDINAL SECTION.



PLAN.

Fig. 151.

Diameter of boiler	5	feet.
Add diameter of tube	3	,,
	8	,,
Multiply by length	12	,,
	96	square feet.
Add one-half of 96 = ...	48	
Divide by . . . . 10)	144	
Horse-power of boiler	$14\frac{1}{2}$	nearly.

The converse operation for finding the dimensions of the boiler when the power is given, would be:—From 10 times the power subtract its ⅓rd part; and the remainder gives the product of the length by the sum of the external and internal diameters.

Example.—Thus, for a boiler 14½ horse-power—

Since 10 × 14½ = ...	145
Subtract ⅓rd of 145	48⅓
	97 nearly.

Thus, 97 is the product of the length by the sum of the external and internal diameters.

In this case, again, we must consult the circumstances of position for determining the suitable length and diameters. The diameter of the tube should not greatly exceed half that of the boiler, because there should be an ample covering of water over all the heated surfaces. And again, it should not be less than 1 or 2 feet, because it must admit sufficient area of fire-grate without excessive length. The fire-grate in the case given, reckoning ¾ths of a square foot per horse-power, should be about 11 square feet; and as a length of fire exceeding 5 or 6 feet would become inconvenient, we must take it at least 2 feet in breadth—that is to say, the diameter of the tube must be 2 feet. The diameter of the boiler might then be 4 feet, and the length would be for these diameters $\frac{97}{2+4} = 16$ feet. Were we to take the diameters as 3 feet for the tube and 5 feet for the boiler, the length would be $\frac{97}{3+5} = 12$ feet.

We believe it will be found practically advantageous to make the length a little more than three times the diameter of the boiler, the diameter of the tube being rather more than half that of the boiler. For 14½ horse-power, according to this proportion, we should have

Diameter of boiler	4 feet 6 inches
Diameter of tube	2 ,, 6 ,,
Length of boiler	14 ,, 0 ,,

In cases where the dimensions of the boiler are considerable, two or more tubes are introduced, as in Fig. 152. The tubes are always placed as low as possible, allowing 4 to 6 inches between them and the outer casing, in order to have their upper and hottest surfaces well covered with water, without interfering inconveniently with the steam space above.

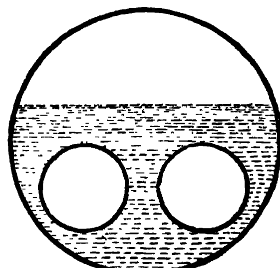


Fig. 152.

For marine steam-boilers, where brick-work setting would be inconvenient, it is usual to arrange the whole heating-surface within the boiler, by means of flues pervading it in all directions (Fig. 153). The water is thus divided into sheets about 6 inches thick; heated on one or both sides by the products of combustion as they pass along the flues. In all such boilers a heating-surface of at least 10 square feet per horse-power, and fire-grate from ½ to ¾th of a square foot per horse-power, should be provided. Nor should the flues be too small in sectional area, nor too much broken up or prolonged, lest the draught or rapidity of movement of air, and consequently of combustion, be interfered with. Of late years, tubular boilers have been very extensively adopted, both for marine and for

land engines. These were first employed principally in locomotive engines, as they afforded a means of securing very large flue-surface within a limited space, and without excessive weight of material. Fig. 154 represents the arrangement of tubes in a locomotive boiler.

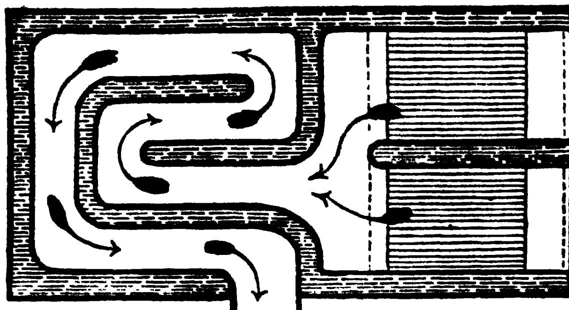
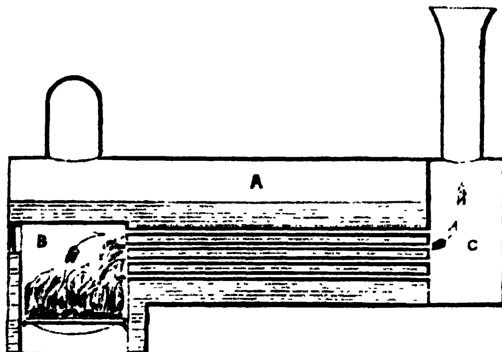


Fig. 153.

The body of the boiler A is cylindrical: at one end B is the fire-box, surrounded by water space; at the other end C is the smoke-box, surmounted by the chimney. In the body are arranged numerous small tubes completely surrounded by water, through which the products of combustion pass in their progress from the fire to the chimney, delivering the greater portion of their heat to the surrounding water. The evaporating power of a boiler of this kind is very great, as we may readily believe on calculating the amount of heating-surface in a boiler of the following dimensions:—



Fire-box inside, 3 feet 6 inches × 3 feet 6 inches × 3 feet 6 inches has 60 square feet actually exposed to the fire, and therefore most valuable as heating-surface. 120 tubes $2\frac{1}{2}$ inches diameter and 10 feet long give 800 square feet of effective flue surface; the whole, including smoke-box, being contained in a space about 13 feet long, 4 feet 6 inches wide, and 4 feet 6 inches high.

We have now described, generally, the different kinds of boilers used for generating steam. They are, of course, subject to numerous modifications in their details and proportions, according to local circumstances and peculiarities of use and situation. They are generally made of wrought-iron plates, from $\frac{1}{4}$th inch to $\frac{3}{4}$ths inch in thickness, according to the magnitude of the work,

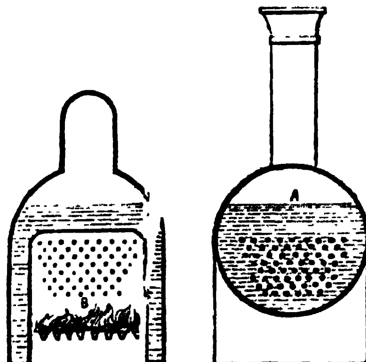


Fig. 154.

and the amount of pressure which they are intended to sustain. These plates are made to overlap each other at the edges, and are fastened together by rivets or round pins of iron passed red-hot through holes provided in the plates, and riveted over so as to form a head. A heavy piece of iron is held against the under side of the rivet-head, and the other end of the rivet is struck repeatedly by heavy hammers, and frequently finished by applying a tool hollowed to the shape of the intended head, and striking the tool by the hammers. The rivet, when the operation is com-

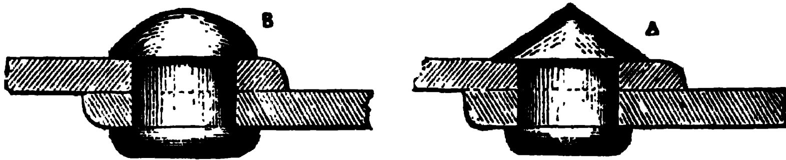


Fig. 155.

plete, is of the form A (Fig. 155) when finished by smart hammering, called *staff-riveting;* and like B when finished by the tool, being then said to be *button-headed.* As the operation of riveting is performed when the rivet is red-hot, not only is the quality of the rivet not impaired by the hammering, as it would be if hammered when cold, but also the contraction or shrinking of the rivet in its length when it cools draws the two plates together with great force, and renders the joint impervious to fluid. When it is found by trial that the joints of the plates, or the edges of the rivet-heads, are not quite tight, as manifested by the leakage of water or steam through any of them, a blunt steel tool is applied to the leaking edge, and struck smartly by a hammer, so as to caulk the joints or force part of the iron into the crevice.

If we suppose, for instance, that an opening exists between the plates at B, and

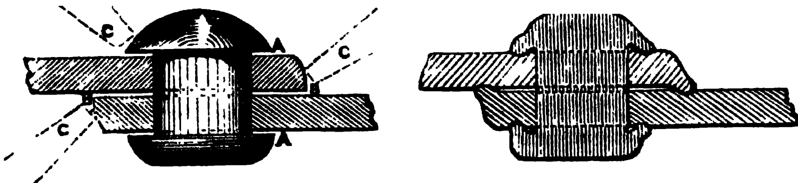


Fig. 156.

round the rivet-head at A (shown greatly exaggerated in Fig. 156), the caulking tool applied at the points marked C forces the iron of the plate edges and of the rivet-heads into the interstices, and thus renders the jointing tight. For jointing the plates at the angles, a peculiar kind of iron, called *angle-iron*, indicated in section at A, Fig. 157, is employed; and where there are considerable flat surfaces of plate exposed to bursting, pressure stays, B, are introduced at proper intervals to prevent the plates from being forced asunder.

For fixing the tubes of tubular boilers in the plates through which they pass, holes are first bored in the plates of a proper size to fit the tubes tightly; and the tubes being cut of the proper length and put in their place, the ends are forced open by means of a conical tool driven by hammering into their mouths. Other methods of fixing tubes and stays are employed; and there are numerous other details of boiler-making of a technical

character, upon which we need not enter. The furnace or fire-grate of a boiler is generally made of numerous bars of wrough or cast-iron, laid side by side so as to form a

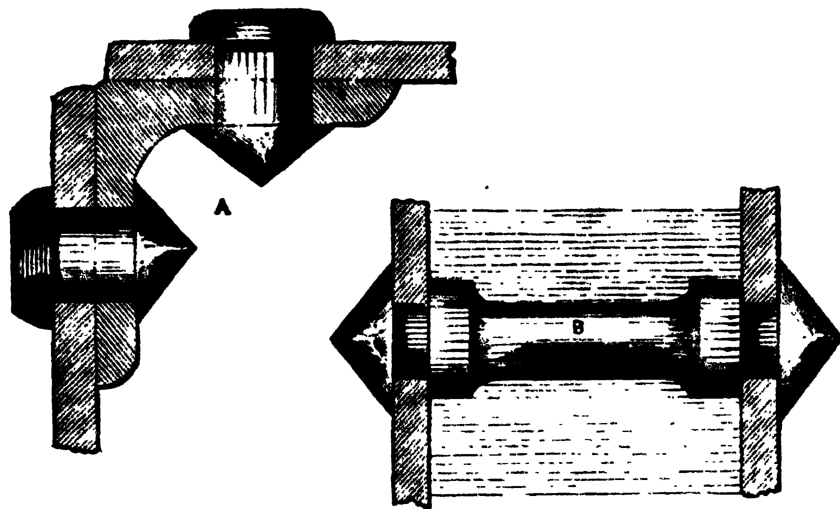


Fig. 157.

grating on which the fuel is placed, leaving spaces about ½ inch wide between the bars for the passage of air upwards to support the combustion, and of the ashes or incombustible refuse downwards into the ashpit. In front of the bars there is generally placed a dead-plate or surface without openings, to receive the fresh fuel, which, lying there for some time exposed to the radiation of the fire beyond, parts with a portion of its gases, and is partially coked before it is pushed onwards to the fire-grate. The gases are ignited in their passage over the hot fire, and produce flame, which plays on the surfaces of the flues. When the supply of air is deficient, large volumes of these liberated gases, having numerous particles of carbon suspended in them, pass through the flues without ignition, and thence through the chimney as black smoke. When this happens, not only is a large and valuable part of the fuel wasted, but the air is inconveniently polluted. The object of smoke-consuming apparatus is to prevent this evil; and the general principle on which all such apparatus is constructed, is either to manage the production of these gases in such a continuous regular manner as that sufficient air may be supplied for their combustion when they are sufficiently heated to ignite—or to supply heated air in some part of the flues, so as to turn into flame there the gases that would otherwise escape unconsumed.

The chimney of a steam-boiler should be of sufficient area and height to produce a good draught or quick current of the heated products of combustion. When the draught is insufficient, the fresh air supplied to the fire is too small in volume, the combustion is retarded, and the flues are filled with smoke instead of flame. A chimney of 30 or 40 feet in height, and having an area of 1 square foot for 10 horse-power, generally gives a sufficient draught. Where height cannot be obtained, as in locomotive engines, the waste steam is made to rush up the chimney with considerable force, and thus to create an artificial draught. The chimney or flue leading to it is provided with a

damper or slide, by which the area of passage may be diminished, and the draught reduced at pleasure. At convenient places in the flues, soot-doors are fixed for giving access to clean them from soot or ashes deposited there.

As all water supplied to boilers is more or less impure, or contains ingredients that become deposited while the water itself is driven off in the form of vapour, it is essential to provide for the frequent cleansing of boilers. In marine boilers, particularly where sea-water is necessarily employed, the deposit of saline ingredients, none of which are volatile, or pass off with the steam, is very large, and of a very troublesome character. It generally settles in the form of a hard, stony crust upon the interior surface of the boiler; and as this crust is a very bad conductor of heat, not only is there a waste of fuel where it exists, owing to its arresting the passage of the heat from the flues to the water, but there is a positive danger from the circumstance that the interior surfaces of the flues are thus over-heated, and the iron of which they are made becomes rapidly oxydized, scaling off in flakes, losing thickness and strength, and becoming liable to disruption from the pressure within exceeding the strength left to sustain it. As salt-water does not deposit rapidly until it becomes super-saturated with salt, it is usual to permit frequently the escape of a considerable quantity of the excessively salt water in the boiler, and to replace it with new water from the sea, less salt and less liable to deposit. For effecting this object, and also for emptying the boiler when required, a pipe and cock, called technically the *blow-off*, is fitted to the lower part of the boiler. This cock should be frequently opened, especially where the water is very dirty or of a saline character, and the worst part of the contents of the boiler, which, being heaviest, lie near the bottom, thus blown out. In order to save the loss of heat occasioned by frequently blowing off the hot impure water from marine boilers, and replacing it by cold, but purer water, an apparatus called the *change-water apparatus* is sometimes employed. It consists of a casing, with numerous small tubes arranged in it as in a tubular boiler. A portion of the contents of the boiler being always permitted to flow through the casing and thence into the sea, a corresponding quantity of purer water is made to pass through the tubes to the boiler; and the latter thus is made to absorb in its passage through the tubes a considerable portion of the heat given out by the former in its passage round them.

Mud-holes are small holes provided in the lower parts of boilers, and fitted with tight covers, which may be removed, when the boiler is not in use, for the admission of a rake to draw out the mud deposited from the water.

The man-hole is an opening sufficiently large to admit a man, provided in the upper part of a boiler, and fitted with a tight cover, which may be removed, when the boiler is not in use, for the admission of a man for cleaning or repairs.

In boilers having considerable flue surface, but not great height of steam-room above the water-level, there is always a danger of priming—that is to say, the water in a rapid state of ebullition is often made to boil over, or blown in considerable quantities into the steam-pipes, and thence into the engine, where it is not only useless, but highly detrimental to the action of the machinery. It is usual, therefore, to provide a boiler with a steam-chest or dome in its highest part, to give greater space for steam and greater height for the mouth of the steam-pipe above the surface of the boiling-water. In general, there should not be less than six inches of water above the flues, nor less than three feet of steam room above the water. In marine-boilers, where the water is surged about by the rolling of the vessel, there should be at least double the height named for water and steam. Should the level of the water be so low that the surface of a flue

is no longer covered by it, the flue is liable to become over-heated, sometimes to redness; the iron is softened and weakened by the excessive heat, and rapidly deteriorated by oxydation and scaling off. But this is not the only danger of an over-heated flue; for on the water again covering it, the enormous volume of steam suddenly generated produces an excess of pressure, and causes an explosion of the boiler. We believe almost every case of explosion can be traced to some circumstance connected with a deficient supply of water; and therefore too much caution cannot be used in watching the condition of the water-level, and providing the proper supply or feed.

In order to ascertain the water-level within the boiler, several kinds of apparatus are employed. The *float* A (Fig. 158) consists of a stone suspended by a wire passing through the top of the boiler, and connected, by a chain passing over a pulley, with a counter-balance weight, sufficient to balance so much of the weight of the float-stone that it shall always

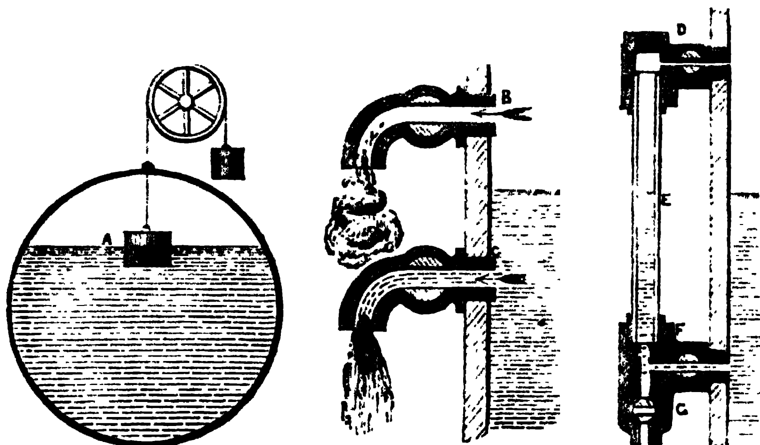


Fig. 158.

lie at the surface of the water. Should the water-level vary, the position of the float-stone, which rises or falls with it, is marked by an index on the pulley.

Gauge-cocks B C are two stop-cocks fitted into the face of a boiler, one above and the other below the proper water-level. When these cocks are opened, steam should blow through the upper, and water through the lower one. The objection to gauge-cocks consists in the circumstance that, in order by them to ascertain the level, the attendant must open them.

The *glass gauge* consists of two stop-cocks, D and F, fitted in the face of a boiler, one above and the other below the water-line. These cocks are connected by a glass tube E, in which the level of the water is distinctly seen. A stop-cock G is provided at the lower end of the glass tube; and this being occasionally opened, the sediment that may collect in the tube or passages of the cocks is blown out by the pressure within the boiler. Should the glass tube burst, the stop-cocks D and F can be closed until a new tube is fitted, or they can be employed as gauge-cocks.

Occasionally the float is connected with a whistle in such a manner that when the level of the water becomes too low, it opens a small stop-cock, which permits steam to blow through the whistle, and thus give audible warning of the danger.

In many boilers an ingenious precaution is taken against the dangerous consequences of insufficiency of water, by the use of fusible metal plugs. A hole is made in the highest part of the fire-box or flue, which is filled up with a plug or rivet of metal fusible at a temperature not greatly exceeding that of boiling-water. So long as this plug is covered with water, its temperature cannot attain the melting point; but should it be left bare, the heat of the fire playing upon it causes it to melt out, and thus to leave a hole, through which the steam escapes into the flue. Not only does the rush of the escaping steam give warning of the circumstance, but it also relieves the steam-pressure within the boiler, and prevents the explosion which might otherwise result from the over-heating of the flue.

In low-pressure boilers, the float occasionally is made to act as a self-feeding apparatus. A (Fig. 159) is a cistern constantly supplied with water, and fitted at bottom with a valve opening downwards into a pipe passing down to nearly the bottom of the boiler; the float B is connected by a wire and lever with the valve, so that when the water-level is too low the descent of the float causes the valve to open, and thus permits the passage of water from the cistern into the boiler. This arrangement can only be adopted when the pressure of the steam within the boiler does not exceed that of the column of water in the feed-pipe. For high-pressure boilers, the height of cistern and feed-pipe would be inconveniently great to overcome the pressure; accordingly, for high-pressure boilers and for marine boilers, the supply of water is effected by means of a force-pump, called the feed-pump, worked by the engine, and regulated by suitable cocks or valves.

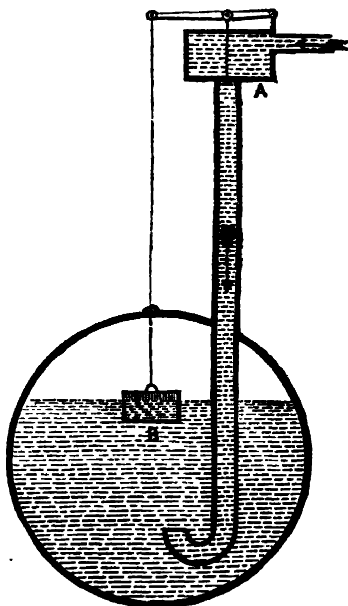


Fig. 159.

The quantity of water required for a boiler may be generally taken at 1 cubic foot per horse-power per hour; or, as 1 cubic foot contains about 6¼ gallons, and weighs about 63 lbs., we may take 1 lb. of water per minute, or 1 gallon every 10 minutes, as the necessary supply for each horse-power. The actual quantity of water required for generating steam to work an engine of 1 horse-power depends much upon the construction of the engine, the extent to which the steam is used expansively, the amount of power derived from condensation, or the amount of resistance to the egress of waste steam, if not condensed; but the above estimate is tolerably correct for non-condensing engines worked without much expansion, and is in excess for condensing engines, and such engines as those in which the expansive force of the steam is taken advantage of, so as to produce great effect with small expenditure of steam.

It has been found by experiment, that 1 lb. of ordinary fuel—coal or coke—is capable of turning from 8 to 10 lbs. of water into steam, according to the capabilities of the boiler. Taking the lower estimate, we should reckon that as 63 lbs. of water are required per horse-power per hour, about 8 lbs. of fuel per horse-power per hour would

be consumed. In condensing engines, working expansively and under the most advantageous circumstances, the consumption of fuel has been reduced so low as from 2½ lbs. to 3 lbs. per horse-power per hour. In non-condensing engines, when the steam is used to a considerable extent expansively, from 5 lbs. to 7 lbs. of fuel per horse-power per hour is nearly the average consumption.

It may be readily conceived, that when water contained in a vessel or boiler of limited strength is subjected to heat in such a manner as to turn part of it into steam, exerting unlimited pressure, there would be constant risk of explosion, unless some measures were taken for limiting the force of the steam, and preventing its pressure from exceeding that which the boiler could safely sustain. Every boiler is, therefore, fitted with one or more safety-valves, which constitute the most important of the boiler fittings.

The principle of the safety-valve is exceedingly simple. As fluids, and therefore steam, press equally in all directions, any part of the boiler-casing, such as 1 square inch, is subjected to the pressure of the steam. If, then, we make a hole in the upper part of a boiler 1 square inch in area, and cover it with a lid, laying on this lid a weight such as 50 lbs., we can apply heat and generate steam, which, as soon as its pressure exceeds 50 lbs. per square inch, will lift the loaded lid, and permit a portion to escape. Should the generating power of the boiler be moderate, the raising of the lid, and escape of a portion of steam, would prevent the pressure from ever exceeding that due to the weight on the lid; but should the steam be generated more rapidly than it can escape through the hole, the pressure must go on accumulating, until the strain to which it subjects the boiler exceeds the strength of the material of which it is made, and an explosive rupture is the consequence. It is, therefore, important to provide a safety-valve, with an opening of sufficient size to permit the escape of steam as rapidly as it can ever be generated, and to load it with a weight not greater than the pressure which the boiler can safely bear. Boilers are generally tested before use, under a pressure very much greater than that with which they are to be used. For condensing engines, the pressure seldom exceeds 20 lbs. per square inch; for ordinary non-condensing engines, 50 lbs. or 60 lbs. per square inch; and for locomotives, it is as high as 120 lbs. and 150 lbs. per square inch above that of the atmosphere. The area of the safety-valve should not be less than ¼th square inch per horse-power

Fig. 160 represents a safety-valve of the ordinary construction. A is a box fixed over a hole B in the upper surface of the boiler, having a truly-faced seating on which the valve C can rest. The stem of the valve passes through the cover of the box, where there is a gland or stuffing-box to prevent the escape of steam round the stem. A pipe D conducts the steam that passes the valve, when it is lifted, to the chimney or elsewhere. The valve is kept down by a lever E, which works on a pin, or fulcrum, at F, and has a sliding weight suspended from it at any point such as G. The arm of the lever is graduated so that the weight can be placed to give such pressure on the valve as may be required. If we suppose, for example, that the area of the valve-opening is 1 square inch, that the length from the centre of the valve-stem to that of the pin F is 2 inches, and that a weight of 10 lbs. hangs at G, 16 inches from F; then the effect of the weight to press down the stem of the valve is as its weight multiplied by the length of lever at which it acts, divided by the length of lever at which the valve acts; that is to say, the pressure on the valve is $\frac{10 \text{ lbs.} \times 16 \text{ inches}}{2 \text{ inches}} = 80$ lbs. As the area of the valve is 1 square inch, this weight is capable of resisting a steam-pressure of

80 lbs. per square inch within the boiler. If the lever be graduated by divisions each 2 inches in length, each of these will correspond to a pressure of 10 lbs. on the valve; that is to say, the weight at—

16 inches gives a pressure of 80 lbs.
14 " " " 70 "
12 " " " 60 " and so on.

We have not reckoned the effect of the weight of the valve and lever, which should generally be weighed, so that the pressure due to them, exclusive of the weight, may be

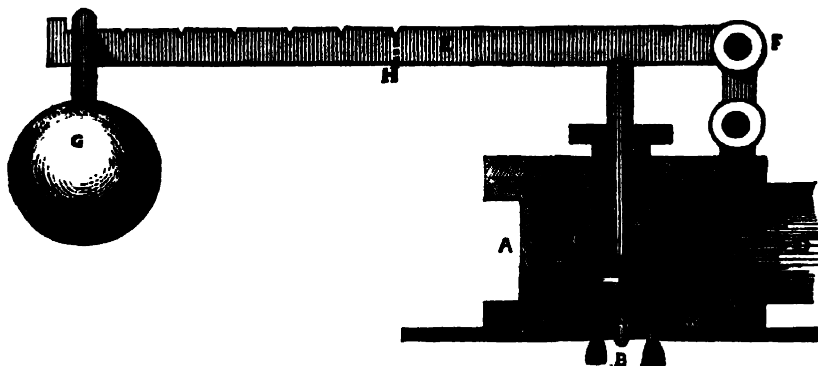


Fig. 160.

estimated before graduating the lever. As a practical example, we will suppose that it is required to make a safety-valve of 4 inches diameter, and load it by a weight and lever graduated to steam-pressures varying from 20 lbs. to 50 lbs. per square inch above atmospheric pressure. We will suppose that the weight of the valve and stem is 5 lbs., that a convenient leverage for the valve is 3 inches, and that the whole lever from F to the end is 30 inches, the lever itself being of uniform depth and thickness, and weighing 9 lbs.

The area of the valve (a circle 4 inches in diameter) is $12\frac{1}{2}$ square inches; the effect of the weight of the lever is the same as if it were collected at its middle point H, 15 inches from F, and its

Pressure on the valve is therefore $\frac{9 \text{ lbs.} \times 15 \text{ inches}}{3 \text{ inches.}} =$ . . . . . 45 lbs.

To which we add the weight of valve and stem . . . . 5 "

Making a total constant weight on the valve = ... 50 ,

And as the area of the valve is $12\frac{1}{2}$ lbs., this gives a constant pressure of $\frac{50}{12\frac{1}{2}} = 4$ lbs. per square inch. For a pressure of 50 lbs. per square inch, or a load of $50 \times 12\frac{1}{2} = 625$ lbs. on the valve, the additional load must be 575 lbs. at a leverage of 30 inches against that of the valve at 3 inches; and therefore a weight of $57\frac{1}{2}$ lbs. at the end of the lever gives the required pressure; because $\frac{57\frac{1}{2} \text{ lbs.} \times 30}{3} = 575$ lbs. on the valve. The same weight, to give a pressure of 40 lbs. per square inch, should act on the valve with a force of $12\frac{1}{2} \times 40 - 50 = 450$ lbs.; and its distance from F must be about 23·48 inches, because

$\frac{57\frac{1}{2} \times 23{\cdot}48 \text{ inches}}{3} = 450$. Now the difference between 30 inches, the leverage for 50 lbs., and 23·48 inches, the leverage for 40 lbs., is 6·52 inches—a division that may be repeated along the lever for 30 and 20 respectively.

We might attain the same result by another process, thus:—Since the constant pressure due to the weight and value is 4 lbs. per square inch, the additional pressure to be derived from the weight of 57½ lbs., to make a total of 10 lbs. per square inch, would be 6 lbs. per square inch, or $6 \times 12\frac{1}{2} = 75$ lbs. in all. The leverage of the weight to produce this load would be found from the simple proportion:—

Weight.		Load on valve.		Leverage of valve.		Leverage of weight.
57½ lbs.	:	75	::	3 inches	:	3·913 inches.

Repeating the same process for 50 lbs. pressure per square inch, we should find the leverage of the weight to be 30 inches. The difference of 30 inches and 3·913 inches, viz. 26·087 inches, being divided into 40 equal parts, each 0·652 inches—because 40 is the difference between 50 lbs. and 10 lbs.—would mark the lever for each lb. of pressure. Every 10lbs. would thus be graduated by intervals of $10 \times {\cdot}652 = 6{\cdot}52$ inches as before.

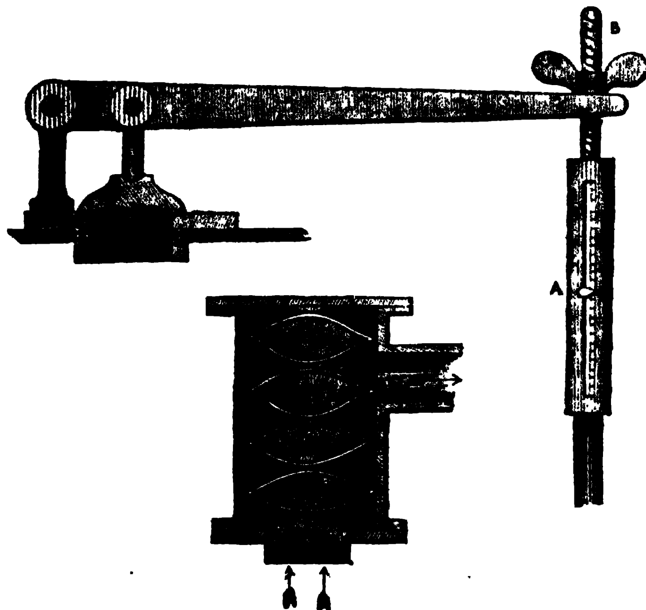


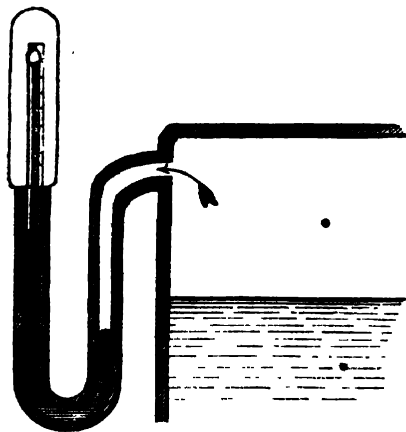
Fig. 161.

In locomotives and boilers where a weight sliding along a lever would be inconvenient, the lever is affixed to a spring-balance A (Fig. 161), graduated to the pressures per square inch due to the spring. By turning a nut B on the stem of the spring-balance, any required pressure can be thrown upon the valve, which is kept down by the spring acting on its lever. Should the pressure within the boiler exceed that to which the balance is adjusted, the valve is opened, and a portion of the steam escapes. The lock-up safety-valve consists of a valve pressed down by a set of strong springs C, the whole enclosed within a box under lock and key. While the engine-driver has command over the spring-balance valve, so as to increase or diminish the load at pleasure, the lock-up valve is inaccessible to him,

and opens whenever he has loaded the open valve beyond the pressure to which the lock-up valve has been adjusted; thus serving as a check upon him in case of his working at a dangerous pressure.

In a boiler for an engine working at very low pressures, there is frequently provided a vacuum valve, which is a safety-valve opening inwards, and admitting air into the boiler, in case the pressure within should fall so far below that of the atmosphere without that there might be danger of collapse.

The steam-gauge is an apparatus generally fitted to boilers for indicating the pressure of the steam. The safety-valve may be employed for this purpose; for if the weight be adjusted on the lever, or the spring of the balance released until the valve begins to open and let steam escape, we know that the weight or spring in that condition is a measure of the pressure. But as this mode of measuring the pressure requires personal attendance, it is better to be provided with some self-acting instrument which shall show at a glance the condition of the steam in respect to pressure; for low-pressure boilers the mercurial steam-gauge is generally employed (Fig. 162). It consists of an iron pipe bent to a siphon form, connected with the boiler, and containing mercury, on which floats a rod of wood extending above the mouth of the tube, and pointing to divisions on a scale. As 2 cubic inches of mercury weigh very nearly 1 lb., the rise of the wooden index through 1 inch in height indicates that the mercury in one limb of the siphon has risen 1 inch and fallen 1 inch in the other, making a difference of level of 2 inches, equivalent to a pressure of 1 lb. per square inch in the boiler. Thus every inch on the scale corresponds to 1 lb. pressure per square inch. For high-pressure boilers, the column of mercury necessary would be inconveniently high, and recourse is therefore had to gauges of other kinds. Among the most effective and ingenious of these may be mentioned that of Bourdon. It consists of a flattened elastic tube of copper or brass, bent into a spiral form. The pressure within the tube tends to bulge it, and uncoil it a little out of the spiral form; and the slight movement thus induced is communicated to an index, which points on a dial-plate to the pressure marked thereon from the result of experiments made for the purpose of determining the proper graduation of the dial.


Fig. 162.

The steam generated in the boiler at such pressure, and in such quantity as may be desired, is conveyed by the steam-pipe to the cylinder, which is a vessel closed at both ends, and fitted with a piston E (Fig. 163), capable of sliding tightly from end to end, and having a rod F passing tightly through one of the end covers, or the cylinder lid. If we suppose A and B two pipes communicating with the boiler, and opening into the cylinder at opposite ends, while two other pipes C and D lead from the ends of the cylinder to the open air, or to any suitable place; conceiving these pipes to be provided with stopcocks, we can see that by opening A and D, while B and C are closed, we admit

steam to press upon the upper surface of the piston E, and force it to the bottom of the cylinder, while the contents of the part below the piston escape by D. On the piston reaching the bottom, if we open B and C while A and D are closed, the pressure acting on the lower side of the piston forces it upwards, while the steam above escapes. Thus, by alternately opening and closing the four stopcocks in proper order, an alternating motion is given to the piston, and the force is communicated by the rod to any suitable machinery without the cylinder. The amount of force so communicated depends on the size of the piston, or number of square inches in its surface on which the steam-pressure acts, the intensity of that pressure, and the velocity at which the piston is caused to travel. If we suppose, for instance, that the diameter of the cylinder is 1 foot, the circular area of which is 113 square inches, that the pressure of the steam is 20 lbs. per square inch, and that the average speed of the piston is at the rate of 200 feet per minute, the power communicated through the rod is equivalent to

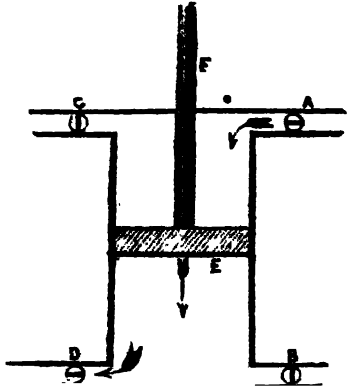


Fig. 163.

$$\frac{113 \text{ sq. ins.} \times 20 \text{ lbs.} \times 200 \text{ ft.}}{33000} = 13{\cdot}7 \text{ horse-power.}$$

Some of the first steam-engines had cocks arranged as we have described, which demanded the continual attendance of some one to close or open them at the proper times. But it suggested itself that apparatus connected to the moving parts of the engine might be so adjusted as to perform this operation; and accordingly great ingenuity has been exhibited in contrivances for alternating the flow of steam to and from the opposite ends of the cylinder. Instead of stopcocks in the pipes, which are subject to considerable and unequal wear from constant working, and thereby become leaky, valves are often employed, similar to the safety-valve, and worked upwards and downwards by means of levers acting on their stems, which project beyond the steam-tight casing in which they are enclosed. Were these valves made in such a way that the steam pressed on their lower surfaces, and tended to raise them from their seats, it would be difficult to keep them tightly down without very considerable force. If, on the other hand, the pressure of the steam acted on their upper surfaces so as to keep them tightly down, considerable force would be required to lift them so as to permit the steam to pass at the proper times. Moreover, as in large engines these valves must be of considerable size to let sufficient steam pass through the openings they cover, and as for effective working they must be raised and lowered very rapidly, it becomes important to reduce as low as possible the pressure upon them, and thus diminish the force necessary for their movement.

The double-beat valve is a contrivance for covering a large area of steam-passage with a valve subjected to moderate pressure. The steam entering at A (Fig. 164), fills the valve-box, in which an annular or ring-shaped valve B is capable of being pressed upwards or downwards by a rod C passing through the cover of the valve-box. When the valve is down, its upper and lower conical surfaces rest on corresponding seats, to which

they are nicely ground; while the valve being raised, the steam flows between these surfaces into the pipe D. The force necessary to raise such a valve, is that of the steam-pressure on the excess of the area of the lower seating above that of the upper, and may be made much less than that on an ordinary valve of equivalent size. We shall suppose, for example, that a valve of 10 inches diameter is subjected to a pressure of 20 lbs. on the square inch. The force to lift it would be

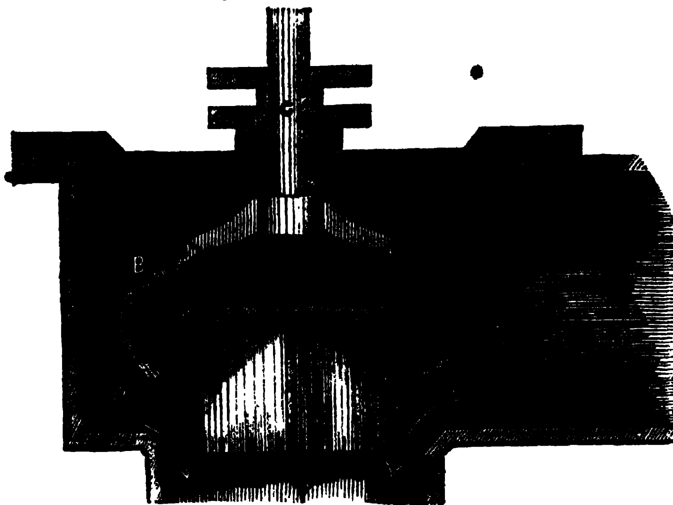


Fig. 164.

(circ. area of 10 ins. diam. =) $78\frac{1}{2}$ sq. ins. × 20 = 1570 lbs.;

and the height through which it must be lifted to give the full passage for steam round it, is $2\frac{1}{2}$ inches; for $2\frac{1}{2}$ × (circumf. of 10 ins. diam. =) $31\frac{1}{2}$ = $78\frac{1}{2}$ sq. ins., the area of a circle 10 ins. in diameter. But were we to use a double-beat valve, having its lower seating $10\frac{1}{2}$ ins. in diameter, and its upper 9 ins. in diameter, the area of the one being $86\frac{1}{2}$ sq. ins. and that of the other $63\frac{1}{2}$, the pressure on the difference, 23 sq. ins., is 23 × 20 = 460 lbs., little more than one-fourth of that on the single-seated valve. Again, the height through which the double-beat valve must be raised is only half that required for the single valve; because when it is raised, passage is provided for the steam both above and below. Thus, to work the double-beat valve, only $\frac{1}{7}$th or $\frac{1}{8}$th part of the force requisite for the single-seated valve is required. Wherever valves are used, especially in large engines, or under great pressures, for alternating the flow of steam, recourse is had to the double-beat valve, or some expedient of a similar character, by which considerable size of passage may be secured without having to lift a great weight, or move it through a great distance.

The frequent raising and lowering of any set of valves, however well balanced, would, however, in quickly-moving engines, be accompanied with noise, and would prove very inconvenient, on account of the complication of machinery required for the purpose, and the greater amount of wear and tear resulting from its use. To avoid these evils, the slide has been contrived; and it is almost universally employed for alternating the flow of steam to or from the ends of the cylinder, except in engines moving very slowly. There are various kinds of slides in use, but they are nearly all contrived on similar principles, with such differences in the details of construction as the peculiar views of makers, or the circumstances of their position, suggest. The most simple kind is called the D-slide, from the circumstance of its form resembling that of the letter D.

Figs. 165 and 166 represent two longitudinal sections of a cylinder fitted with a D-slide. The steam enters by a pipe B from the boiler into a cavity called the *slide-jacket*, on the opposite side of which are three openings, the upper and lower, called the *ports*, communicating by tubes or passages with the upper and lower ends of the cylinder respectively; and the middle one communicating by a cavity E with an opening at the side, by which steam can escape. These three openings are partially covered by a hollowed plate of metal, the *D-slide*, which, as its name implies, can be made to slide up or down by means of a rod C passing tightly through the jacket. The hollowed part of the D-slide is made to embrace the middle passage and either of the ports, so as to let steam escape from the cylinder, while it leaves the other port open for the ingress of steam to the cylinder; and as for every ascent and descent of the piston in the cylinder, a corresponding ascent and descent of the slide is effected by means of apparatus connected with the moving parts of the engine, the complete successive alternation of the steam is maintained without the expenditure of more power than is necessary to overcome the friction of the slide over the *facings* in which the ports are situated. The face of the slide, and the surface on which it rubs, are made very true and smooth in the first place; and when they are not subjected to undue wear by the ingress of dirt or grit, their contact remains steam-tight for a long period. The slide we have described is called the short D-slide, and is generally used in locomotives and engines which have not long cylinders. But when the cylinder is of considerable length, the passages from the ports to the ends of the cylinder are also long; and having to be filled with steam at every stroke or alternation of the piston, which is ineffective in producing power, considerable loss is occasioned from this waste of steam in

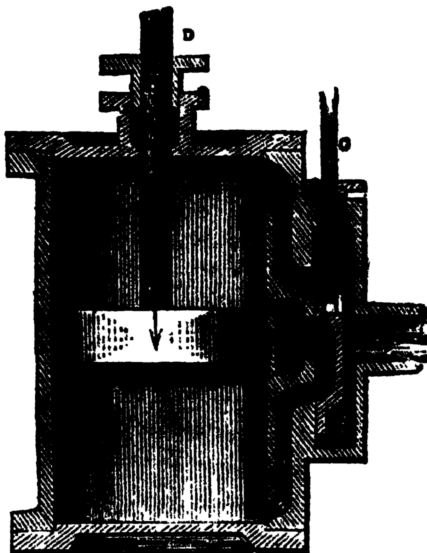


Fig. 165.

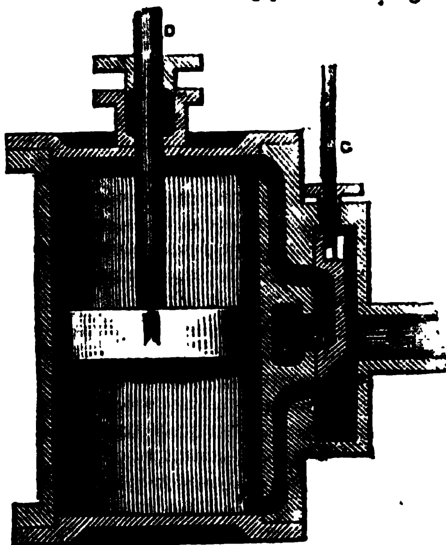


Fig. 166.

merely filling the passages. To obviate this defect, the slide is sometimes lengthened, so that the passages from the ports to the cylinder are proportionally shortened. It is found convenient for the construction to make this kind of slide of a hollow cylindrical form, fitting into a cylindrical jacket at each end, and being smaller in diameter at the middle. The steam entering at B (Fig. 167) fills the cavity surrounding the slide, and gets access to the upper or lower port as the slide is moved upwards or downwards, while the steam passes from either of these ports into the cavities at the upper or lower ends which communicate through the tubular body of the slide, and from one of which the waste-pipe C conveys the steam which has done its work.

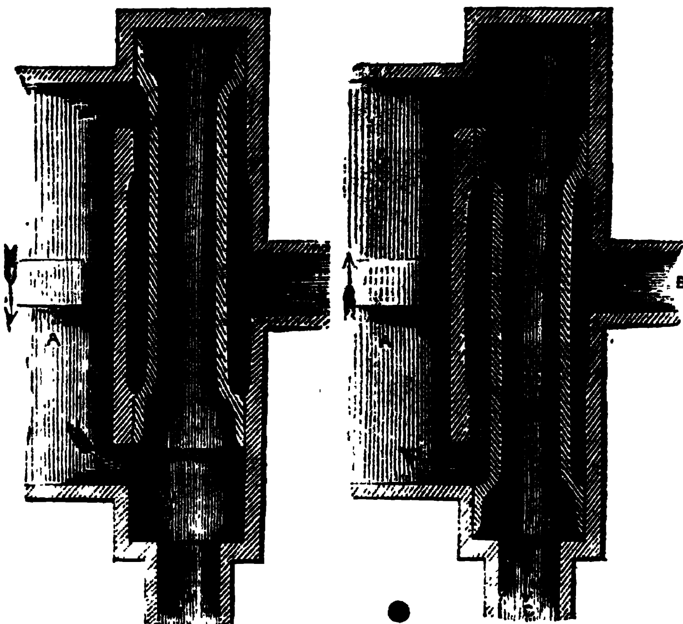


Fig. 167.

We might mention many other varieties of slides; but all being constructed on similar principles to those we have described, we need not discuss them in detail. We will now proceed to describe the practical construction of a cylinder, piston, and slide, such as would be suitable for a non-condensing engine.

Referring to Figs. 165 and 166, which give a general view of the cylinder and slide, we have to inquire into the proportions of the parts and the details of their construction in such a manner as to be economical, durable, and efficient.

The cylinder is made of cast-iron, bored in a suitable lathe so that the interior is as nearly as possible perfectly cylindrical. The covers are also of cast-iron, having a projecting part turned to fit into the ends of the cylinder, to which they are secured by bolts and nuts. The mouths of the cylinder are generally bored somewhat larger than the rest, so that if after some years' wear it become necessary to bore out the cylinder afresh, thereby making its diameter a little larger, the same covers may still fit it. The flanges or projecting rims of the cylinder and the faces of the covers which lie against them are turned very true; and if well smoothed require only a little thin flour-paste to be spread over them to render the joints impervious to steam when the bolts are screwed tightly up. The length of the cylinder is determined by the length of stroke or movement of the piston: it should be such as to allow from $\frac{1}{4}$th to $\frac{1}{2}$ an inch clearance

between the piston and the cover at each end of the stroke. Theoretically, the less clearance the better, because any space left between the piston and cover at the end of the stroke has to be filled at every stroke with steam uselessly. But, practically, it may be difficult to determine the length of stroke with perfect accuracy; the wear of the machinery may alter it a little, and water or dirt may get into the cylinder, so that it is necessary to allow a little space, such as we have named, for clearance. When the steam first enters a cold cylinder it becomes condensed into water; and not unfrequently during the working of the engine, the boiler primes or boils over, sending water along with the steam into the cylinder. But as water is practically incompressible, its presence between the piston and the cylinder-covers at each end of the stroke would be quite as detrimental as the presence of a mass of iron or any hard material, unless an exit were provided for it; for the sudden approach of the piston would be arrested by the water, and either the cover would be forced off or the parts connecting the piston to the rest of the machinery would give way. Accordingly, cylinders are often fitted with relief-valves—small safety-valves loaded by weights or springs, and communicating with each end of the cylinder, so that whenever, by the presence of water, the pressure becomes increased to a dangerous extent, the valve is opened, and permits the water to escape. In the absence of relief-valves, *pet-cocks* or small stopcocks are fitted for the same purpose. On first admitting steam to the cylinder, these are left open to permit the escape of the water arising from condensation; and during the working of the engine they may generally be left a little open, especially when the presence of water is manifested by a sharp blow, heard when the piston strikes upon it. The thickness of the cylinder depends on its diameter, and the pressure to which it is subjected; and the strength of the flanges, covers, and bolts and nuts must be determined on the same ground. For such details it is difficult to give precise rules, as experience and study of well-proportioned works can alone give the power of determining them.

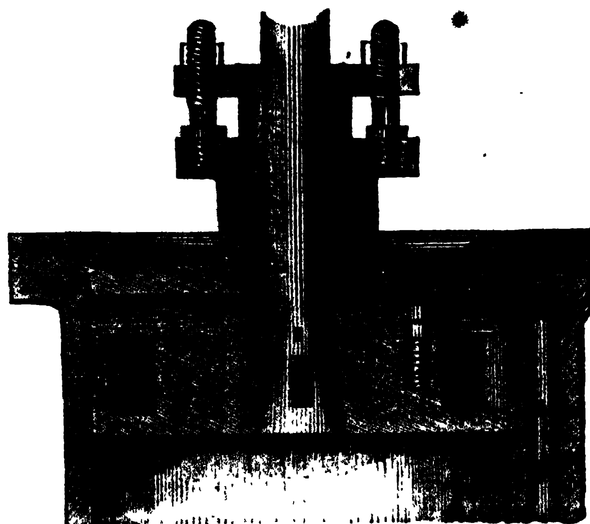


Fig. 168.

The piston is constructed in various ways, one of which, being simple and effective, we will describe:—The body of the piston consists of a disc and boss A, the outer edge of the disc fitting the cylinder, and the boss having a central conical hole, in which the piston-rod B is secured by means of a key, or thin bar of iron slightly tapered in width, driven through a slot in the boss and rod, so as to tighten the conical end of the rod in the corresponding conical hole. To the body of the piston is secured by screws a cover

C fitting the cylinder, and leaving between it and the edge of the disc below a groove, which contains the packing-ring D. This ring is made to fit the cylinder, and is cut obliquely across at some point of its circumference E, a parallelogram-shaped hole being cut out of the middle, and filled with a piece of metal truly fitted to it. By thus cutting the ring across at E, it is permitted to expand in diameter; and the slits made at E are covered by a plate F inside the ring, so that no steam can pass by them from one side of the piston to the other. Several bent pieces of steel-plate G are placed between the ring and the boss of the piston, so as to push the circumference of the ring outwards.

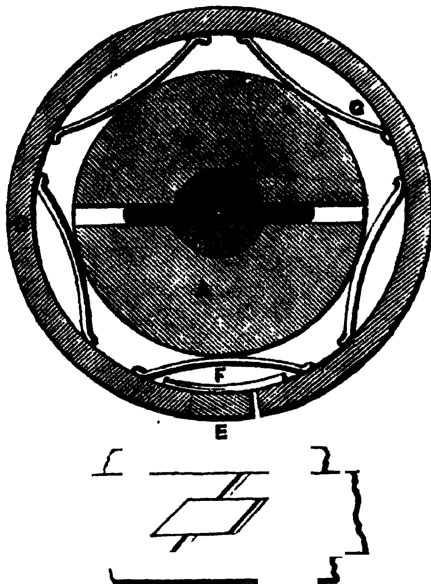


Fig. 169.

As the inside of the cylinder and the edges of the piston and its cover become worn by constant rubbing, the packing-ring is made to expand, and still to work tightly in the cylinder, without permitting the flow of steam past the piston. Sometimes, for small pistons, the packing-ring is merely made thicker at the side opposite its slit; and being at first slightly larger than the cylinder, so that it must be compressed when pushed into it, its own elasticity makes it expand to fit the cylinder even after considerable wear, without the necessity for steel springs within it (Fig. 170).

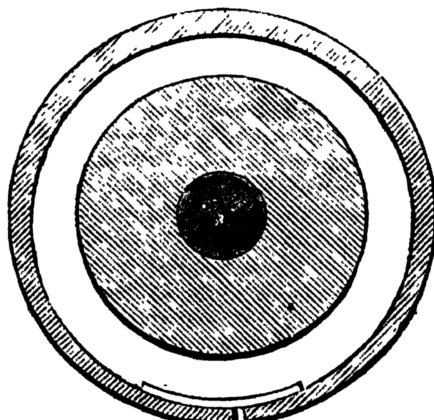


Fig. 170.

The piston-rod, in passing through the cylinder cover, is surrounded by a cavity called a stuffing-box, and filled with soft twisted hemp and tallow, called *packing*, which is compressed in the cavity by means of a *gland*, forced down upon it by tightening screws. By the use of this packing, while the rod travels upwards and downwards, steam cannot pass round it; for even if the rod be worn somewhat irregularly, the elasticity of the packing serves to prevent the leakage of steam.

The slide-rod, and, indeed, all rods about an engine for moving valves or parts within cavities containing steam or water, have to pass through packing of this kind.

For opening or closing the communication between the boiler and the cylinder, so

as to admit steam from the one to the other, to diminish the quantity or entirely arrest it, a stopcock is generally employed for small engines, and for larger ones a shut-off valve. The stopcock, being sufficiently well-known, we need not describe. The *shut-off* or *stop-valve* is generally made as indicated in Fig. 171. The steam-pipe A communicates with the valve-box, having a pipe B proceeding from it. The mouth of this pipe is fitted with a conical edging or seating, to which a conical-edged valve C is nicely fitted. Through a stuffing-box D in the cover of the valve-box passes a screwed rod, connected by a free joint to the valve C; when the rod is turned round by a handle E, so as to screw it inwards through the gland D, the valve is pressed firmly down on its seating, and thus all communication from A to B is cut off. By unscrewing the rod, the valve is raised from its seating as much as may be required for the passage of steam.

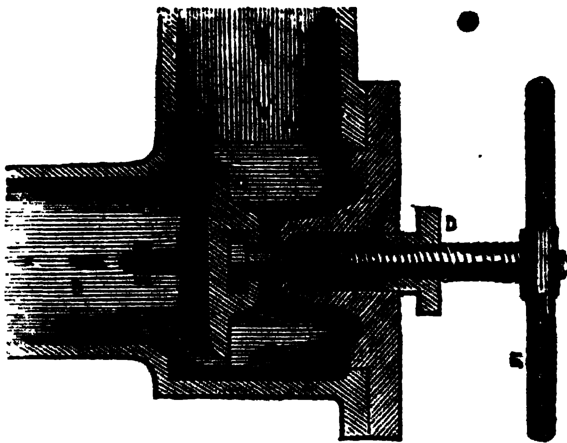


Fig. 171.

The throttle-valve is for the purpose of choking or throttling the passage of steam in a pipe, so as to regulate the quantity passing through it in a certain time, without perfectly arresting it. It consists of a disc A (Fig. 172) mounted on a rod passing across the pipe through a stuffing-box at one side. On turning the disc edgeways towards the current, the steam is allowed to pass; but when it is turned across the pipe, none can pass except such a small quantity as can leak round the edges of the disc. This valve is almost universally applied to engines which are fitted with a governor; and by its means the speed of the engine is regulated with great nicety, as more or less steam is permitted to pass the valve.



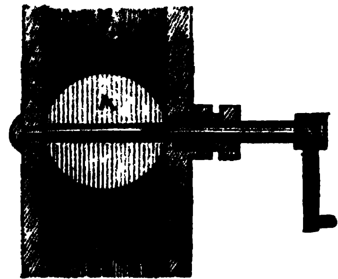


Fig. 172.

The feed-pump of an engine is for the purpose of supplying the boiler with water to take the place of that which is boiled off, and passes away in the form of steam after working the engine. It consists of a barrel O (Fig. 173) through a stuffing-box, in the upper part of which a plunger G is worked alternately upwards and downwards.

At the lower end of the barrel is a valve-box, containing a suction-valve B, covering a pipe A by which the water is drawn from a convenient reservoir, and a discharge-valve D, past which the water has to flow in its progress to the boiler by the feed-pipe E. When the plunger is raised, the valve D being kept down in its seat by the pressure of water in the boiler (equivalent to that of the steam) communicated through the feed-pipe E, the vacuum left by the rise of the plunger is filled by water entering from A and raising the valve B for its passage. On the descent of the plunger, the water being forced out of the barrel, presses down the valve B, but raises the valve D, and flows onwards to the boiler. At some convenient part of the feed-pipe E there is always fitted a stopcock, or shut-off valve, for completely cutting off communication between the pump and the boiler in case of the valves being deranged. But water being almost totally incompressible, it would be extremely hazardous to close this communication while the pump is in action; for in that case the barrel must be burst open, or some part of the machinery that works the pump must be broken. It is, therefore, usual to provide also a relief-valve, constructed exactly like a safety-valve on the feed-pipe, to permit the efflux of the water when its ordinary passage is closed. The cover F of the feed-valve box should be capable of being readily removed to give access to the valves; and it is often made of considerable size, hollowed out to contain air, which is compressed by the influx of water during the descent of the plunger, and reacts to force the water onwards to the boiler while the plunger ascends. In all cases, indeed, where water under considerable pressure is exposed to the recurring action of a propelling force, as in the feed-pump, an air-vessel should be provided to act as a spring relieving the blow on the water, and regulating its motion to a gradual flow instead of a sudden movement.

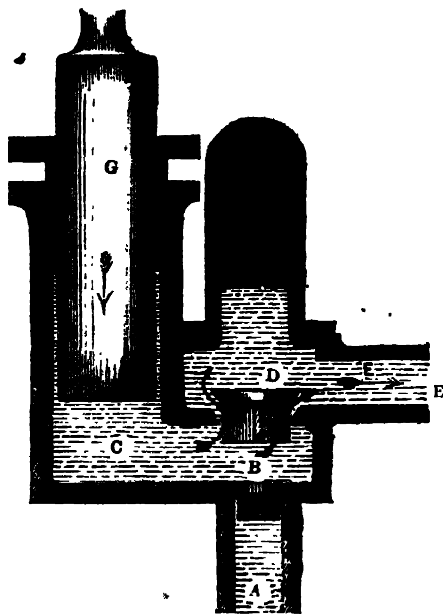


Fig. 173.

Having now described the cylinder in which, and the piston on which, the steam acts to put the machinery in motion—the *slide* by which the alternation of the course of the steam is effected—the *throttle-valve* by which its quantity is regulated—and the *feed-pump* by which the necessary supply of water is maintained in the boiler: we have to inquire how the reciprocating motion of the piston is converted into the rotary movement required for driving machinery, and how this rotary movement produces the reciprocating motion of the slide and feed-pump, and governs the action of the throttle-valve.

To the end A (Fig. 174) of the piston-rod there is jointed the connecting-rod A B, having an eye at B working on the *crank-pin*, or pin fixed to the *crank*—an arm B C projecting from the main shaft or spindle C. The crank-pin can move round in a circle,

the diameter of which is exactly equal to the length of stroke, or distance through which the piston travels in the cylinder. As the piston descends from F to G, making the down-stroke, the crank is caused to revolve from E round to D, one half-revolution. Again, while the piston ascends from G to F, making the upstroke, the crank revolves from D round to E, another half-revolution. Each revolution, then, of the crank requires a double stroke of the piston; and to effect it, the upper and lower portions of the cylinder have each to be filled and emptied of steam; or as their capacities are equal, the cylinder has for each revolution to be twice filled and emptied. It is obvious that at every different point of its revolution the crank is acted on by a different force, owing to the varying obliquity of the connecting-rod. At the two extreme points, D and E, where the crank is in a line with the connecting-rod, the effect of the piston to cause it to revolve is reduced to nothing; for it merely pushes or pulls it against the central shaft. These points are technically called the *dead centres*, because there the force of the piston is dead or ineffective. But to make up for the total want of action at those points, we find that at some other points the effect of the force passing through the connecting-rod to turn the crank is greater than the pressure on the piston, in consequence of the obliquity of its action.

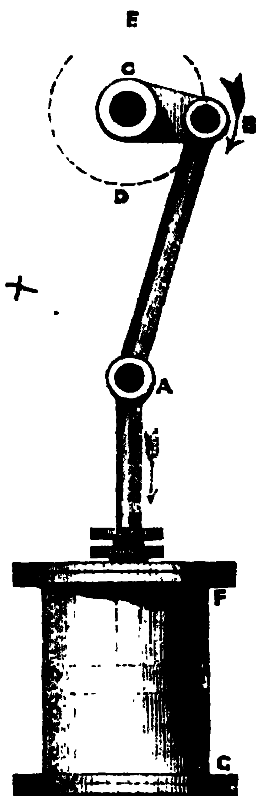


Fig. 174.

Again, the piston, during a revolution or double stroke, passes through a distance equivalent to twice the diameter of the crank-circle; while the crank-pin passes over the circumference of that circle, more than 3 times its diameter. The influence of the pressure on the piston to turn the crank may be best conceived by a graphical delineation of the force in the following manner:—If we divide the circle described by the crank-pin into any number of equal parts (Fig. 175), and draw a straight line A B equal to the half-circumference divided into corresponding parts, A B represents the distance through which the crank-pin moves during half a revolution, developed into a straight line. If at any point, such as 2, we draw a line 2 C representing the crank, and a line 2 P representing the connecting-rod, and draw Q 2 touching the circle at 2, and therefore representing the direction in which the crank-pin is moving at the point 2, while P C represents the direction of the piston, we may take any length P D representing the force of the piston (as, for instance, if the pressure on the piston were 5 tons, we might take P D = 5 inches), draw D E perpendicular to P D, P F parallel to Q 2, and E F perpendicular to P F. Then, on the principle of resolved forces, the length of P E represents the force transmitted through the connecting-rod, and P F the force tending to turn the crank, while D E measures the side thrust on the piston-rod, and E F the longitudinal strain on the crank, pushing the shaft against its bearings. If, now, at the point 2 in the line A B we erect a perpendicular 2 R_2 equal in length to P F, and at the points 1, 3, 4, &c., erect others deduced by the same kind of construction, we can trace

a curve through their summits, the height or ordinate of which at any point measures the force turning the crank-pin at the corresponding point of its circumference; and therefore the area or space enclosed between the curve and its base A B, which may be considered to be made up of an indefinite number of these ordinates, measures the total force expended on the crank during a half-revolution. If, now, we take a line G H equal to the diameter of the crank-circle or the stroke of the piston, and dividing it into any number of equal parts, erect ordinates each equal to P D, and therefore representing the constant force of the piston, the area of the rectangular figure G H K L thus formed will, in like manner, measure the total force of the piston during one stroke. It will be found that the area of this figure is exactly equal to that of the curvilinear figure, as might be predicted from the knowledge of the mechanical principle, that by no combination of machinery can we create or annihilate force; and that consequently whatever power the piston during its straight stroke impressed upon the crank, is found in the crank during its circular movement. By altering the length of the connecting-rod, as compared with that of the crank, we alter the figure of the force-curve, but we do not change its area; and whether the connecting-rod be long or short, the power conveyed through it during the half-revolution is constant. The other half-revolution being effected under similar circumstances, would give a *force-curve* precisely like that of the former.

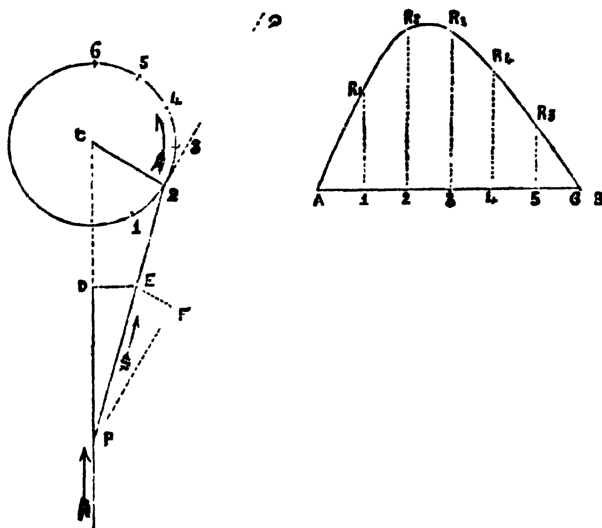


Fig. 175.

In this investigation we have observed that the oblique action of the connecting-rod causes a lateral thrust on the piston-rod, measured by the line D E. It will be found that the longer the connecting-rod, the less will this lateral thrust be; and as it is a force not only useless to the machinery but positively prejudicial, as it tends to bend the piston-rod or force it out of its straight path, it is advantageous to reduce it to as small a quantity as possible, and to provide means for counteracting its influence. For this reason, a long connecting-rod, three or four times the length of the crank radius at least, should be employed, and the end of the piston-rod should be made to move in guides so as to prevent it from being deflected. As the piston at the dead centres has no influence in causing the crank to revolve, it is necessary to provide some means of making up for this deficiency. On the crank shaft there is fixed a large heavy wheel, which revolves

with it, and acts as a reservoir of force to carry the crank round the dead centres and otherwise to equalise the movement. A large mass of matter, such as the fly-wheel, weighing sometimes many tons, cannot be put in motion at a high velocity without the expenditure of great force; but when it is in motion it requires as great force to arrest it. In the motion of a crank connected with a fly-wheel, while the crank is receiving its most advantageous impulse from the piston at such points as 2, 3 or 4, it communicates some of its overplus to the fly-wheel, which is there stored up in the form of momentum or active power, to be given out upon the crank and any machinery connected with it, when it is at the points 1 and 6 receiving no power from the piston.

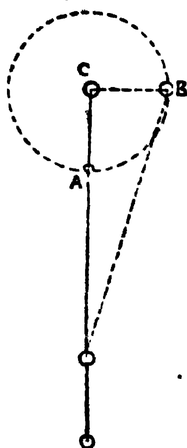


Fig. 176.

In cases where a fly-wheel, on account of its weight and bulk, cannot be applied—as in marine and locomotive engines—the engine is made in duplicate, with two cylinders, pistons, connecting-rods, and cranks, the cranks being fixed on one shaft at right angles to each other. While the one crank is on its dead centre, as C A (Fig. 176), and receiving no rotary impulse; the other crank C B is nearly at its best position for receiving the force of its piston. As the whole power of an engine has to pass through its connecting-rod, the joints which connect it with the piston-rod and the crank require to be made of great strength, and with precautions against friction and wear. The pins of the crank and piston-rod, on which these joints work, are made of wrought-iron, for the sake of strength; and as the friction of like metals upon each other is found to exceed that of different metals, the eyes at the ends of the connecting-rod are lined or *bushed* with brass, gun-metal, or some soft metal, such as tin alloyed with copper. The special construction of those eyes depends upon circumstances, different engineers having preferences for different forms.

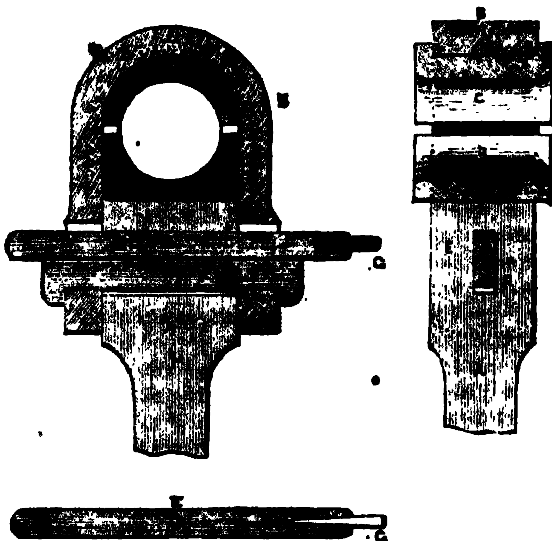


Fig. 177

In Fig. 177 is represented what is called the *strap-eye*. The end A of the connecting-rod is squared, so that a wrought-iron strap B, bent to horse-shoe form, can slide on to it. Between the arch of the strap and the flat end of the rod, are inserted the upper brass C and the lower brass D, generally made of gun-metal, sometimes lined with

soft metal. The brasses have projecting lips or flanges at each side to prevent them from moving sideways within the eye. Through the sides of the strap and the head of the connecting-rod is cut a slot, into which are fitted two pieces of iron, the key E and the gib F, each slightly tapered, so that when the key is driven gently into its place by a hammer, it acts as a wedge, pulling the strap down, and thereby tightening the brasses on the pin to which they are fitted. A little space is left between the edges of the brasses to permit their closer approach as they or the pin become worn in the hole; and the slot for receiving the gib and the key is extended upwards in the strap and downwards in the connecting-rod to permit the key to be driven. Lest the key should be shaken loose by the motion of the machinery, it is often split open at the small end, and a wedge G is driven in to spread it laterally.

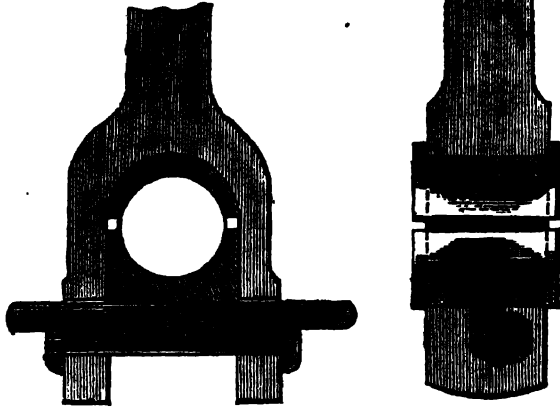


Fig. 178.

The fork-end (Fig. 178) is made by forming the end of the connecting-rod like a fork, fitting it with gun-metal bushes, and tightening them by means of a gib and key. The bushed-eye (Fig. 179) is formed by shaping the end of the rod into an eye, in which

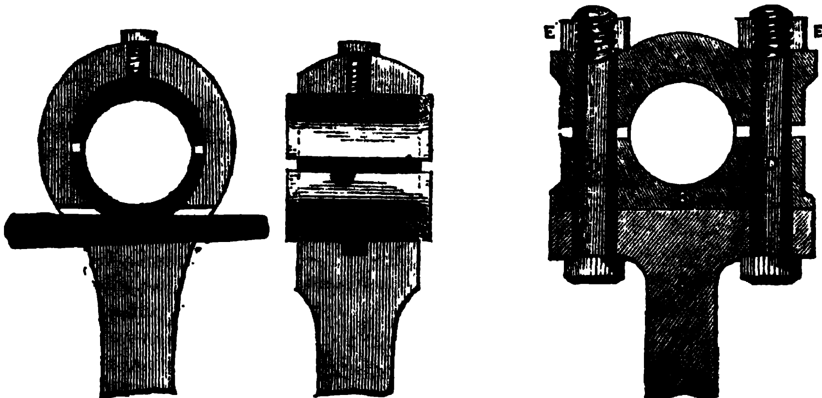


Fig. 179. Fig. 180.

gun-metal bushes are fitted, capable of being tightened by a key passing through a slot in the rod, and bearing against the lower bush, which is notched to receive it. Sometimes the eyes are made as represented in Fig. 180. The end of the rod A is spread out,

on which are fitted two brasses made with a hole to fit the pin, and secured by means of bolts and nuts D E passing through the end of the rod, and holes in the brasses fitted accurately to receive them.

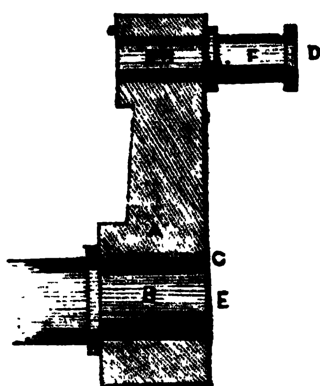


Fig. 181.

Such are the modes principally used of forming the eyes of connecting-rods, or of any joints through which considerable strain has to be communicated.

The crank A (Fig. 181) is made sometimes of cast-iron for stationary engines, but generally, for marine engines, of wrought-iron, having a hole fitted to B the round end of the shaft, on which it is fixed, and prevented from turning by a key C or tapered piece of iron, driven tightly into a slit, formed partly in the crank and partly in the shaft. Generally, the round hole in the crank is made slightly smaller in diameter than the round end of the shaft; the crank is heated so as to expand it, and permit it to be driven on the shaft; and as it cools it tends to contract, and thereby becomes very firmly bound on to the shaft, the end of which is rivetted or hammered over the hole. The crank-pin D is made to fit truly into a hole in the crank, in which it is sometimes secured by a key or pin driven through it transversely. The distance E F between the centre line or axis of the shaft and that of the pin is called the *throw* of the crank, and it is exactly half the

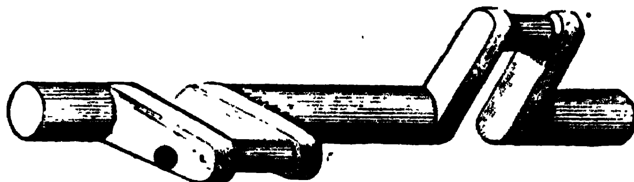


Fig. 182.

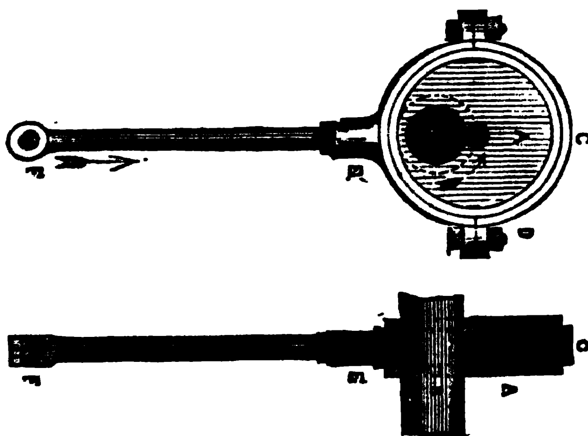


Fig. 183.

length of the *stroke* of the piston. For locomotive engines, and, indeed, for many of the largest marine engines, where double cranks are required, it is usual to forge both cranks and the shaft in one piece, as indicated in Fig. 182. By this means great strength and simplicity are secured, and all the risk of the loosening of parts put together by keys or otherwise is avoided.

As, by means of a crank, the reciprocating motion of a piston is converted into a continuous rotary motion of the shaft, so the continuous revolutions of the shaft may, by means of a crank, be converted into a reciprocating movement for the slide or feed-pump; but for this purpose, instead of a crank, an eccentric is generally employed. It consists of a circular disc A (Fig. 183), having a hole B, not in its centre, through which the shaft passes.* Round the disc is fitted a ring C, generally made in halves, secured to each other by bolts and nuts at D; and to one side of the ring is attached the eccentric rod E F. The eccentric disc, or *sheave*, being firmly fixed on the shaft, is caused to revolve with the latter, and its centre is thus made to describe a circle round the centre of the shaft. The whole sheave thus becomes a crank-pin of extended diameter; and as it can slip freely round within the ring, the end F of the eccentric-rod is caused to move upwards and downwards, during every revolution, through a distance equal to the

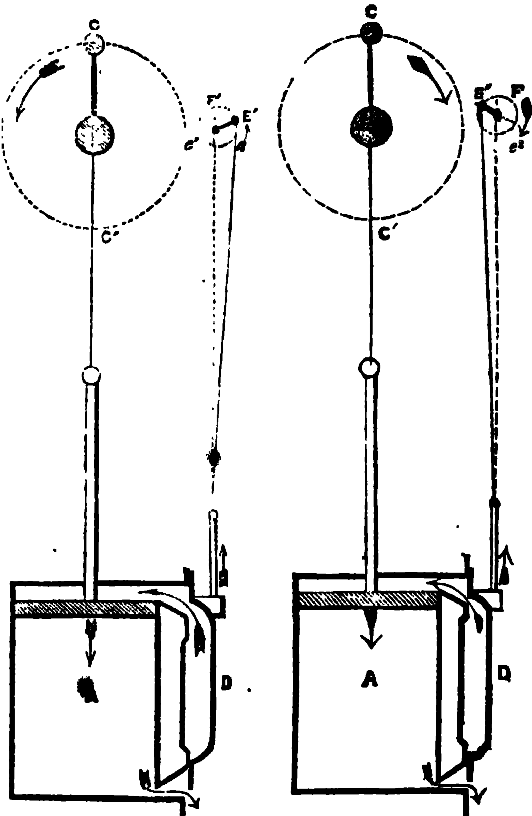


Fig. 184. II.

diameter of the circle through which the centre of the sheave revolves. The radius of this circle is called the *throw* of the eccentric; and its diameter, or the distance through which F is caused to move during a half-revolution, is called the stroke of the eccentric-rod. The setting of the eccentric upon the shaft, or the fixing of its position with respect to that of the crank, is a matter of nice adjustment for causing an engine to act well, as we shall endeavour to describe. Let A (Fig. 184) represent the outline of a cylinder and piston, with the ports at one side, and a long D-slide D fitted to them,

C the circle in which the crank-pin revolves, and E that in which the centre of the eccentric revolves in connection with it. We suppose the piston at the top stroke, and the crank-pin at C. If it is intended that the crank shall revolve in the direction of the arrow in I, the eccentric revolving in the same direction, the slide should be just opening to admit steam above the piston, and to permit its exit from below it. The centre of the eccentric must, therefore, be at some such position as E',—so that, as it continues to revolve, it may continue to open the slide to steam above and to eduction below for some time, and then be ready at e' when the crank reaches C' to reverse the movement. Again, if the rotation of the crank be in the opposite direction, as in II, the eccentric centre must for the like reason be at E', a point on the side of the vertical central line opposite to that occupied by E'. In an engine always moving in one direction, the eccentric can be fixed on the shaft in such a manner as to give the proper movement to the slide; but when it is desired that the engine should move in the opposite direction, some expedient must be devised for altering the position of the eccentric on the shaft through an angle measured by twice E' F', or E'' F'', so as to throw its centre as much to one side as it was to the other side of the crank pin. The simplest mode of effecting this object is to make the eccentric sheave loose on the shaft, with a piece A B (Fig. 185) projecting from its face, and a piece, or stop, C D pojecting from the shaft. When the shaft moves in one direction, the eccentric remains at rest until the end C of the stop comes against B, when it is caused to revolve with the shaft. But, if the shaft rotate in the opposite direction, it leaves the eccentric behind, until D come round to A, when it is again caused to revolve,

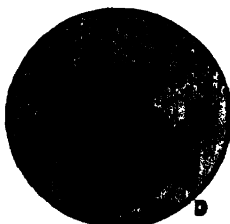


Fig. 185.

but having its centre changed with respect to any fixed point on the shaft, by a quantity determined by the extent of slip before the opposite edges of the stops come in contact. When this arrangement is adopted, the end of the eccentric-rod is generally made of the form indicated in Fig. 186, having what is called a *gab* or round-bottomed notch A fitted to the pin of the slide rod B. When the eccentric-rod is pushed or drawn aside, so as to relieve the slide-rod pin from the gab, the slide can be moved upwards or downwards by hand, independently of any motion of the eccentric-rod, and the movement of the steam above or below the piston thus changed at pleasure. C is a guard or stop to prevent the withdrawal of the rod too far from the pin. The slide-rod is generally made capable of being worked by a lever conveniently situated for the hand of the attendant. When he wishes to reverse

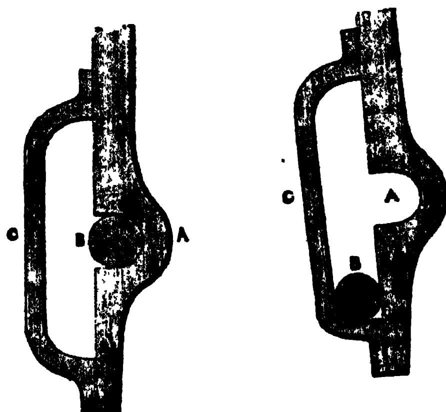


Fig. 186.

the motion of the engine, he quickly withdraws the gab from the slide-rod pin, and by means of the hand-lever moves the slide in the direction opposite to that in which it was formerly moved by the eccentric. He thus throws the steam-pressure on the opposite side of the piston—if it was formerly ascending it begins to descend, and conversely—the movement of the crank and shaft is thus reversed, the stop that drives the eccentric is brought round to the opposite side; and when, after a few alternations of the slide by hand, the reversal of the engine is fairly established, the gab of the eccentric-rod is permitted again to drop over the slide-rod pin, and the contrary motion continues.

When there are two engines working together, such as marine or locomotive engines, the number of operations required for reversal makes it a rather difficult matter. There are two slides, two eccentrics, and two sets of hand-levers; both gabs have to be thrown out of gear, and both levers worked for a time by hand, while great care is demanded on the part of the attendant lest he work them in such a way as to neutralize each other, or oppose the reversing effect which he desires to produce. To overcome these difficulties, an ingenious arrangement, called the *link motion*, has been introduced in marine and locomotive engines. For each of the engines there are two eccentrics fixed side by side on the shaft, and their rods are jointed to an arc A (Fig. 187) with a circular slot in it, in which the pin B of the slide-rod can freely slide. The two eccentrics are fixed on the shaft in such positions that one is adapted for the motion of the shaft in one direction, while the other suits its motion in the opposite direction. When the rod of the one is nearly in a direct line with the slide-rod, it gives it its reciprocating motion, while the other merely causes the arc to oscillate without affecting the motion of the slide; but when the arc and rods are pulled aside, so as to bring the slide-pin under the other eccentric rod, its motion is given to the slide, and the engine is thus reversed. With such an apparatus, then, one simple movement of a hand-lever, connected with the arcs of both engines, causes their immediate reversal. But this is not the only advantage of the link motion. It will be readily seen that the middle point A of the arc being brought round to the slide-rod pin, the latter will be left nearly at rest, for the opposite ends of the arc being moved nearly in opposite directions by the eccentrics will merely oscillate round A, as a fulcrum or centre. By bringing the arc to this position the engines are stopped, because the slide being at rest, admits no alternation of steam above or below the piston. Again, by shifting the arc so as to bring the pin to any point on either side of A, more or less movement of the slide in either direction is produced at pleasure, and thus the quantity of steam passing through the ports into the cylinder may be varied, and consequently the speed of the engine, according as the slide is caused to expose a greater or less amount of opening at the ports for its admission.

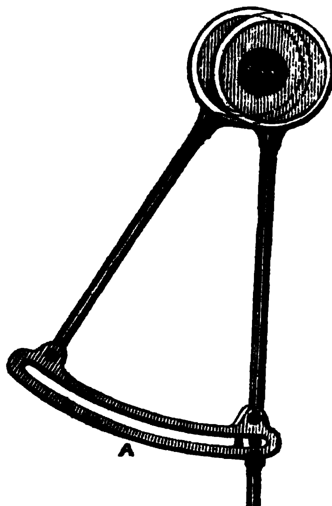


Fig. 187.

When valves are used instead of the slide, for alternating the flow of steam to the cylinder, it is not unusual to move them by apparatus called *cams*; A (Fig. 188) is a

pipe leading from the boiler, B a port to the cylinder, and C a valve, either simple or of the double-beat kind, closing the passage from A to B. The valve-rod C D, passing through a stuffing-box in the cover of the valve-box, terminates in a roller D, which bears upon a cam F fixed on a shaft E, caused to rotate by the engine. This cam is a disc partly circular, and with part of it F projecting to a greater distance from the centre. As long as the roller D bears upon the circular portion, the valve C remains down upon its seat; but as the projecting part of the cam is brought by its revolution under the roller, the valve-rod is pushed up and the valve lifted to permit the

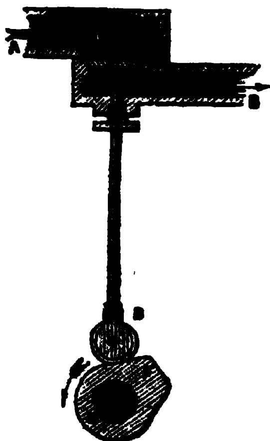


Fig. 188.

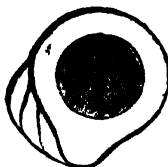




Fig. 189.

passage of the steam. By varying the extent of the projecting part, the valve can be kept open during a greater or less portion of a revolution of E. When it is desirable to vary this period, the cam is sometimes made with numerous steps of various extents, on any of which the roller may be made to bear at pleasure (Fig. 189).

For engines revolving slowly, this mode of working valves is very valuable, as it gives the power of admitting steam to the cylinder during a less or greater part of a stroke, and thereby of taking advantage of its expansive power within the cylinder. But when the rate of revolution is rather rapid, it is difficult to make the cam and roller capable of working without noisy and injurious blows, resulting from their rapid alternations.

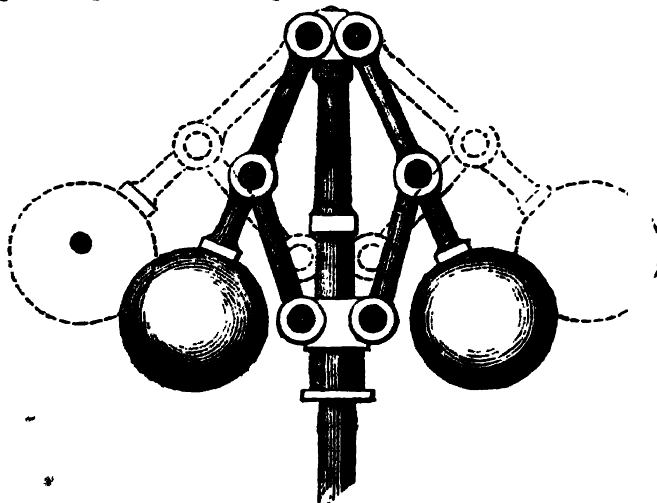


Fig. 190.

In stationary engines, which are subjected to continual variations of work, and yet are required to move with great regularity, it is essential to

provide some means of governing the speed. The most simple and efficacious apparatus for this purpose is the conical pendulum or governor, invented by Watt (Fig. 190). We have already described it in general terms as applied to windmills.

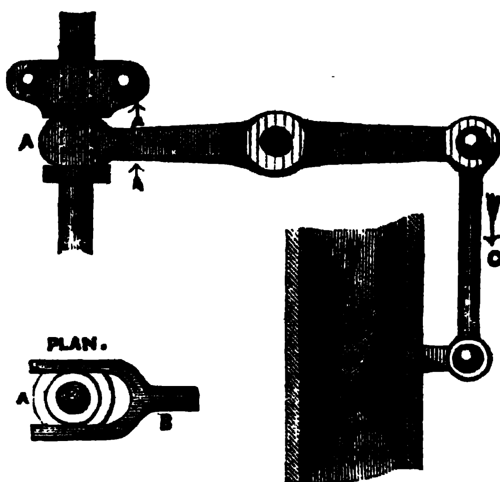
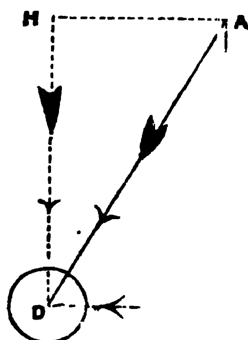


Fig. 191.

For steam-engines it is used in a similar manner. The vertical spindle is put in motion by the engine, and revolves quickly or slowly according to the velocity of the engine. When it revolves rapidly, the balls fly outwards and raise the grooved brass which slides on the spindle A (Fig. 191), and thus moves a forked lever B, which, by proper rods and levers C, causes the throttle-valve D to turn round in the steam-pipe, and check the passage of the steam to the cylinder. When the engine revolves slowly, on the other hand, the balls fall in, the brass sinks, and the throttle-valve is presented edgeways to the steam, and permits a more free passage.



As in the pendulum of a clock, the length from the point of suspension to the bob must be regulated to beat seconds, or half-seconds, or any other intervals that may be required; so in the conical pendulum or governor, the length of the arms that carry the balls must be regulated by the speed at which they revolve. In discussing this question, we need only consider one ball, as each is regulated by the same law, and they are generally made in duplicate for the sake of balancing the apparatus, and to give it symmetry. The force which tends to throw the ball outwards from the vertical spindle, is the centrifugal force of its revolution, or its tendency in obedience to the first law of motion to proceed in the straight line E F (Fig. 192), touching the circle in which it revolves, rather than to be continually diverted from its straight to a circular path. The force which opposes the centrifugal force, and causes the ball to be continually deflected from the straight

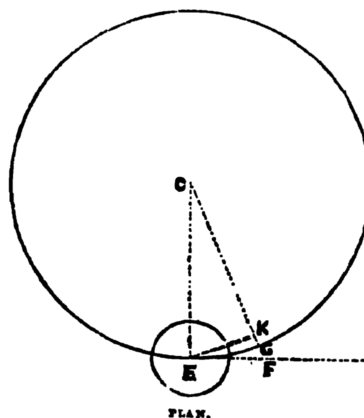


Fig. 192.

line, is the weight of the ball tending to push it close home to the vertical spindle. If we take any position of the ball, such as D, A B being the vertical spindle, and complete the parallelogram of forces A B D H, while the line D H or A B represents in quantity and direction the weight of the ball, and D B drawing it down, or A H the centrifugal force pushing it outwards, A D represents the resultant of those two forces as a tension on the rod by which the ball is suspended. If we know the velocity with which the ball moves in its path, and the radius of that path, we can estimate its centrifugal force in comparison with its weight, and can make the limb A D of such a length that these two forces shall be properly proportioned for a certain velocity.*

The length of the arm of the governor, measured from the point of suspension to the centre of the ball, may be found from the following rule:—

---

* *Note.*—E G being part of the circular path, which may be taken as small as we please, and E K perpendicular on C F, F K is nearly bisected in G, and E K is nearly equal to E F, and K F nearly parallel to C E. Also C E : E F :: E K : K F $= \frac{E K \cdot E F}{C E}$, or nearly $\frac{E F^2}{C E}$.

A body projected from E with such a velocity as would in a small period Δt cause it to describe E F if acted on during that period by a constant deflecting force parallel to C E, giving it during Δt a velocity which would cause it to describe uniformly F K in the time Δt would be deflected through F G, the half of F K in the time Δt.

If $f =$ the deflecting force per second (measured by the velocity acquired), $f \Delta t^2$ is the space traversed during Δt at the velocity acquired during Δt, and if v be the velocity per second of the body in E F, and therefore E F $= v \Delta t$, and $r =$ C E, the radius (measured in inches),

$$f \Delta t^2 = F K = \frac{E F^2}{C E} = \frac{v^2 \Delta t^2}{r} \therefore f = \frac{v^2}{r}$$

Taking $n =$ the number of revolutions per minute, $\frac{n}{60} =$ number per second, and $2 \pi r =$ the circumference traversed in each revolution, $\therefore v = \frac{2 \pi r n}{60} = \frac{r n}{9 \cdot 545}$

Hence, $f = \frac{r^2 n^2}{(9 \cdot 545)^2 r}, = \frac{r n^2}{91 \cdot 1 \text{ \&c.}}$, very nearly.

The force of gravity is measured by a velocity of $32\frac{1}{6}$ feet or 385 inches acquired per second, and taking $w =$ the weight of the body, $f : w :: \frac{r n^2}{91} : 385$

$$\therefore f = \frac{w r n^2}{91 \cdot 1 \text{ in.} \times 385} = \frac{w r n^2}{35344} \text{ very nearly.}$$

Now, as f is represented by B D, while w is measured by A B, and as B D $= r$

$$A B : B D :: w . f :: w : \frac{w r n^2}{35344}$$

$$\text{or} \quad A B = \frac{35344}{n^2} \frac{B D}{B D} = \frac{35344}{n^2} = \left(\frac{188}{n}\right)^2$$

It is convenient in practice to make the angle B A D about 30° when the governor is at its average speed, when A D $= 2$ D B, and as A D$^2 =$ A B$^2 +$ D B^2, we find

$$A D = \frac{2}{\sqrt{3}} \left(\frac{188}{n}\right)^2 = \left(\frac{200}{n}\right)^2 \text{ very nearly.}$$

Divide 200 by the number of revolutions per minute, and multiply the quotient by itself for the length in inches.

Example.—Required the length of a governor arm suited to 50 revolutions per minute: $\frac{200}{50} = 4$, and $4 \times 4 = 16$ inches.

To find the speed suited to a governor of a given length of arm.

Divide 200 by the square root of the length (in inches), and the quotient will be the number of revolutions per minute.

Example.—Required the proper speed for a governor having an arm 16 inches long.

The square root of 16 is 4, and $\frac{200}{4} = 50$ revolutions per minute.

The rods and levers connecting the governor with the throttle-valve should be capable of adjustment, and it is useful to have an adjusting counterbalance to the centrifugal force of the balls, by changing which the regulated speed of the engine can be varied at pleasure. Although the governor we have described is a most valuable accession to an engine, yet it is not a perfect regulator; for its

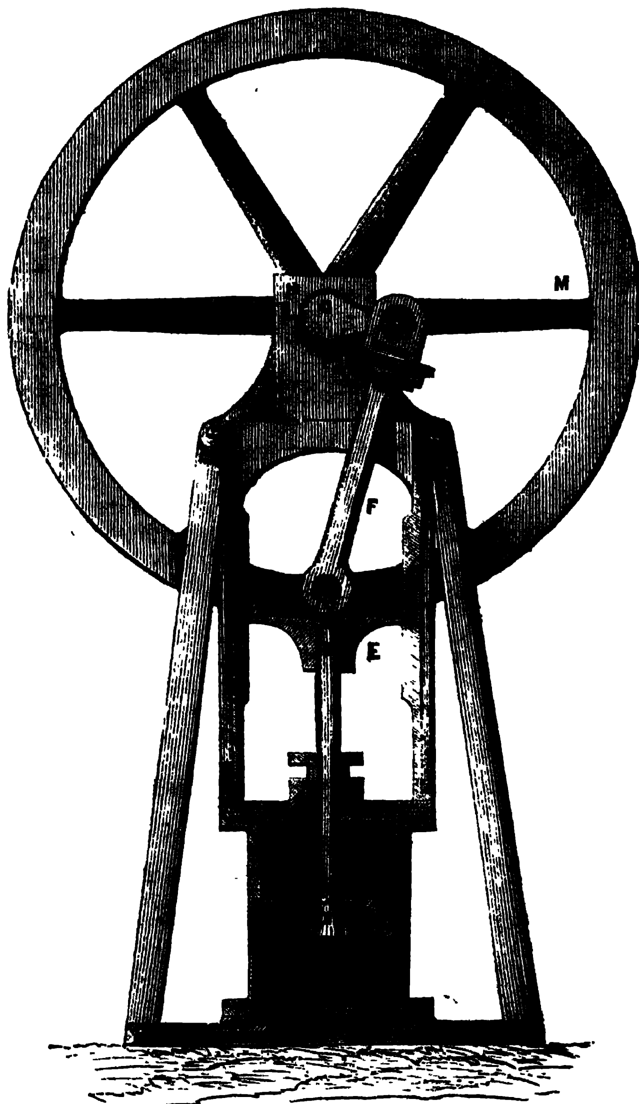


Fig. 193.

very mode of action implies that the velocity of the engine must have undergone a change, before the governor can have begun to move the throttle-valve. But within certain limits the variation in speed of an engine thus regulated is inconsiderable, and there is no apparatus so simple and durable that is capable of maintaining like regularity.

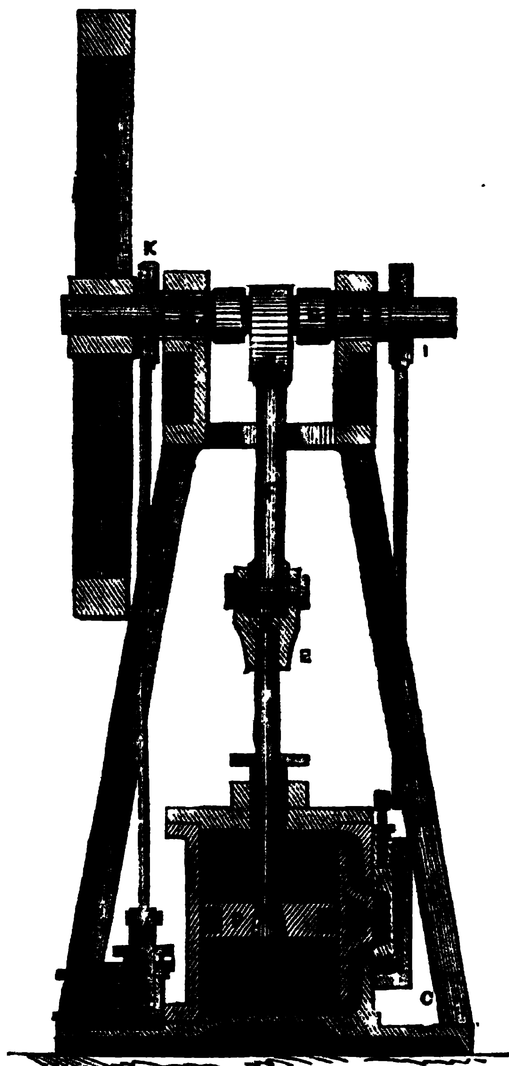


Fig. 194.

Another kind of governor has been applied, not without success. The lever of the throttle-valve is connected by a rod with a small piston working in a cylinder; the engine puts in motion a small pump, which forces air or water into this cylinder, and escapes from it by an aperture, the size of which can be regulated at pleasure. If the speed of the engine increase, the small cylinder is filled more rapidly than it can empty itself, and the piston thus raised closes the throttle-valve. Should the engine move more slowly, the small cylinder becomes emptied more rapidly than it is filled by the pump; and the piston descending, opens the throttle-valve so as to admit more steam to the main cylinder.

Having now described separately the principal parts of which a non-condensing engine consists, we will discuss some of the modes in which these parts can be most conveniently arranged.

Figs. 193 and 194 are sections, transverse to each other, of a vertical, direct-acting non-condensing engine; and Fig. 195 a plan of the same. A is the foundation-plate, forming the bottom of the cylinder B, which is secured to it by bolts and nuts.

C, the slide and steam passages. D, the piston and rod, terminating in a cross-head E, which works between guides, and to which is jointed the connecting-rod F. G, the crank; and H H, the bearings in which the crank-shaft revolves. I, the eccentric and rod for working the slide; and K, an eccentric for working the feed-pump L. M, the fly-wheel. This engine is of very simple construction, and has no parts that are unnecessary to the efficient working.

Figs. 196 and 197 are views of what is called a table-engine. The cylinder is elevated on a table; the piston-rod terminates in a cross-head, having a roller at each end working in a guide, and a connecting-rod descending on each side of the cylinder to a shaft made with two cranks below, working in bearings, and carrying a fly-wheel and eccentrics for the slide and feed-pump. The governor is drawn by bevil gearing from the crank-shaft.

In the locomotive engine, the action is precisely similar to that of the direct-

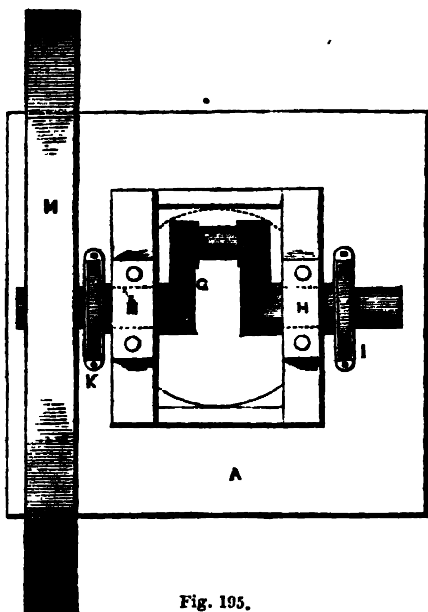


Fig. 195.

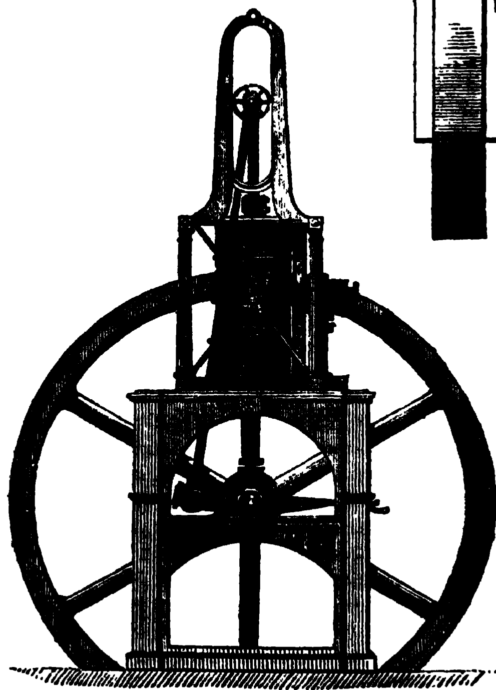


Fig. 196.

acting vertical engine, although the cylinder is laid horizontally, and the framing is combined in such a manner with the boiler as to secure strength and lightness.

Methods of Estimating Power.—In estimating the power of a non-condensing engine, we have only to ascertain the pressure on the surface of the piston causing it to move, and the distance passed over by it under this pressure in a given time, or the velocity with which the pressure acts. The product of the pressure by the velocity gives the power; and if the former be taken in lbs., and the latter in feet per minute, we have to divide their product by 33,000 to reduce it to the standard of horse-

power. For example, if we knew that the piston moved over 200 feet every minute under an average pressure of 1650 lbs. on its whole surface, the effect would be the same as if $1650 \times 200 = 330{,}000$ lbs. moved over 1 foot per minute; and as 1 horse-power is reckoned equivalent to 33,000 lbs. moved over 1 foot per minute, the power in the case assumed is $\frac{330{,}000}{33{,}000} = 10$ horse-power.

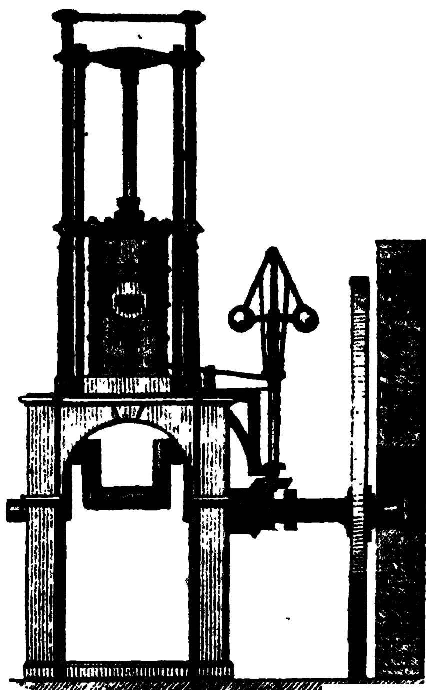


Fig. 197.

But in estimating the pressure on the piston, we are not entitled to assume it as that of the steam in the boiler, which may be measured by a steam-gauge. In the first place, there is always some length of pipe between the boiler and the cylinder, and the steam passing through this pipe loses some portion of its heat. Referring to the table of temperatures and corresponding pressures, we can see that the reduction by a very few degrees of the former, produces a very considerable diminution of the latter. While it is usual to cover the steam-pipe with felt, or some such non-conducting casing, in order to prevent a loss of this kind as far as possible; yet with every precaution there is a diminution of pressure to the extent of several pounds per square inch, especially when the initial pressure is considerable. Moreover, before the steam enters the cylinder, it has to pass the throttle-valve, which may be partially closed by the governor, and the steam is thus, as it were, *wire-drawn*, or made thinner, less dense, and consequently capable of pressing with less force on the piston. Were the piston at rest, however small might be the passage for the steam, yet, like water finding its level, its pressure would very quickly become uniform throughout every cavity to which it might have access. But as the piston is in motion, the steam has to flow along the pipe and passages with sufficient rapidity to follow up the piston in its progress; and if its course be arrested or impeded, that portion of it beyond the impediment must necessarily be of less density than that before it. Again, as the alternate flow of the steam above and below the piston is controlled by means of valves or a slide, the apertures covered by them cannot be opened or closed instantaneously, and there must therefore be at every alternation moments of transition, during which the passages are throttled and the steam wire-drawn. When the slide is moved by an eccentric, the opening and closing of the ports is gradual, and the amount of passage open for the steam is continually changing. If the slide and eccentric be so adjusted that the upper port is just beginning to open when the piston is at the top, it will continue to open for some time during the descent of the piston, until it attains its extreme width. It then begins to close; and

before the piston reaches the bottom it must be closed some time by the slide, which has moved onwards a sufficient distance to be ready for opening the bottom port to admit the steam below the piston. The slide and eccentric are generally so adjusted that each port is closed when the piston has passed through ⅔rds of its stroke.

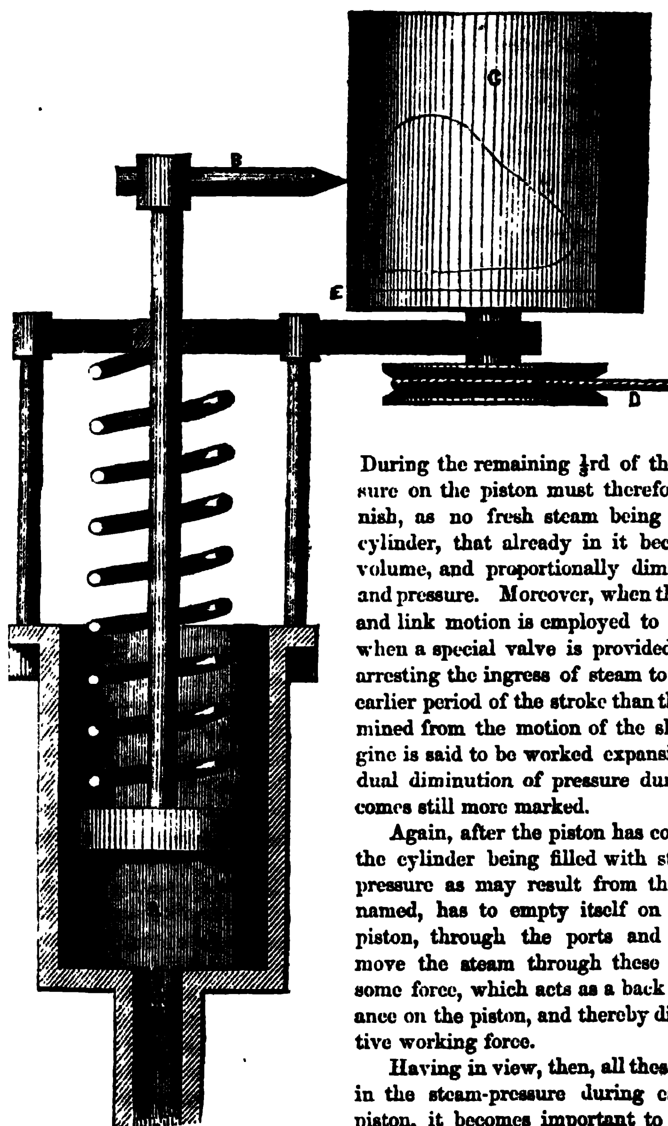


Fig. 198.

During the remaining ⅓rd of the stroke, the pressure on the piston must therefore gradually diminish, as no fresh steam being admitted into the cylinder, that already in it becomes expanded in volume, and proportionally diminished in density and pressure. Moreover, when the double eccentric and link motion is employed to work the slide, or when a special valve is provided for cutting off or arresting the ingress of steam to the cylinder at an earlier period of the stroke than that which is determined from the motion of the slide alone, the engine is said to be worked expansively, and the gradual diminution of pressure during the stroke becomes still more marked.

Again, after the piston has completed its stroke, the cylinder being filled with steam of such final pressure as may result from the causes we have named, has to empty itself on the return of the piston, through the ports and waste-pipes. To move the steam through these passages demands some force, which acts as a back pressure or resistance on the piston, and thereby diminishes its effective working force.

Having in view, then, all these causes of change in the steam-pressure during each stroke of the piston, it becomes important to ascertain what is the mean or average pressure throughout, which may be reckoned as the working pressure, or the actual force applied to each part of the surface of the piston to move it against the resistance of the machinery on which it acts.

The *indicator* is a simple and beautiful instrument, by which this element of power can be ascertained with the greatest accuracy. A (Fig. 198) is a small cylinder open at top, fitted with a piston, and communicating by a pipe and stop-cock with the upper or lower part of the main cylinder. The piston is pressed down into the cylinder by a nicely adjusted spiral spring, and a pencil B is fixed to the piston-rod. C is a roller round which a piece of paper is wound; on the axis of this roller is fitted a pulley D, connected by a string with some of the moving parts of the engine. The roller is also fitted with a spring, like the main-spring of a watch, in such a manner that after being pulled round in one direction by the motion of the engine communicated through the string, it is made to recoil by the spring. If we suppose the stop-cock closed, the piston, being pressed on by the spring and the atmosphere, will remain at the bottom of the cylinder; and the pencil being stationary at its lowest point E, while the roller is made to rotate backwards and forwards, will describe a line on the paper which would appear straight on its being unfolded from the roller. But if, while the roller continues its motion, the stop-cock be opened, then the piston will be subjected to the pressure of steam in the main cylinder, and will be forced upwards in opposition to the pressure of the spring and of the atmosphere, and the pencil will trace a line on the paper varying in height as the piston rises and falls. But, farther, if the spiral spring be so adjusted that we know exactly how many pounds will compress it an inch, and if we know the area of the piston, we can exactly measure the amount of pressure on it by the height to which the pencil is raised above the neutral line E, where it remains when subjected to no upward pressure. And thus the position of the pencil on the paper, or the mark left by it at any point, furnishes the measure of the pressure on the main piston of the engine at the corresponding point of its stroke. On unfolding the paper from the roller we should find a figure (Fig. 199) described on it by the pencil, which, when properly analyzed, gives us the means of reckoning the varying pressure on the piston, and often points out defects in some of the adjustments, and suggests modes of remedying them.

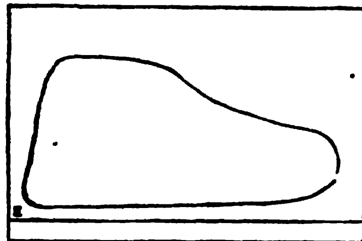


Fig. 199.

If we suppose that the area of the indicator-piston is 1 square inch, and that the spring is adjusted so that it requires a force of 10 lbs. to compress it 1 inch in length, or 1 lb. to compress it $\frac{1}{10}$th of an inch, we can form a scale of tenths of inches, and apply it to the indicator diagram at a number of points A, B, C, &c. (Fig. 200), equally distant, and measure off the lengths of ordinates A L, B M, &c., drawn through these points, and thus estimate the pressures acting on the piston of the indicator-cylinder at equidistant points of the stroke through which the paper is made to travel. These pressures correspond exactly with those to which the main piston of the engine has been subjected during its stroke, because the small cylinder of the indicator communicates freely with the cylinder of the engine. If we suppose the indicator to be fixed at the top of the cylinder, the upper part of the curve *a*, L, M, &c., R, Z, is that traced during the descent of the piston when the steam is pressing on it. The lower part of the curve Z Y, &c., S *a*, is that traced during the ascent of the piston when the steam is escaping from the cylinder. Were the indicator fixed to the bottom of the cylinder, we should get

corresponding curves for the steam-pressures there. Generally, when the slide-gear is properly adjusted, these figures are very nearly alike; and, if so, the upper part of the

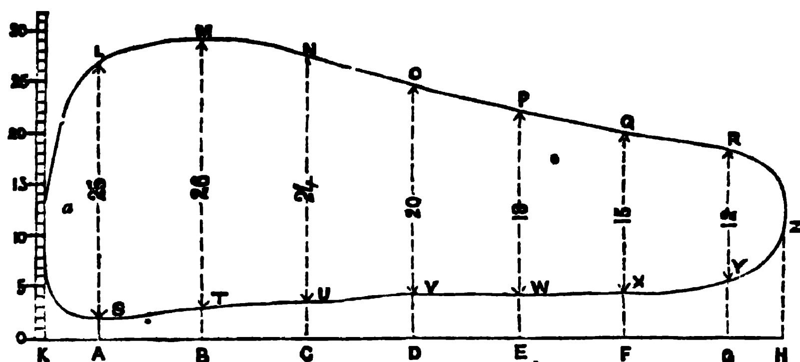


Fig. 200.

curve may be taken as that traced by the active pressure either above or below the piston, while the lower part of the curve may be taken as representing the corresponding resistance of steam during its egress from the cylinder. Now, as the total height C N of any ordinate measures the total pressure on one side of the piston when it is at the point of its stroke corresponding to C; and as the part C U of the same ordinate represents the resisting pressure on the piston at the same point of its stroke; the difference U N, or the part of the ordinate intercepted between the upper and lower limbs of the curve, measures the effective pressure on the piston clear of all resistances. The same applies to all the ordinates; and as we may suppose the whole curved space made up of numerous equal, narrow, vertical strips, each measured in height by an ordinate, we reckon the area of the figure contained within the curve as an expression of the power developed by the piston during its stroke. Or, having taken a considerable number of pressure ordinates and found the average, we consider this the mean effective pressure on the piston. For example, the average of those marked in the figure, found by adding them into one sum, and dividing it by their number, is $\frac{140}{7} = 20$ lbs., the mean effective pressure on every square inch of the piston. In taking the average in this way, the most correct method is to take the first and last ordinates, A L and G R, at a distance from the respective ends of the stroke *a* and Z, half that which separates the ordinates generally, A B or B C.

If we suppose the engine from which this figure was taken had a cylinder 12 inches in diameter, a stroke of 15 inches, and that the crank was making 80 revolutions per minute, we can readily calculate the effective power of the engine thus:—Each revolution of the crank requires an up-stroke and a down-stroke of the piston, or a travel through twice 15 inches, viz. $2\frac{1}{2}$ feet; and as 80 revolutions are made per minute, the piston travels over $80 \times 2\frac{1}{2} = 200$ feet per minute. Again, the area of the piston 12 inches diameter is 113 square inches, and as this is pressed on with an average load of 20 lbs. on every square inch, the total pressure on it is $113 \times 20 = 2260$ lbs. We have, there-

fore, 2260 lbs. moved over 200 feet per minute, equivalent to $2260 \times 200 = 452000$ lbs. moved over 1 foot per minute, which give $\frac{452000}{33000} = 13\frac{3}{4}$ horse-power nearly.

The complete rule for finding the power may therefore be thus stated:—Measure the ordinates (at least 6 or 8 in number) contained within the indicator figure, sum them up, and divide by their number for the mean pressure; multiply the area of the piston (in square inches) by the mean pressure (in lbs. per square inch) by twice the length of stroke (in feet) and by the number of revolutions per minute, and divide the product by 33000 for the horse-power.

Example.—On the indicator being applied to an engine, having a cylinder 30 inches diameter, a stroke of 4 feet, and making 27 revolutions per minute, 8 ordinates of the figure were found to be respectively 34, 34, 34, 33, 24, 18, 14, and 9: required the power of the engine.

Sum of 8 ordinates = 200, which, divided by their number 8, gives 25 lbs. as the mean pressure.

Area of cylinder, 30 ins. diameter	=	707 square inches.
Multiply by mean pressure .		25 lbs.
Total mean pressure on piston . . .		17,675 lbs.
Double the stroke		6 feet.
		141,400
Number of revolutions		27 per minute.
Divide by	33,000)	3,817,800
Horse-power		$115\frac{1}{2}$ nearly.

But in making these calculations, it must not be forgotten that we only reckon the force with which the piston moves. In communicating this force to the crank-shaft, and thence to the machinery driven by the engine, there are losses from friction and other causes for which some allowance must be made. The piston rubs along the surface of the cylinder, its rod rubs through the stuffing-box; so with the slide. The end of the piston-rod rubs on the guides, which save it from yielding to the oblique action of the connecting-rod; the connecting-rod eyes rub on their pins, the crank-shaft rubs in its bearings, the eccentric and its rod also present rubbing surfaces, the fly-wheel encounters considerable resistance of air to its rotation, and the feed-pump demands power for its working. All these resistances vary with the conditions of the rubbing surfaces, the accuracy or inaccuracy of their fitting, their state of lubrication, and other circumstances; so that it is difficult to state any constant deduction to be made from the calculated power on account of them. An engine in a very good state should thus waste not more than $\frac{1}{8}$th to $\frac{1}{10}$th of its power, while one in a bad state may lose as much as $\frac{1}{3}$rd. It may generally be fair to reckon the loss at $\frac{1}{4}$th or $\frac{1}{5}$th of the calculated power. Thus, in the example given, the calculated power being $115\frac{1}{2}$, we take the real power about 90, deducting rather more than $\frac{1}{5}$th for losses.

The indicator figure is not only a measure of power, it is also a picture of defects, and may often furnish useful hints as to the proper mode of improving the action of the engine. If A B (Fig. 201) represent the stroke, F H a line drawn at a distance A F below equal to 15 lbs., and if we suppose steam at 30 lbs. above atmospheric pressure

is admitted to the cylinder during half the stroke, then suddenly cut off, we draw A C = 30 lbs., C D half the stroke; take E midway in P H, and fill in the hyperbolic curve D E. Then will the figure A C D E B represent the best possible effect that could be got under the conditions given; for the straight line A C represents the sudden rise of the pressure from that of the atmosphere to 30 lbs. above it at the beginning of the stroke; the straight line C D represents the continu-

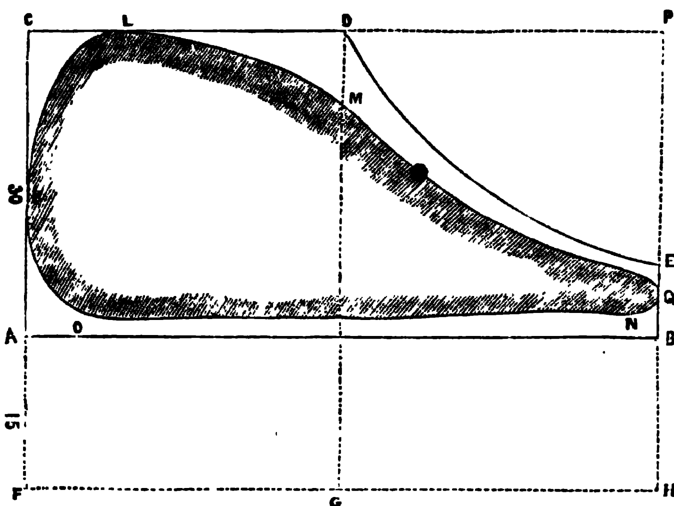


Fig. 201.

ance of that pressure during half the stroke; the curve D E indicates the gradual reduction of pressure as the steam enclosed in the cylinder expands to fill its increasing capacity; the point E midway in P H marks the pressure at the end of the stroke, half that at the beginning or middle, because the capacity of the cylinder at the end has been doubled while the quantity of steam within it has remained constant. The straight line E B marks the sudden fall of the final pressure to that of the atmosphere when the exit port is opened; and the straight line B A represents the constant resistance of the atmosphere to the issue of the steam during the return stroke of the piston. Such might be the theoretical figure. The practical figure inscribed within it must necessarily fall short in some respects of that which is theoretically perfect. For instance, at the beginning of the stroke, the port opening not suddenly, but gradually, produces a curved line from K to L, the piston having travelled some distance before the full pressure is attained; the gradual closing of the port or expansion valve, and some diminution of pressure from the cooling of the steam, or from leakage past the piston, are indicated by the inclined line L M. The cooling varies the expansion curve M Q from the true hyperbola, and the gradual opening of the exit port causes a curved turn from Q to N instead of a sudden drop E B. The line O N above the line of atmospheric pressure A B, indicates some additional resistance to the issue of the steam dependent on limited area, or bad form of opening, or leakage from the steam side of the piston, and the turn at O K marks the gradual closing of the exit port and opening of the inlet port for the succeeding stroke. A careful analysis of a figure produced by an engine not working satisfactorily, will thus point out causes of loss, and suggest means of remedying them, by widening the ports, readjusting the eccentric and slide, clothing the cylinder to prevent cooling, and such other arrangements as may be found advan-

tageous. In the hands of an experienced engineer, indicator diagrams become highly suggestive of merits and defects, and often furnish more information as to the economical working of an engine or the reverse, than continued observation of its structure and action could supply.

In estimating the power of a non-condensing engine without reference to its indicator figure, we may generally make a very near approximation to the truth by the following mode of reckoning. We assume the engine to be in fair working order, and fitted with an ordinary slide, cutting off the steam at about ⅔rds of the stroke; that the steam-pipe is not of great length, and well clothed with non-conducting material; that the ports are well proportioned, and the piston and slide tight. We farther suppose the piston to travel at the velocity of 200 feet per minute, which is found to be practically a fair working rate; and that a horse-power to be effective, after all allowances for friction, &c., should be estimated at 44.000 lbs. moved 1 foot per minute, or 220 lbs. moved 200 feet per minute. We observe the pressure in the boiler, and deduct from it ¼th for loss by cooling in the steam-pipe and expansion in the cylinder, and 2 lbs. for resistance to exit and other losses, the remainder being reckoned as the mean effective pressure. Multiplying this by the area of the piston, and dividing by 220, we get a fair estimate of the power.

Example.—An engine having a cylinder 30 inches in diameter, is worked at a pressure of 36 lbs. in the boiler: required its power.

From pressure in boiler		36 lbs.
Deduct one-fourth	9 lbs.	
And	2 lbs.—	11 ,,
Mean effective pressure		25 ,,
Multiply by area of 30 inches		707
Divide by		220)17675
Horse-power		80

In general it is the business of engineers to provide engines of certain powers without special reference to the pressure at which they should be worked. By employing very high pressures, the size, weight, and cost of an engine are certainly reduced; but, on the other hand, some danger is incurred, and the tear and wear is considerable. By using very low pressures, again, the cylinder necessarily becomes large, the engine generally cumbrous and heavy, and little advantage can be taken of the expansive power of the steam. We consider a boiler pressure of 40 to 50 lbs. to be a fair average on which to estimate the engine-power; and would suggest the following rules for calculating the power of a given engine, and the diameter of cylinder necessary to produce a given power.

1. To find the power of an engine when the diameter of the cylinder is given.

Rule.—Divide the diameter (in inches) by 3, and square it for the horse-power.

Example.—Required the power of an engine having a cylinder 15 inches in diameter.

$$\frac{15}{3} = 5, \text{ and } 5 \times 5 = 25 \text{ horse-power.}$$

2. To find the diameter of cylinder necessary for a given power.

Rule.—Multiply the square root of the power by 3; the product is the diameter of the cylinder in inches.

-What should be the diameter of a cylinder for 100 horse-power?

Square root of 100 = 10, and 3 × 10 = 30 inches.

The length of stroke must depend on the number of revolutions made by the crank in a given time. It is convenient to assume that the piston, in engines going at a fair average speed, shall travel over 200 feet per minute. Sometimes it moves at the rate of 250, and even as much as 300 feet per minute; but, upon the whole, 200 is a convenient and economical speed. This is the product of twice the stroke by the number of revolutions; and hence its half, 100, is the product of the stroke by the number of revolutions. If, then, either the length of stroke or the number of revolutions be given, the other may readily be thus found:—

1. Given the length of stroke to find the speed.

Rule.—Divide 100 by the stroke (in feet), the quotient is the number of revolutions per minute.

Example.—What is the speed of an engine having 2 ft. 6 ins. stroke?

$$\frac{100}{2\frac{1}{2}\text{ feet}} = 40 \text{ revolutions per minute.}$$

2. Given the speed to find the stroke.

Rule.—Divide 100 by the number of revolutions per minute, the quotient is the length of stroke in feet.

Example.—What must be the stroke of an engine making 35 revolutions per minute?

$$\frac{100}{35} = 2{\cdot}57 \text{ feet, or about 2 ft. 7 ins.}$$

The dimensions of the steam passages should be proportioned to the area of the cylinder; for while the piston travels at its quickest speed, increasing rapidly the space to be filled with steam, the passages should admit the steam with sufficient velocity to sustain the pressure on the retreating piston. It is found, practically, that the area of the steam-pipe may be advantageously $\frac{1}{36}$th of that of the cylinder, or the diameter of the one $\frac{1}{6}$th of that of the other. Thus, for an engine having a cylinder 30 ins. diameter, the steam-pipe should be 5 ins. diameter. When it is intended that the piston should travel more rapidly than the average rate of 200 feet per minute, the steam-pipe should be proportionally large. In such cases, its diameter may be advantageously reckoned at $\frac{1}{5}$th of the diameter of the cylinder. The ports which admit the steam into the cylinder should always present an area of passage considerably greater than that of the steam-pipe, for during a great part of every stroke they are partially closed by the slide. The exhaust-pipe, which conveys the steam from the cylinder, should be larger than the steam-pipe; for the waste steam should be permitted to become rapidly expanded in volume, in order that its back pressure on the piston may be diminished as much as possible.

The size of the feed-pump should always be greatly in excess of that which is absolutely required for supplying the amount of water boiled off in steam for the engine. Occasionally, the valves of the pump leak, there may be leaks in the boiler, some of the water may pass over in priming, and a good deal may be wasted in blowing off. The pump should, therefore, be capable of supplying at least 3 times as much water as is actually due to the steam supplied to the cylinder. The pump is generally arranged so as to make one stroke for each revolution of the engine, or each double stroke of the piston. If, then, we take a case where the diameter of the cylinder is 30 inches, the stroke 4 feet, and the average pressure 30 lbs. above that of the atmosphere—that is to say, the steam at 3 atmospheres—we find that, during each revolution, the cylinder twice

filled with steam uses a volume of about 68,000 cubic inches. Steam at 1 atmosphere of pressure being about 1600 times the volume of water, its volume at 3 atmospheres is $\frac{1}{3}$rd of that, or about 533 times that of water. The water necessary to generate 68,000 cubic inches of steam at 30 lbs. pressure, therefore, amounts to 128 cubic inches; and as the feed-pump should be capable of supplying thrice this quantity, it must deliver at each stroke 384 cubic inches. Should we make the stroke of the pump 2 feet, half that of the piston, its area must be $\frac{384}{24} = 16$ square inches, or its diameter $4\frac{1}{2}$ inches.

As a general rule for the dimensions of the feed-pump in non-condensing engines, we may offer the following:—

Multiply the square of the diameter of the cylinder (in inches) by the length of stroke (in feet), divide by 90, and the quotient is the product of the square of the diameter (in inches) of the pump, by its length of stroke (in feet).

Example.—Required the size of the feed-pump for an engine having a cylinder 30 inches diameter and a stroke of 4 feet.

$\frac{30 \times 30 \times 4}{90} = 40$, the product of diameter squared by stroke. If we take the stroke of the pump 2 feet, then $\frac{40}{2} = 20$ is the square of the diameter, or the diameter is about $4\frac{1}{2}$ inches, because $4\frac{1}{2} \times 4\frac{1}{2} \times 2 = 40$ nearly.

Many treatises on the steam-engine give rules for the dimensions of all the principal parts of an engine. We think, however, that these rules are not in many cases practically available, because a difference in general design and arrangement, or in average pressure and speed, occasions very considerable variations in the proportions of the parts. Careful study of well-made engines, and actual experience in their construction and working, form the true sources of information as to their due proportions.

The *Condensing Engine* in all main points resembles the non-condensing engine; but it requires some additional parts in order that the vacuum produced by condensing the steam may be employed as a source of additional power. If we suppose that the steam on leaving the bottom of the cylinder, instead of flowing out into the atmosphere, which resists its egress with a pressure of 15 lbs. per square inch, were conducted into a vessel totally void of air or steam, this resisting force would be entirely removed, and the effect of the steam pressing on the upper side of the piston would be increased by that quantity. If the vacuum in the vessel were not perfect—that is to say, if there were contained in it some rare fluid, such as air or steam, or a mixture of both, greatly attenuated, and capable of pressing with a force of only 2 or 3 lbs. on the square inch—the pressure of the steam on the piston would be increased by a quantity 2 or 3 lbs. less than 15 lbs. per square inch. Generally, if we reckon the pressure of steam in the boiler as its absolute pressure, not its excess over the atmosphere, and deduct the pressure of fluid in the vacuum vessel, the difference will be the effective pressure on the piston. Thus, with steam in the cylinder exerting a pressure of 10 lbs. above that of the atmosphere, or having an absolute pressure of 25 lbs. per square inch, while the vacuum vessel contains a fluid pressing with a force of 2 lbs. per square inch, the effective pressure on the piston is $25 - 2 = 23$ lbs. per square inch. The condition of the fluid in the vacuum-vessel as to pressure is generally measured by a barometer.

The upper part of the vessel A (Fig. 202) is connected with a glass tube C, about 30 inches long, dipping into a cup of mercury B. Were the space in A an absolute vacuum, the atmospheric pressure on the surface of the mercury in the cup would force

the mercury up the tube to the height of about 30 inches, because a column of mercury 30 inches in height presses with a force of 15 lbs. on the square inch. But if A contained fluid pressing with 2 lbs. per square inch, the mercury would attain a height of only 26 inches; because the pressure of a column 26 inches high is 13 lbs., and the additional 2 lbs. of fluid pressure make up 15 lbs., the atmospheric pressure.

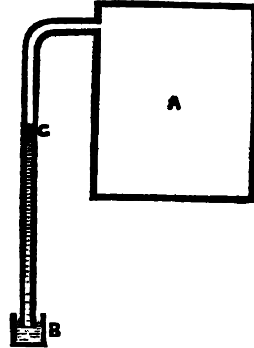


Fig. 202.

If, then, we know the pressure of steam in the cylinder (above that of the atmosphere), and the height of the mercurial column in the barometer, we find the effective pressure on the piston by adding the steam pressure to half the height (in inches) of the column.

It is remarkable that some of the earliest efforts made for obtaining power from steam were directed to the construction of apparatus in which its condensation, as well as its elasticity, should afford the force required. At the end of the 17th century, Captain Thomas Savery succeeded in constructing an engine for raising water by means of steam. A vessel A (Fig. 203) connected by a steam-pipe and cock B with a suitable boiler, communicated with a vertical water-pipe C, in which were fitted two valves D and E opening upwards. Steam being admitted into A forced out the air contained in it by the upper part of the water-pipe, and occupied its place. The steam-cock B being closed, and a stream of cold water made to pour over A, the steam within it became condensed, and formed a partial vacuum; the pressure of the atmosphere acting on the water at the bottom of C, forced it up the pipe through the valve D and into A, so as to fill the void space. The steam-cock being again opened, the pressure of steam on the water in A forced it through the valve E and up the pipe. The vessel A was thus successively filled and emptied by the alternate closing and opening of the steam-cock, and the water raised through the height of the pipe C.

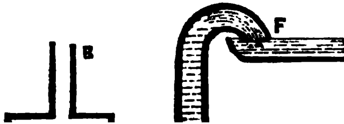



Fig. 203.

A few years afterwards, Thomas Newcomen applied steam to give motion to a piston in a cylinder. The cylinder A (Fig. 204), communicated by a pipe and cock B with a boiler generating steam at low (or nearly atmospheric) pressure. A piston C, fitting the cylinder, was connected by a chain with one end of a beam or lever D, to the other end of which E was attached a pump-rod with a heavy weight F. The ends of the beam were

made arcs of circles, so that while the chains were wound on them or unwound from them, the piston and pump-rod might move in straight lines. Steam being admitted into the cylinder below the piston, so as to balance the atmospheric pressure on its upper surface, the weight F caused it to ascend; and when it reached the top, the steam-cock was closed, and a water-cock G was opened to admit a jet of water into the midst of the steam. The coldness of the water rapidly condensed the steam, and formed a partial vacuum below the piston, and the atmospheric pressure forced it down to the bottom, raising the weight F. The water-cock being now closed, and the steam-cock opened, the ascent of the piston was repeated.

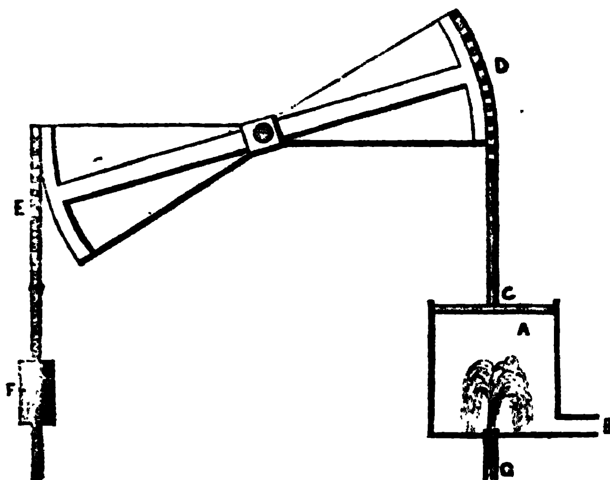


Fig. 204.

Numerous inventors contributed to the improvement of Newcomen's engine, and about the beginning of the 18th century it had become a practically useful, but not economical apparatus for pumping. The principal objection to the use of this, as well as of Savery's engine, consisted in the circumstance that the alternated flow of cold water into the steam-cylinder cooled it down and greatly diminished the force of the steam as it entered, a great portion of its heat being ineffectively expended on the cold metal of the cylinder. It was reserved for the illustrious James Watt, shortly after the middle of last century, to introduce those improvements in the arrangement and construction of the steam-engine, which have rendered it the most useful and economical of all known sources of power. His principal improvement on Newcomen's engine was the use of a separate vessel for condensing the steam, with an air-pump for removing the water of condensation and the air liberated from the water. But his ingenuity was likewise devoted to the improvement of every detail of arrangement and construction; and, having found the steam-engine in many respects rude, ineffective, and costly in its working, he left it an apparatus as nearly perfect as any human work can be. Since his time, changes have been made in the arrangements of the parts to suit peculiar circumstances of operation, and new forms have been devised for particular purposes; but in all their leading features, the steam-engines of the present day are essentially the products of Watt's fertile genius. The condensing engine, as improved by Watt, is of two kinds :—

Single-acting, where the steam is permitted to press on one side of the piston only, so as to cause it to make a single stroke; the return stroke being effected by a counterbalance weight.

Double-acting, where the steam presses alternately on each side of the piston, and its reciprocating movement is converted by a crank into rotary motion.

The single-acting engine is well suited for such an operation as pumping, where the reciprocating movement of the pump-bucket corresponds with that of the piston. The double-acting engine, again, is adapted for driving machinery. In both these kinds of engine, where the steam is condensed, there are required a condenser and an air-pump, which we will now describe.

The condenser is a vessel B (Fig. 205), generally made of about ¼th of the capacity of the steam-cylinder, with which it communicates by the pipe D. The steam, after acting on the piston, instead of escaping into the atmosphere as in non - condensing engines, flows by this pipe into the condenser, which is placed in a cistern of cold water, and has a pipe and cock, I, for the admission of a jet of cold water to condense the steam. This jet is called the *injection*, and its quantity is regulated by means of the cock, worked by a rod passing upwards, with a handle in some place convenient for the attendant. The bottom of the condenser communicates by a passage, fitted with a valve G, with the air-pump A. The bucket or piston P of the air-pump is fitted with valves opening upwards, and is moved upwards and downwards by a rod connected with some part of the engine, and passing through a stuffing-box in the air-pump cover. Near the top of the air-pump there is a passage Q, fitted with a discharge valve opening into the *hot-well* K, from which the feed-water is pumped to the boiler, the overplus or waste being discharged by a waste-pipe. The cistern in which the condenser and air-pump are placed is kept constantly filled with cold water by a pump called the *cold-water pump*, supplying it by a pipe N at the bottom, while the heated water overflows by a suitable waste-pipe. While the steam from the cylinder flows into the cold condenser, and meets the cold water diffused through it by the injection jet, it becomes condensed into water, and falls with the injection-water to the bottom, occupying very little volume compared with that which it occupied while in the state of steam, and leaving the space of the condenser a partial vacuum. But water always contains a quantity of air mingled with it, which passes over with the steam from the boiler to the cylinder, and thence to the condenser, and the injection-water also parts with a portion of the air it contains, so that after a time the condenser would become filled with the air so liberated, and the water of injection and condensation, unless means were taken to remove them.

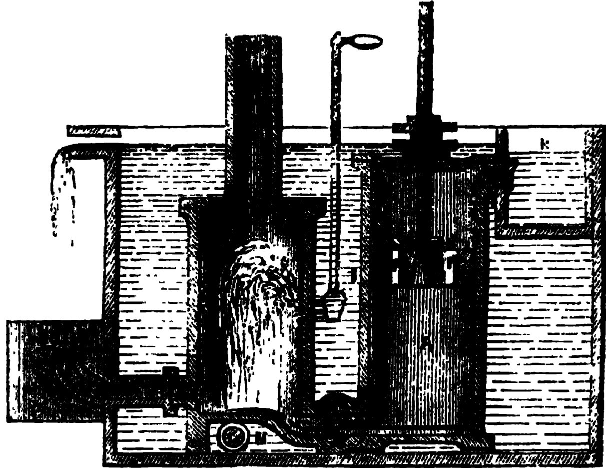


Fig. 205.

As the air-pump bucket descends, the discharge-valve Q being closed, prevents any air or water from entering the space left by the descent of the bucket; but when it has reached the bottom and begins to ascend, the bottom-valve E, and the bucket-valves P permit the air and water from the condenser to flow into this space. On the ascent of the bucket, the bucket-valves P are closed by the pressure of air and water above them, and the contents of the pump are discharged through the valves Q into the hot-well. Thus by the alternate descent and ascent of the air-pump bucket, when its capacity and the amount of injection are properly proportioned, almost a perfect vacuum is maintained in the condenser; and the effective pressure of the steam on every square inch of the piston is increased by nearly 15 lbs. above that which it would be were the steam permitted to escape uncondensed into the atmosphere.

The condenser is generally fitted with a *blow-valve* H, which comes into play on starting the engine thus. The engine having been stopped, the condenser and air-pump may have become quite filled with water through the injection-cock; and on starting the engine again, no vacuum could be produced while they are thus *water-logged*. But by opening a small valve called the *blow-through valve*, a communication is made between the steam-pipe from the boiler, and the eduction-pipe leading from the cylinder to the condenser. The pressure of the steam in the boiler is thus brought to bear on the water in the condenser, and forces it out by the *blow-valve* H, the steam occupying its place. On shutting the blow-through valve and opening the injection-cock, the steam in the condenser is condensed and a vacuum formed, so that the engine may be started. As long as there is even a partial vacuum in the condenser, the atmospheric pressure on the blow-valve keeps it closed.

The bottom-valve, bucket-valves, and discharge-valves of the air-pump, are frequently made of vulcanized caoutchouc sheet, cut into discs and laid over gratings. Water or air forced through the perforations in the gratings, raise the flexible discs of caoutchouc, and pass round their edges; but neither air nor water can return, for the atmospheric pressure forces the caoutchouc discs firmly down on the gratings, and thereby effectually closes their openings.

It is a peculiar property of all vapours, that, besides their sensible heat, or the temperature to which they raise the thermometer, they contain a great amount of latent heat, not measured by the thermometer, but by its effect when the condition of the vapour is changed. The latent heat of steam, when its temperature or sensible heat is 212°, is estimated to be about 1000°. This does not mean that the latent heat could raise a thermometer 1000°, but simply that a pound of steam at 212° being condensed by its mixture with 1000 lbs. of water at any temperature, such as 60°, could raise the temperature of the whole mass of water 1°. In other words, if it were found that the combustion of a certain weight of fuel could raise the temperature of a given mass of water from 211° to 212°, it would require 1000 times that quantity of fuel to convert the water into steam, having still the sensible temperature of 212°. This great latent heat is something essential to the condition of water in a state of vapour, for as soon as any portion of it is removed by bringing the steam into contact with a cold substance, a part of the steam is immediately condensed into water; and the remainder expanding to fill the space thus left void, loses density and pressure as it gains volume. In estimating the quantity of injection-water necessary for condensing the steam of an engine, we must therefore bear in mind that it is not alone the sensible temperature, but also the latent heat of the steam which we have to absorb by the cold water injected. Let us assume that 1 cubic foot of water, having been converted into steam in the boiler,

and having acted on the piston in the cylinder, flows into the condenser at a temperature of 212°, and containing 1000° of latent heat, and that it there mingles with a quantity of water at 62° sufficient to condense it and produce an ultimate temperature of 112° throughout the mixture. The total heat of the steam being 1212°, has to be reduced to 112°—that is, the steam has to lose 1100° of temperature; the injection water entering at 62° and being raised to 112° has to gain 50°. The quantity of injection-water must therefore be 22 cubic feet, for 22 cubic feet raised 50° are equivalent to 1 cubic foot reduced 1100°, because $22 \times 50 = 1100$.

The temperatures we have assumed are such as would frequently occur in practice; and generally it will be found that the quantity of injection-water required for an engine is from 15 to 25 times the quantity required for feed.

The capacity of the air-pump is generally $\frac{1}{8}$th of that of the cylinder, the stroke being usually $\frac{1}{2}$, and the diameter $\frac{1}{2}$ that of the cylinder. The power necessary to work the air-pump of a condensing engine is about $\frac{1}{10}$th of the total power.

The indicator applied to a condensing-engine produces a figure similar to those we have already discussed. But as during one stroke of the piston the communication to the condenser is open, the pencil traces a line below that of atmospheric pressure. Thus if A (Fig. 206) be the line of atmospheric pressure, another line B, drawn at the pressure of 15 lbs. below A, would be the line of absolute vacuum, or of no pressure. The lower limb of the figure would more nearly approach this line, the more perfect the vacuum in the condenser. The area of the figure represents the power during a stroke; and the mean effective pressure is found by taking the average of the lengths of numerous ordinates drawn within the figure as before.

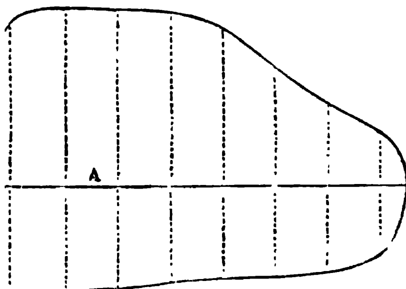


Fig. 206.

Non - condensing engines are generally of the form called *beam-engines.* The piston-rod does not act directly on the crank in the case of the double-acting engine, nor on the pump in the case of the single-acting engine, but on one end of a lever or beam, working on central bearings, from the other end of which the crank or pump is worked by a connecting-rod. By jointed rods connected with pins on the beam, the air-pump is worked with half the stroke of the piston; the feed-pump and the cold-water pump are also worked with their proper strokes. As the piston-rod, and also the air-pump rod, pass through stuffing-boxes in the cylinder and air-pump covers respectively, and must therefore move in straight lines, while the parts of the beam to which they are connected vibrate in circular arcs, some arrangement is required for controlling the motion of the rods so that it shall not partake of the circular vibration. This could be effected by making them slide between rectilineal guides, but not without considerable friction and some difficulty in arranging the guides in a suitable manner. We are indebted to

Watt for a simple and elegant manner of effecting this object by the apparatus called the *parallel motion*.

If A D_1 and B C_1 (Fig. 207) be two equal levers vibrating in circular arcs D_1 D D_2, C_1 C C_2 respectively, their extremities being connected by a rod D_1 C_1, it will be found that E, the middle point of this rod, moves from E_1 to E_2 in a line which is very nearly straight. Again, if F G, one of these levers, be prolonged to H so as to be double the length, and a parallelogram G H M L be formed of jointed rods so that F K and L are always in one straight line, then K being controlled so as to move in a straight line or nearly so, L will also move in nearly a straight line. Applying this principle to the beam of an engine, K is the point to which the air-pump rod is connected, and L that to which the piston-rod is connected; G M and H L are called *parallel motion links*, and M N, the subsidiary lever which controls the movement of K, is called the *radius rod*. In the case of single-acting engines, where the end of the beam opposite to that worked by the piston works a pump, a parallel motion is also fitted there to give rectilineal motion to the pump-rod.

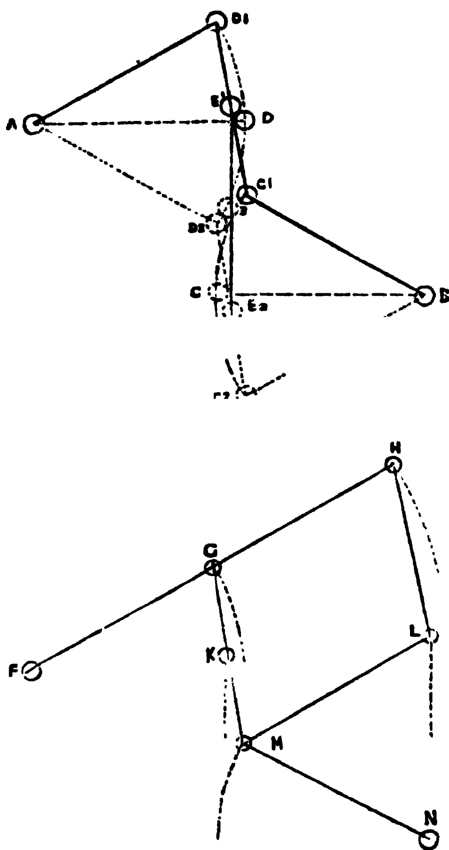


Fig. 207.

In marine engines, where it is desirable to keep the weight as low as possible, the arrangement of beam is different. Instead of one beam above the cylinder and crank, two beams are fitted below, one on each side of the cylinder, and the parallel motion employed in that case is somewhat different from that which we have described, although the principles embodied are very similar.

The alternate movements of the steam to and from the opposite ends of the cylinder are effected in condensing engines in the same manner as non-condensing engines, by a slide worked by an eccentric, or by valves worked by cams. In single-acting engines valves are generally employed, and they are worked by levers struck by pins or tappets fixed to the air-pump rod, as it ascends and descends along with the piston.

The single-acting beam-engine is almost exclusively employed for pumping. The

form of engine most commonly used for this purpose is what is called the Cornish engine, because at the mines in Cornwall these engines have been employed with great economy and efficiency for draining to great depths.

The steam is admitted above the piston at considerable pressure, and the supply is cut off when the piston has performed a small part of its down-stroke, so that the rest of its stroke is effected by the expansion of the steam. When the piston has reached the bottom of the cylinder, the steam is permitted to pass from its upper to its lower side, so that it does not press it in either direction, but the weight of the pump-rods at the other end of the beam, loaded if necessary with additional weight, causes the piston again to ascend to the top of the cylinder. The steam is again admitted above the piston, while that which is below it flows into the condenser; and thus the alternation is continued, the valves which control the movements of the steam being worked by levers and tappets from the air-pump rod.

Fig. 208 represents a double-acting condensing beam-engine working a crank. Here the weight of the piston and air-pump bucket and their rods at one end of the

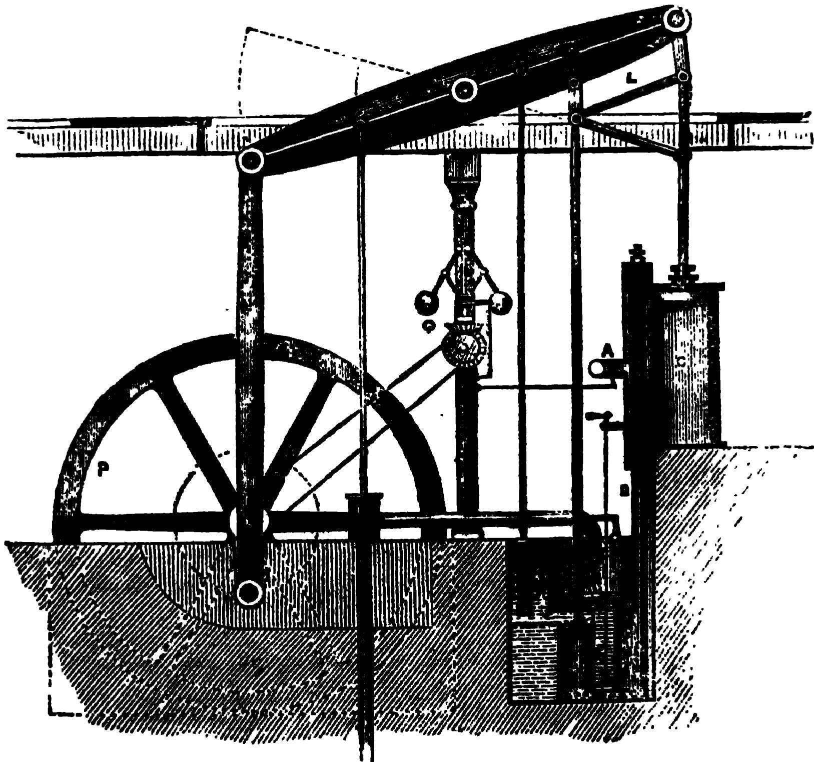


Fig. 208.

beam is balanced by that of the connecting-rod at the other end, and the steam is alter-
na

condenser, so that each end of the beam is alternately pushed upwards and pulled downwards by the pressure on the piston. A the steam-pipe from the boiler, supplying steam to the slide-jacket or space round the slide B. C the cylinder fitted with piston, and piston-rod connected to one end of the beam M by the parallel motion L. E the condenser, communicating with the slide-jacket by the eduction-pipe D. F the injection-cock, admitting water into the condenser by a jet from the cold cistern in which it is placed. G the air-pump, worked by a rod from the parallel motion, and discharging into the hot-well H, in which is placed the feed-pump worked by a rod from the beam, so as to draw water from the hot-well and propel it by a feed-pipe to the boiler. K the cold-water pump, worked by a rod from the beam, drawing water from a well or stream, and supplying the cold cistern for condensation. N the connecting-rod, worked from one end of the beam, and giving rotary motion to the crank O. P the fly-wheel, fixed on the crank-shaft, and revolving with it, with the necessary momentum for bringing the crank over the dead centres, or highest and lowest points of its revolution, where the connecting-rod is ineffective to turn it. Q the governor, caused to revolve by a strap or band, and suitable gearing connecting it with the crank-shaft. The steam-pipe A is fitted with a throttle-valve connected with the governor by levers and rods, so that the velocity of the engine is controlled by the throttling of the steam, as already described.

Fig. 209 represents a condensing marine engine suited for driving paddle-wheels. A, the slide-jacket supplied with steam from the boiler. B, the cylinder fitted with

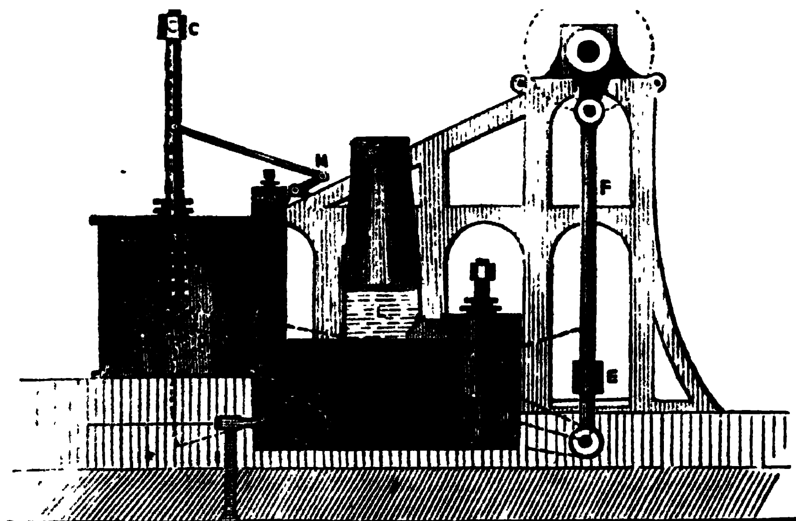


Fig. 209.

piston and rod. C, the piston cross-head, from the ends of which, side rods descend on each side of the cylinder to the extremities of two side beams D. H, the condenser supplied with injection-water, by the injection-pipe and cock I, passing through the ship's bottom. K, the air-pump, worked by side rods from pins in the side beams D,

and discharging into the cistern-head L, whence the supply of water for feed is drawn, and from which a discharge-pipe conveys the waste water through the ship's side into the sea. E, the cross-head of connecting rod, connected by side rods at each end to the beams D. F, the connecting rod, causing the revolution of the crank G, on the shaft of which is fixed the paddle-wheel. M is a parallel motion for constraining the cross-head C to move in a straight line. The feed-pump is generally placed on one side of the air-pump, and a bilge-pump (for emptying the bottom of the vessel of water leaking into it) on the other, both being worked by the cross-head which moves the air-pump bucket. The general arrangement of the marine engine is such as to keep the principal weight at as low a level as possible, lest the stability of the vessel should be endangered by too much weight above the floating line. These engines are generally in duplicate, placed side by side longitudinally in the vessel, the crank-shafts working the paddle-wheels being laid transversely.

The conditions of a marine engine suited for driving a screw-propeller, differ considerably from those of the engines we have hitherto described. The shaft of the screw-propeller is necessarily at a low level, and lies longitudinally in the vessel to which it is fitted, and it must be made to revolve at considerable velocity. When this system of propulsion was first tried, it was usual to employ ordinary marine engines placed transversely in the vessel, and to fit on their shaft a large cog-wheel, driving a pinion on the shaft of the screw-propeller below it. By this arrangement the low level of the latter was secured, as well as its high velocity; but the weight, bulk, shake, and noise of the toothed gearing were found objectionable. Screw-propeller engines are generally made now with very short stroke, so that the number of strokes made in a given time is much greater than in ordinary engines of equivalent power. The cylinders and air-pumps are arranged horizontally; and the piston-rods act on the cranks placed at a low level, without the intervention of side beams.

Combined Engines are those which embody the principles of both the condensing and non-condensing engines. In these a small cylinder and a large cylinder are placed side by side, and their piston-rods are connected so as to work together. High-pressure steam admitted to the small cylinder, works its piston as in a non-condensing engine; but instead of being permitted to escape into the atmosphere, is conveyed to the larger cylinder, in which it expands to greater volume but lower pressure, and acts on a greater area of piston. Thence the expanded steam flows into a condenser, as in the ordinary condensing engine.

This arrangement gives the opportunity of securing a large amount of useful effect from the steam, and thereby economizing fuel in the boiler. If we suppose, for example, that steam with an absolute pressure of 75 lbs. is admitted into the smaller cylinder during $\frac{1}{4}$th of the stroke, and allowed to expand during the remainder of the stroke, its final pressure, allowing for a little loss by cooling, will be about 18 lbs., and the mean pressure throughout the stroke will be about 42 lbs. The steam at 18 lbs. being now admitted into the larger cylinder during $\frac{1}{2}$ the stroke, will have a final pressure of about 8 lbs., and a mean pressure of about 13 lbs. throughout the strokes. The back pressure of rarefied air and vapour in the condenser may average 2 lbs. throughout the stroke, and the mean effective pressure on the larger piston would then be $13 - 2 = 11$ lbs., while that on the smaller is $42 - 13 = 29$ lbs. per square inch. Now, if the area of the small piston be to that of the large in the proportion of 11 to 29, the total mean pressure on each will be alike; and the power of the engine will be double that of either, with an expenditure of steam at 75 lbs. pressure, or 60 lbs. above

that of the atmosphere, only sufficient to fill ⅕th of the capacity of the smaller cylinder. By cutting off at an earlier period of the stroke, and using a still larger second cylinder, still greater economy of steam may be attained.

In determining the power of a condensing engine, it is necessary to know the pressure of the steam employed, the amount of vacuum produced in the condenser, the size and velocity of the piston, and the various losses of force occasioned by the friction of the machinery, and the resistance of the air-pump, feed-pump, and cold-water pump. Assuming that, generally, the speed of the piston is about 200 feet per minute, and that the pressure in the boiler does not exceed 5 or 6 lbs. per square inch above that of the atmosphere, we may take the following rule as to power:—

Given the diameter of the cylinder to find the power.

Rule.—From the diameter (in inches) subtract 6, square the remainder, and divide by 20 for the power.

Example.—What is the power of a condensing engine having a cylinder 54 inches diameter?

$$54 - 6 = 48, \text{ and } \frac{48 \times 48}{20} = 115 \text{ horse-power.}$$

Given the power to find the diameter of the cylinder.

Rule.—To the square root of 20 times the power add 6, for the diameter in inches.

Example.—Required the diameter of cylinder for 115 horse-power.

$$20 \times 115 = 2300, \text{ square-root} = 48 \text{ nearly, and } 48 + 6 = 54 \text{ inches.}$$

These, however, merely furnish rough guesses at the power of any engine. The proper method of ascertaining the real power is to apply the indicator, when the engine is only moving itself, so as to ascertain the power necessary to overcome the friction and resistance of its working parts; and to deduct this quantity, with some allowance for additional strain, from the power indicated when the engine is in full work. No definite rule can be offered for estimating these allowances, as varieties in construction and workmanship introduce great differences among experimental results. In general, it is safe to reckon not more than ⅘ths of the indicated power as really effective to move machinery, the remaining ⅕th being absorbed in friction and moving the pumps.

Rotary Engines.—In all steam-engines, except those that are single acting, a good deal of mechanism is necessary in order to convert the reciprocating rectilineal motion of the piston into the continuous circular motion of the crank. It has, therefore, been a great object with many mechanics to devise a rotary engine, or one in which the steam pressure shall at once give the required rotary motion, without the intervening machinery of connecting-rod, beam, parallel motion, and crank. Many of the arrangements devised for this purpose present great ingenuity, and it is not improbable that some may prove ultimately successful; but hitherto no rotary engine has proved so far satisfactory as to warrant its adoption in the place of those already in use.

A singular error regarding rotary steam-engines has crept into some of the best books on the subject; and as it is calculated to discourage inventors who may apply themselves to devising engines of this kind, we will endeavour to point it out. It is asserted that steam applied to give direct rotary motion to a piston is less effective, or loses part of the power which it would produce if applied to move a piston in a straight line. If we suppose E D (Fig. 210) to be a piston fitted to an annular cylindrical vessel B G A F, so that it may be moved round the centre C by the pressure of steam admitted behind it, we have to ascertain whether a certain quantity of steam applied to a piston thus arranged will generate as much power as it would produce when applied to a piston in the ordinary way. Some authors (among them Tredgold, in his large

work on the steam-engine) say that "the quantities of steam being equal, the power of rotary action will be less than that of rectilineal action;" and this fallacy is but too generally admitted among engineers. That it is a fallacy may be very easily shown by a practical example. Let us suppose that the piston D E measures 10 inches each way, and therefore presents a surface of 100 square inches, and that it is acted on by steam having a pressure of 20 lbs. per square inch; so that the total pressure on its surface is 100 × 20 = 2000 lbs. Now, considering C D as the arm of a lever, of which a portion E D is loaded with pressures distributed uniformly over it, we know that their combined effect to turn the lever round its fulcrum C is the same as if the whole pressure were collected into one force at H, the centre of gravity of the part D E. We have, therefore, effectively a lever or arm of the length C H, pushed round C by a force of 2000 lbs. applied at H. If we take C E = 9 inches, then as E H = 5 inches ($\frac{1}{2}$ of 10 inches), C H = 14 inches; and during one revolution, H passes over a distance equivalent to the circumference of a circle having a radius of 14 inches—that is to say, over 88 inches, or 7⅓ feet. Hence, the work done during a revolution is 2000 lbs. × 7⅓ feet = 14,667 lbs. moved over 1 foot. Also the quantity of steam required to fill the annular space passed through by the piston is thus found, C E being 9 inches, and C D being 19 inches:—

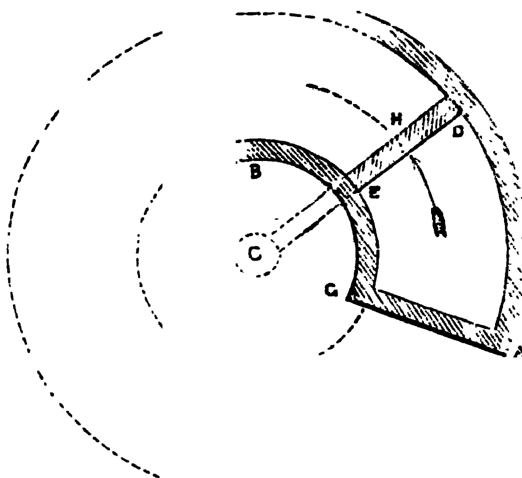


Fig. 210.

Area of outer circle (38 inches diameter) . .	=	1134·12 square inches.
Area of inner circle (18 inches diameter) . .	=	254·47 ,,
Area of annular space (difference)		879·65 ,,
Multiply by breadth		10 inches.
Volume of annular space		8796·5 cubic inches.

Were we to apply this quantity of steam in an ordinary cylinder, whatever be its dimensions, we should produce equivalent power. Let us, for example, take a cylinder 1 foot diameter, having an area of piston 113·1 square inches, subjected to a pressure of 113·1 square inches × 20 lbs. = 2262 lbs., moving through a stroke of 6·488 feet (a length which requires equivalent volume of steam), we find the work to be 2262 lbs. × 6·488 = 14,667 lbs. moved over 1 foot as before. Instead of reckoning the whole revolution of the rotary piston, we might take any portion of its revolution, and we should find the power developed by the steam used exactly equal to that which would be pro-

duced by the same quantity of steam, in any corresponding portion of a cylinder where the piston moves rectilineally. Without reference to any special numerical example, we observe these general laws. Whatever be the form of the rotating piston D E, steam pressing uniformly on its surface produces the samo effect to turn it round its centre C, as if the whole pressure were collected into one force acting at the centre of gravity H of the surface of piston; and the work done by the steam during any part of a revolution is equivalent to the total pressure in the piston, multiplied by the distance traversed by the centre of gravity H, or the portion of the circumference of a circle of which C H is the radius; and, further, the volume of steam required during the given portion of a revolution is (by the well-known law of mehsuration of annular solids) measured by the area of the piston, multiplied by the distance passed over by its centre of gravity. In the case of a piston moving rectilineally, the work done by the steam, and the volume of steam used during any portion of a stroke, are measured in precisely the same way, and bear the same relation to each other. There is, therefore, no theoretical objection to the application of steam in such a way as to produce direct rotary motion. That there are considerable practical difficulties in the arrangement of the parts, and their construction so as to present steam-tight rubbing surfaces, and to avoid undue friction and unequal wear, is doubtless true; but were these difficulties fairly surmounted, we should be in possession of an engine where simplicity, and economy of weight and bulk, might enable us to apply steam-power in many cases where it is not now applicable without inconvenience.

Before leaving the subject of rotary engines, we may mention that steam has been applied successfully to produce rotary motion on the same principle as that of Barker's mill, or the turbine applied to water-power. Steam of considerable pressure, passing through openings on the sides of several tubular arms mounted on an axis, causes them to revolve in the direction opposite to that in which the steam issues, on the same principle as the movement of a rocket, where the issue of the elastic gases, generated by the combustion of the charge at one end, leaves an unbalanced pressure to act on the other end, and thus to force it onward through the air; but we believe this mode of applying steam-pressure, though exceedingly simple, is by no means sufficiently economical to warrant its general adoption.

APPLICATIONS OF STEAM-POWER.

The chief purposes to which steam-power is applied are the following:—

I. Pumping water for the drainage of mines and docks, or at water-works for the supply of towns.

II. Driving machinery for raising ore from mines, for moving heavy weights, or for agricultural and manufacturing purposes.

III. For locomotion on railways.

IV. For propelling steam-vessels.

We will briefly discuss the modes in which steam-power is generally made available for these different uses, and the forms of steam-engines most advantageously applied in each case.

I. Pumping.—In pumping water, until of late years, single-acting engines were almost universally applied. We may most readily account for this, not on the ground of

any advantage derived from the use of single-acting engines, but from the circumstance that the earliest form of engine that was rendered practically available was that of Newcomen, which was single-acting, and suited only for working pumps; that Watt's improvements were first applied to engines of this kind, and that his engines were introduced at mines to supersede the labour of men and horses formerly applied to pumping; that these engines were of first-rate quality, effective and durable, and naturally impressed the miners with a preference for their form and arrangement; and that engineers in mining districts applied themselves rather to the perfecting of forms already in use than to the introduction of new forms. So strong indeed has been the preference for single-acting engines, especially those of the kind employed in Cornwall, when applied to pumping, that few double-acting engines have ever been employed for this purpose until very recently. We believe, however, that the results have shown decided advantages in favour of the double-acting engines, and that ere long they will supersede the more cumbrous and less advanced form of those that are single-acting.

The pumps employed in mines are of the lifting or forcing kind, having a stroke seldom exceeding 8 or 10 feet. When the mine is deep the water is raised by stages, each 150 to 200 feet in height. The lowest pumps discharge into a reservoir about that height above the bottom of the mine; the second set of pumps draw from that reservoir, and discharge into the next higher; and so on, until the water is finally delivered at such a level that it may be permitted to flow off by natural drainage. As apparatus is generally required at mines for stamping and crushing the ore, it is not unusual to deliver the water at such a height above the general level of the ground, as may permit its use for driving a water-wheel, from which motion is given to the ore-crushing machinery. But as power is also required for raising the ore, it is generally more advantageous to employ a double-acting engine, besides the main pumping engine, for this purpose, as well as for the preparation of the ore. The power required for raising water depends upon the quantity raised in a given time and the height to which it is raised. If we suppose that it is necessary to raise 100 cubic feet of water per minute 150 fathoms, or 900 feet, since each cubic foot of water weighs $62\frac{1}{2}$ lbs., the total to be raised is $100 \times 62\frac{1}{2} = 6250$ lbs. per minute. This load raised through 900 feet is equivalent to $6250 \times 900 = 5625000$ lbs. raised 1 foot, and would require $\frac{5625000}{33000} = 170$ horse-power, without allowance for friction and excess of power to give the velocity of movement to the column of water. To estimate this quantity, we should add at least 10 per cent. or $\frac{1}{10}$th, making the power 187-horse, which should be the effective power, after making all allowances for the friction of the engine and the various losses occasioned by the cooling or leakage of steam, and the working of the air-pump and feed-pump.

The performance of the Cornish engines has received special attention from the circumstance that a number of mine proprietors placed their engines under the superintendence of engineers, and had monthly reports published as to the quantity of water lifted, the quantity of coals consumed, and other particulars. Six of the most effective engines reported on presented the following average results:—

Diameters of cylinder from 60 to 90 inches.

Mean pressure of steam per square inch, 10 lbs.

Length of stroke from 8 to 10 feet.

Average number of strokes per minute, 7.

Diameter of pumps, 9 to 18 inches.

Average length of pump-stroke, 7 feet 6 inches.

Average work, reckoned in lbs. of water lifted 1 foot high by the consumption of 1 bushel of coals, 40,500,000 lbs., or the work of 1,227 horse-power.

In pumping docks, the quantity of water to be lifted in a given time is generally great, but the height inconsiderable, and the pumps may therefore be of large diameter. As a practical example, we may take the case of a rectangular dock, 200 feet long, 40 feet broad, and 20 feet deep, the water being lifted 2 feet above the highest level to give a fall for its drainage. The total quantity to be lifted is $200 \times 40 \times 20 =$ 160,000 cubic feet, or $160{,}000 \times 62\frac{1}{2} = 10{,}000{,}000$ lbs. At the beginning of the work the water has to be lifted only 2 feet, at the end it has to be lifted 22 feet, and the mean height of lift is therefore 11 feet, the work done being equivalent to $\frac{10{,}000{,}000 \text{ lbs.} \times 11}{33{,}000}$ $= 3333$ horse-power; or allowing $\frac{1}{10}$th for friction and velocity, about 3660 horse-power. If the work be done in 1 hour or 60 minutes, the power of the engine must be $\frac{3660}{60} = 61$ horse-power, clear of all losses. As in a case like this, the work at first is very light, and becomes greater as the level of the water is lowered, and the height to be lifted consequently increased, it is advisable to have several pumps, all of which may be kept in action at first, but which may be thrown out of action successively as the load on them becomes increased, the strain of the engine being thereby not greatly varied.

In water-works for supplying towns, not only has the water to be lifted to such a height that it may command the highest level to which it has to rise, but it has also to be conveyed through great lengths of pipe, often extending many miles. It is convenient to arrange, as near as possible to the pump, a high reservoir of such altitude as may be sufficient to cause the necessary flow in the great length of pipe connected with it. This is effected by pumping the water through an elevated siphon, open at the top to permit the issue of air that may be mingled with the water. The pump, in making its stroke, has thus to put in motion only the column of water contained in the ascending limb of the siphon, instead of the whole mass contained in the pipes. But in order still farther to relieve the pump and engine from the strain required to put in sudden movement even this mass of water, the pump is provided with a capacious air-vessel, which is a dome-shaped vessel, the upper part of which contains air, and the lower part communicates freely with the discharge-pipe of the pump. When the pump discharges its contents, the air in the vessel is compressed by the influx of water below it; and while the pump is making its return stroke, the elasticity of the compressed air continues the flow of the water that had been forced into the vessel. The air thus acts as a spring, yielding to the force of the discharge from the pump, and sustaining the pressure on the flow-pipe at intervals when the pump is inactive. When the pump is double-acting, or discharges at both the up-stroke and the down-stroke, the intervals of inaction are only those occupied by the turn of the stroke, and the air-vessel has less to do than when the pump is single-acting. Two single-acting pumps, of which the one discharges while the other is drawing, produce a like effect; upon the whole, we believe that for simplicity, economy of cost and of working, a good double-acting engine, with double-acting pump, is preferable to two single-acting engines and pumps, each of half the power, or to one single-acting of equivalent power. We know of no advantage possessed by the single-acting engine, which cannot be fully secured in the double-acting engine. It has so happened that many single-acting engines have been made of large dimensions and long stroke, and that the expansive power of the steam has been

employed by cutting it off at an early part of the stroke, to such an extent as to secure great economy of fuel. But in the double-acting engine, particularly when the duplicate cylinder, with high-pressure and low-pressure action, is applied, the same economy of fuel can be secured, and the bulk and weight of the whole are greatly reduced.

II. **Driving Machinery.**—The engines so employed, whether to raise ore from mines, to draw railway trains up steep inclinations, or for agricultural and manufacturing purposes, are universally double-acting. Where the power required is small, or where fuel is cheap and water scarce, non-condensing engines are generally preferred. But, on the other hand, where there is a plentiful supply of water, and where fuel is expensive, condensing engines are employed; as by a certain expenditure of fuel, there is certainly a greater amount of power generated when the steam is condensed. For large manufactories, cotton, flax, and flour mills, breweries, and like establishments, the beam-engine is generally employed. Its advantages consist in the accessibility of all its parts in case of damage or repair, and the steadiness and regularity of its movement, resulting from its massiveness and solidity. The substantial and imposing look of a large beam-engine is certainly, however, with many persons, an argument for its use, where engines of less weight and bulk might be applied with quite as great advantage. The marine engine, having received great attention to its perfect construction, has often with advantage taken the place of ordinary beam-engines in manufacturing establishments; and we believe the only objection to their general use is, that they are rather more expensive in the first place. Where there is a deficiency of water for condensation, it is necessary to provide large reservoirs in which the water discharged from the air-pump may have time to cool before it is again used in the condenser. Warm water, when used for injection, must either be admitted in quantities so great as to impede the engine by the additional work thrown on the air-pump,—or, if limited in quantity, can only effect partial condensation of the steam, so that the piston is impeded by the imperfection of the vacuum.

The power of an engine required for manufacturing purposes, necessarily depends on the kind of work to be done, and its amount. A manufactory generally contains numerous machines, like and unlike; and unfortunately very few accurate experiments have been made as to the power required for working them. It is stated that in the cotton manufacture, one horse-power is sufficient for 100 spindles of cotton yarn, and the machinery necessary for the preparation of the cotton; and, again, that one horse-power will work 12 power-looms. But, every day, improvements are being made in the construction of such machinery, and changes are introduced among the methods of preparing the material; and it therefore becomes difficult to assign any determinate method of estimating the power.

In the iron manufacture, steam-power is generally applied to the production of the blast of air for the smelting furnace. The air is condensed by a blowing cylinder or pump, till it exerts a pressure of 2 to 3 lbs. per square inch above that of the atmosphere, and from 1500 to 1800 cubic feet per minute are forced into the furnace. When for simply melting or heating iron in a cupola, reverberatory furnace or forge, the pressure of the air need not be so great as in smelting operations, it is supplied by a fan or wheel, with several blades revolving rapidly in a case. The rapid rotation of the fan causes the air occupying its central part to fly off to its circumference by centrifugal force; and openings being provided at the centre and the circumference of the fan-case, the air rushes in at the former and out at the latter with considerable velocity, and is conveyed by air-drains or tubes to the furnaces. The various processes to which the

iron is subjected—rolling, shearing, flatting, punching, turning, planing, and the like—are all conducted by the application of steam-power. And so with operations on other metals, on wood, and generally all crude materials, the facility of deriving adequate power from the application of heat to water enables us to employ machinery instead of manual labour in a multitude of ways, so as not only to execute works that could not have been attempted without having such a force at command, but to do so with an economy and ease that could not otherwise be attained. In all these manufacturing operations it is essential to the safety as well as to the accurate working of the machinery, that the speed of the prime mover should be uniform.

Where there are numerous machines driven by the same power, it may often happen that several are in operation at once, while at another time none may be at work. To overcome the resistance caused by suddenly bringing a heavy machine into action, as in the case of a set of rollers for preparing iron, and at the same time to prevent the dangerous increase of speed that would result from suddenly throwing it out of work, it is essential that there should be a good governor and a very heavy fly-wheel. The fly-wheel acts as a reservoir of force in the one case, ready to give out a portion of its momentum when the additional resistance comes upon it; and as a reservoir of resistance or inertia, in the other case, ready to absorb a portion of the unnecessary power. The governor, again, permits the supply of additional steam to the cylinder as soon as the velocity of the engine begins to undergo diminution, and checks or throttles the supply whenever the speed rises above the average.

When the power required is small, or under circumstances where there is a difficulty in supplying water for condensation, or where there would be a great disadvantage in having any complicated machinery to be attended to, it is usual to employ non-condensing engines; these being more simple are less costly in the first place, less liable to derangement and wear, and more easily understood and managed. It is true that for a given amount of power, the expenditure of fuel is somewhat greater than where the steam is condensed; but with a good boiler generating steam at a pressure of 40 to 60 lbs. if the cylinder be made of considerable size, and the steam cut off at an early part of the stroke so as to act expansively, the excess of fuel expenditure is almost inappreciable. For agricultural purposes, in the colonies, or generally in districts removed from engineering establishments, simplicity is of more importance than even economy of fuel. Many of these non-condensing engines, from 5 to 15 horse-power, are made portable, so that the power may be readily brought to the work instead of the work being brought to the power. The portable engine consists of a cylindrical boiler, perforated with flues or tubes like that of a locomotive, mounted on wheels, with a direct-acting engine laid horizontally upon the boiler, or arranged in some convenient way for lightness and economy of space. For thrashing and winnowing corn, draining or irrigating land, brick and tile making, and field operations generally, these engines are very serviceable, especially in cases where the work of a farm is not enough to keep an engine constantly employed, and where the same engine may do duty throughout a considerable district. We believe, however, that as agriculture approaches more to the condition of a manufacturing art than it has hitherto done, the fixed engine will gradually supersede those that are portable; the farm buildings will become the head-quarters of the various operations; the materials will be brought thither to be operated on; and agricultural processes will be carried on with the regularity and precision of other branches of art. Of late years, many persons of high standing and energetic have devoted themselves to the furtherance of this object; and their example

is being generally followed throughout the country. His Royal Highness Prince Albert has erected a very complete farming establishment near Windsor. A fixed engine puts in motion a train of machines for thrashing, dressing, and grinding corn, cutting chaff, bruising beans and oats, steaming the food of cattle, and other processes, all systematically and economically arranged; and similar plans are adopted in many other places at home and abroad.

III. **Locomotives.**—In no application of steam-power has greater ingenuity been manifested than in locomotive engines. The great essentials of a locomotive are, lightness, compactness and strength of construction, rapidity of action, and facility of management. Some of the earliest attempts at these engines were necessarily rude and imperfect; but the rapid extension of the railway system, and the immense advantages derived from speed in the transmission of passengers and goods, have hurried on improvements so quickly, that in a very few years the locomotive has risen to a high degree of perfection. For some time it seemed difficult to contemplate the possibility of covering the land with a net-work of iron-ways, extending over many thousand miles in length; and accordingly attention was chiefly devoted to the construction of locomotives for running on ordinary roads. Some of the engines made for this purpose were tolerably successful in their operations, and embodied in their construction a great amount of ingenuity, which has not been without its use in leading to the more perfect locomotive of the present time. Even when railroads began to be formed, the employment of steam on them was a doubtful question, and their most sanguine promoters scarcely dared to hope for a speed exceeding 10 miles an hour, or to expect that passenger-traffic would equal in importance the conveyance of heavy goods. It was feared that the friction of an iron wheel on an iron road would not be sufficient to move onward the locomotive itself, much less a heavy train of carriages; and accordingly it was proposed to apply levers to act as legs and feet propelling the carriage, or to lay down a continuous rack, in which a toothed wheel driven by the engine might work, or an endless chain to be wound round a revolving barrel. It was not until the actual trial was made, that the simple expedient of causing a pair of wheels to revolve on the rails was adopted without reserve.

The boiler of a locomotive has already been described; the engines are simply two, direct acting, laid horizontally side by side under the boiler, working cranks at right angles to each other on a shaft which crosses under the boiler transversely, and has the driving-wheels fixed at each end. For each revolution of the engine, therefore, there is one revolution of the driving-wheels. Sometimes these are made 6 to 7 feet in diameter, and have therefore a circumference of 20 feet. If we suppose the engines to make 200 revolutions per minute, or $200 \times 60 = 12000$ per hour, as the whole circumference of the driving-wheel is brought to bear on the rail during each revolution, if there be no slip, the locomotive must advance 20 feet for each revolution—that is to say, $12000 \times 20 = 240000$ feet, about $45\frac{1}{2}$ miles per hour. When the rails are damp and slippery, the wheels occasionally slip round without taking sufficient hold of them to propel the train. In such cases a little sand strewn on the rails generally causes sufficient friction to commence the movement of the train; and its momentum once established, the locomotive has only to keep it up against the resistance of the air and the friction of the running wheels. When the locomotive is applied to the propulsion of heavy goods'-trains, especially up inclinations, it is necessary to obtain more friction than could be derived from one pair of driving-wheels. For this purpose the locomotive is fitted with one and sometimes two additional pairs of driving-wheels, of the same size as those

worked by the engine. These wheels are coupled by connecting-rods working on pins projecting from their arms, so that they all revolve simultaneously, and thus present double or treble the amount of rubbing surface on the rails. The locomotive has attached to it a tender, which is an iron tank mounted on wheels, divided into two compartments, one containing fuel and the other water for the supply of the engine. The water-tank is connected by a flexible tube to the feed-pump of the locomotive, by which the pump draws its supply, and forces it into the boiler. In order to save fuel, the water in the tank is heated by steam blown through it from the boiler, while the train is stopping, and the steam is blowing off to waste by the safety-valve. The engine-driver and stoker stand on a stage at the fire-box end of the boiler, and have under their eye the water and steam-gauges. They have conveniently arranged levers for working the steam-valve so as to permit more or less steam to enter the cylinder; for moving the link-motion of the slides, so as to reverse the motion of the engines; for adjusting the pressure on the safety-valve, the supply of feed-water, and the break on the wheels when it is necessary to stop or move more slowly. They have convenient means of oiling all the working parts, and of opening pet-cocks in the cylinders to permit the issue of water, and the cock for sounding the steam-whistle as a signal of their approach. Altogether the locomotive is perhaps the most perfect apparatus that has been designed by engineers; and the materials and workmanship applied in its construction are necessarily of the best kind, to withstand the constantly reiterated shocks of the movement, and to convey power so great through parts so light and apparently so complex.

IV. **Propulsion of Vessels.**—Although numerous modes of applying steam to the propulsion of vessels have been proposed, two only have met with general adoption—viz. the paddle-wheel and the screw-propeller. The action of paddle-wheel engines is precisely similar to that of locomotives: the paddles in the one case occupying the place of the driving-wheels in the other, and having float-boards successively immersed in the water and withdrawn from it as the wheels revolve. The paddle of a steam-vessel is, indeed, an undershot water-wheel reversed in its action; that is to say, instead of the wheel being put in motion by the current of water, the wheel put in motion by the steam-engine causes a current of water by its revolution; and the reaction of the water propelled by it backwards, forces the wheel forwards, and with it the vessel to which it is attached. As the float-boards enter and leave the water, they act on it obliquely, tending in some measure to press it down in front and raise it behind. In sea-going vessels, where the wheels often act on the undulating surface of the water, this obliquity of action becomes a very considerable resistance, and tends to retard the engines and give them severe shocks. To obviate this defect, paddles are frequently made in such a manner that while passing through the water they hang nearly vertical. Such a wheel is called a *feathering* paddle, because the float-boards *feather*, or enter and leave the water edgeways like the oar in the hands of a practised rower. The power required for a steam-vessel depends upon its form and tonnage, and the speed at which it is moved.

Steam vessels are generally made very long in proportion to their breadth, and finely wedge-shaped at each end, so as to cut through the water with as little resistance as possible. In vessels of similar form and proportions the power required to produce a given speed is nearly as the tonnage. But a very slight increase of speed demands a very considerable augmentation of power, as may be thus estimated:—To double the speed of a vessel it is necessary to push aside double the quantity of water in a given time, and to impel this water with double the velocity, and therefore to encounter four times the resistance;

and, as the speed of the engines must be at least doubled, the power expended in a given time must be at least eight times. Generally the power may be taken as the cube of the speed, or rather more. If, for example, a pair of engines working to 100 horse-power propelled a vessel at the rate of 8 knots (nautical miles) per hour, we should have to work the engines up to 200 horse-power to attain a speed of 10 knots per hour; because while the cube of 8, or $8 \times 8 \times 8 = 512$, the cube of 10 is 1000, nearly double of 512, and therefore the power in the one case must be double of that in the other. The same law applies in the case of vessels propelled by a screw.

The principle on which the screw acts as a propeller may be best understood by considering the action of a windmill reversed. The screw generally consists of two inclined blades (Fig. 211) projecting from an axis mounted in bearings in what is

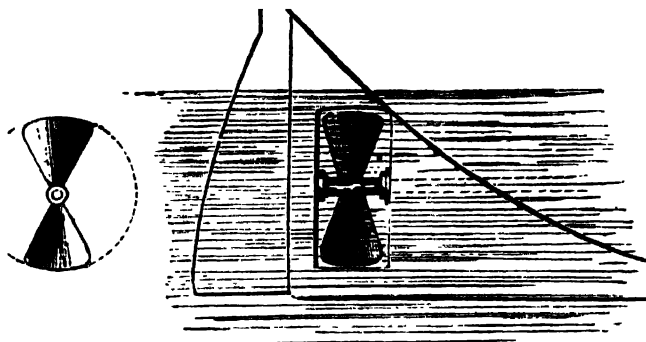


Fig. 211.

called the dead-wood of a vessel, or the part of the stern immediately before the rudder. The screw-shaft works through a water-tight tube fixed in the dead wood, and is put in motion by the engines fixed in the body of the vessel. The whole screw is immersed under water, and the blades are of such size that, looking endways on the shaft, they appear to occupy each about one-sixth of the circle which circumscribes them.

The *pitch* of the screw depends on the obliquity of the blades to the plane of the circle, and this obliquity increases from the circumference towards the centre, because the parts nearer the centre travel with less circumferential velocity, and yet the water passes them with equal longitudinal speed. When we say that a screw has twenty-feet pitch, we mean that if the screw-blade were continued through a complete revolution round its axis, any straight line drawn parallel to the axis would cut two portions of the blade at points twenty feet apart. Did the screw in revolving through the water put the latter in motion at such a rate as precisely to glide along its blades in straight lines parallel to the axis, every revolution of the screw would move any portion of water exposed to it through a length of 20 feet. But, practically, the water is almost at rest, and the screw worms its way through it, having to put in motion the vessel to which it is connected, and a certain *slip* occurs, or the speed with which the screw and vessel travel through the water is less than that due to the obliquity of the screw by about 25

or 30 per cent. If we suppose that a screw of 20 feet pitch makes 60 revolutions per minute, any point in its surface without slip would move through $20 \times 60 = 1200$ feet longitudinally per minute, or $1200 \times 60 = 72000$ feet $= 12$ knots an hour nearly. But allowing twenty-five per cent., or one-fourth, for slip, the actual speed would be nine knots per hour.

Although screw-propellers have not as yet been applied in such a manner as to produce speed equivalent to paddles from a given consumption of fuel, yet the advantages resulting from their small bulk and weight as compared with paddles, from their being arranged in a place almost inaccessible to shot, and in such a manner that they do not interfere with the sailing qualities of the vessel, are such as to favour their general adoption in the navy as well as in the mercantile marine.

It would be beyond the limits of a work like this to enter upon the consideration of the various forms and details of engines suited for special purposes. We have endeavoured only to present a few leading outlines of their general arrangements, and of the construction of parts more or less common to all. Peculiarities of site, of purpose, and even of taste, dictate peculiarities of arrangement. There cannot be said to be any *beau-ideal* or perfect example of a steam-engine; but moderate attention given to the leading features of effective engines will better enable the practical student to appreciate advantages, and detect errors of design and construction, than the study of the best drawings or descriptions.

THE COMMUNICATION OF POWER.

The communication of power is really the communication of motion, for power implies change, and change of place is motion. A column supporting a weight, communicates the pressure of the weight to its foundation; and a string suspending a load from a hook, communicates its tension to the hook; but in neither of these cases is *power* communicated, for no change is effected: time forms no element in the question, the same strain being conveyed to the point of support in an instant as there is conveyed in a century. But if the column or string be in motion like the piston-rod of a steam-engine pushing or pulling against a resisting force, not only is the pressure acting on the piston conveyed through the rod as a simple strain, but it is also conveyed through a certain distance in a certain time, or at a certain velocity, and can by its motion effect changes on materials presented to it, proportioned to the amount of power developed, and the time during which it acts. The *power* of any mechanical arrangement, therefore, means its capability of effecting change; while its *work* means the quantity of change effected. The most simple kind of change which we can contemplate is that of position; and we therefore take change of position or motion as the measure of one element of work done. The most simple notion as to quantity of change, is that of mass or weight of material moved; and we therefore take weight as the measure of another element of work. Lastly, we estimate the capability of effecting change—that is to say, power—by the time required for the work done. The greater the mass moved, the greater the distance over which it is moved, and the less the time occupied in the motion, the greater the power developed.

An engine of 10 horse-power, means an engine capable of moving 10 times the mass that can be moved by an engine of 1 horse-power over a given distance in a given time, or of moving the same mass over 10 times the distance in the same time, or of moving

the same mass over the same distance in $\frac{1}{10}$th of the time. Power, indeed, is simply pressure multiplied by velocity; and if we know the pressures and velocities communicated through any trains of machinery, we compare their powers by comparing the products of these factors.

It is a simple law of mechanics, admitting of no exception, that whatever be the nature or complication of any mechanical arrangement by which power is conveyed, whatever be the power applied to move it, the same power would be given out from every part of it, provided no waste occurred from friction. And the more perfect the construction of the machinery in respect of smoothness of rubbing surfaces, the more nearly is this law practically fulfilled.

If we take any of the simple mechanical powers, such as the lever A B (Fig. 212), capable of vibrating on the fulcrum F, we observe that when A F is equal in length to F B, a weight of 10 lbs. hung from A, balances the same weight hung from B; or when A F is twice F C, a weight of 10 lbs. at A balances 20 lbs. at C.

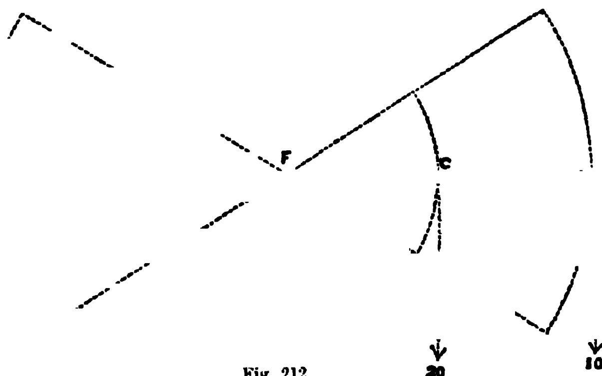


Fig. 212.

Such is the law for mere pressure or weight without motion; but when we consider the question in respect of power, we have to imagine that the lever vibrates on its fulcrum, so that every point of it describes a portion of a circle round the centre F. We may suppose one such vibration effected in a second, and that the arc described by A measures 2 feet in length, then will the arc described by C be 1 foot long; and 10 lbs. at A, moving through 2 feet in 1 second, develops the same power as 20 lbs. at C moving through 1 foot in 1 second, because $10 \times 2 = 20 \times 1$; the products of the pressures by the velocities are equal in both cases. We might trace the same law through all the simple mechanical powers, and through all possible combinations of them; that is to say, through all possible arrangements of machinery.

The practical application of this law in calculating the proper strengths and proportions of parts of machines is exceedingly simple.

In a machine, for instance, moved by 1 horse-power, whenever we can ascertain the velocity of any part, we can find at once the pressure or strain passing through that part. One horse-power is reckoned as 33,000 lbs. moved 1 foot per minute; if, then, some part of the machine in question were found to move over 10 feet in 1 second, or 600 feet in 1 minute, we should at once know that the strain on that part is equivalent to a pressure of 55 lbs., for 55 lbs. $\times$ 600 feet $=$ 33,000 lbs. $\times$ 1 foot. As a practical example, let us suppose that in some part of a machine worked by 5 horse-power, there is a wheel 7 feet in diameter revolving 30 times in a minute, and that we desire to estimate the pressure on the rim

of that wheel, or the resisting force which would have to be applied to balance its rotary force.

The circumference of a wheel 7 feet in diameter is 22 feet, and this moving 30 times round in a minute has a velocity of $22 \times 30 = 660$ feet per minute; the power, 5 horse, is quivalent to $33,000 \times 5 = 165,000$ lbs. moved through 1 foot per minute, or 250 lbs. moved through 660 feet per minute. The strain on the rim of the wheel is therefore equivalent to 250 lbs. Generally, the rule for estimating the strain (in lbs.) of any part is to divide the power (expressed in lbs. moved over 1 foot per minute) by the velocity of the part (in feet per minute).

The kind of motion most conveniently conveyed through machinery is rotary motion; and the chief subject of our inquiry will be as to how this can be conveyed and converted into motions of another kind, such as vibratory or reciprocating movements.

A shaft or spindle is a rigid bar of metal, or sometimes of wood, caused to rotate round its axis, and capable of conveying the rotary motion given to it along its whole length, and giving it off at any point to machinery connected with it. The lengths of shafts are limited, in consequence of the difficulties attending their construction in great lengths. Large shafts, such as are used for the paddles of marine engines, are sometimes 20 or 30 feet long; but smaller shafts seldom exceed 10 or 12 feet in length. When greater lengths are required, the shafts have to be coupled or connected together.

Fig. 213 represents a simple coupling for round wrought-iron shafts. A and B, the ends of the two shafts, being nicely rounded, are inserted into a cylindrical box bored to fit them, and having a groove or keyway cut along its interior surface corresponding to grooves cut in the shafts. These

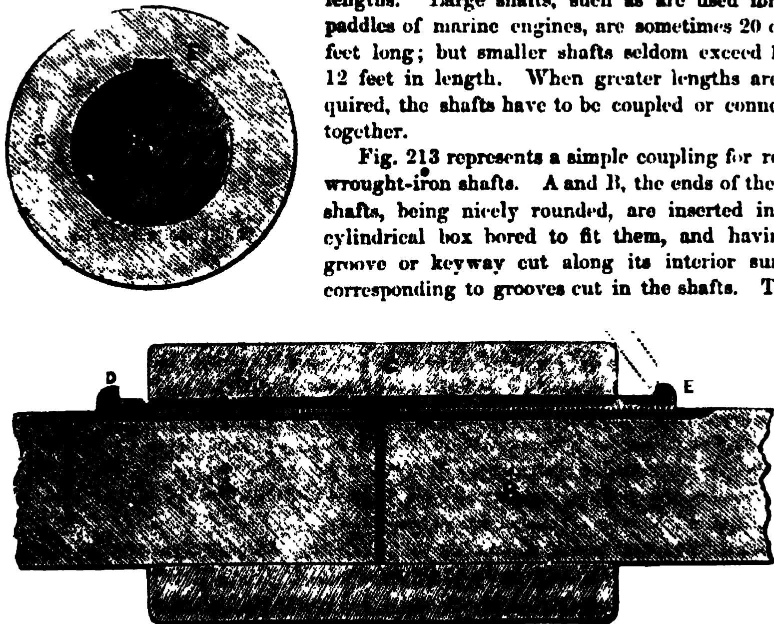


Fig. 213.

grooves are filled by keys, D E, made slightly taper, and driven tightly in from each end. One of the shafts A being caused to rotate, must carry the coupling round with it, and also the other shaft B, because the keys prevent either shaft from slipping round within the coupling, or the coupling from slipping round either. In making such a coupling of cast-iron, the following proportions will be found practically suitable:—External diameter of coupling double that of shaft; length of coupling three

times the diameter of shaft; width of key, one-fourth the diameter of shaft; mean depth of key, one-half its width.

The keys should be made with *gib-heads*, as shown in the figure, so that they may be driven out when required by applying a piece of iron rod to the projecting part and striking it with a hammer, as marked by the dotted lines F, or by driving a wedge between the gib-head and the coupling.*

Some engineers in coupling the shafts, halve them over each other, as in Fig. 214, and

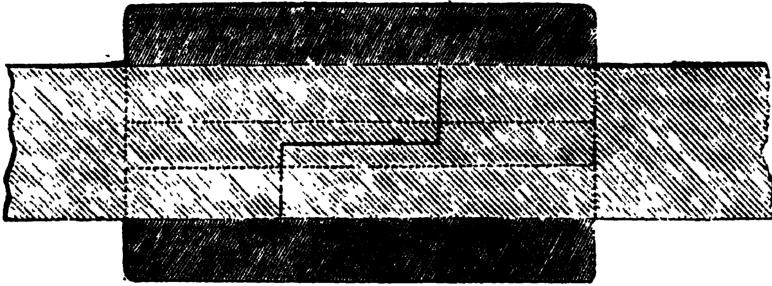


Fig. 214.

insert a tapered key in a groove formed in the coupling only, its inner surface fitting the shafts. Such a key, called technically a *hollow key*, from the hollowing or concavity of its inner surface, tightens the coupling very firmly on the shafts. The *face-*

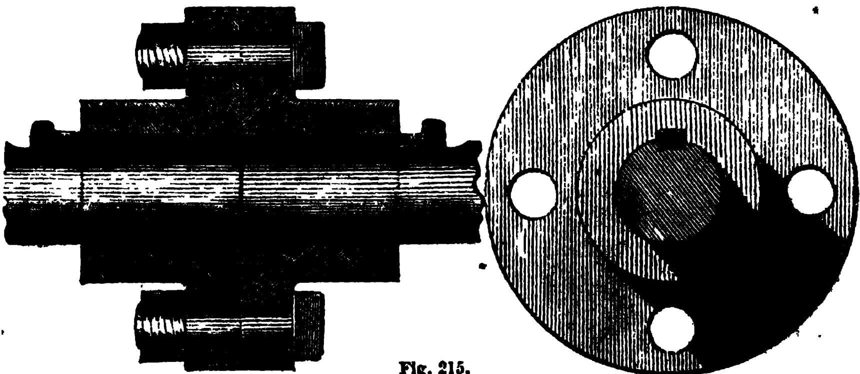


Fig. 215.

coupling consists of two discs, with projecting bosses keyed on the ends of the shafts facing each other, held together by four or more bolts and nuts passing through corresponding holes in the two discs.

* A "key," used for fixing a coupling or any other piece of machinery on a round shaft, is a piece of iron or steel of parallel breadth, but slightly tapered in thickness, so that on driving it into the "keyway" or recess provided for it, it acts as a wedge, pressing the opposite surface of the shaft against that of the hole. A "feather" is a key of equal breadth and thickness throughout, fitted into a recess in the shaft, and projecting from it into a slot formed in the hole of the coupling or other boss, which can thus slide along the shaft longitudinally, but must turn with it.

Sometimes when it is desired that by no accident shall the movement of any length of shafting be reversed, a ratchet-coupling is applied. *a* (Fig. 216) is a toothed boss keyed firmly on the shaft *d*, and *b* a similar toothed boss capable of sliding longitudinally on the shaft *c* along a *feather* or parallel key, which prevents it from turning round independently of *c*. So long as the shaft *d* revolves in one direction it carries the other shaft *c* round with it: but should the rotation of *d* be reversed, the pressure on the inclined faces of the teeth causes the boss *b* to slip out of gear with *a*, and thus to uncouple the two shafts.

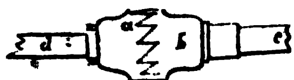


Fig. 216.

When it is desired to couple or uncouple two lengths of shafting at pleasure, a *clutch*, like that represented in Fig. 217, is generally employed.

A, the driving shaft on which is keyed firmly a half clutch C, having two projections from its face; B the driven shaft, the end of which rests in the hole of C, and on which is fitted, by a *feather* or parallel key, the other half clutch D, having also projections on its face similar to those on C. A groove is formed in D, and fitted with a ring E, put together in halves, and having pins projecting from its opposite sides, which may be taken hold of by hand or by a lever. The half clutch D being capable of sliding on the shaft B, can, by means of the ring, be pushed in or out of gear with C; and as the feather prevents it from turning on its shaft, when the projections on it are inserted between those in the other half clutch, the shaft B is caused to revolve by the rotation of A. Sometimes, when it is desired that the strain passing through any shaft shall be limited, recourse is had to *friction-couplings*. A very common mode of forming a friction-coupling is to key on one length of shaft a boss with a conical hole bored in its face, and on the other to fit by a feather a conical boss fitting nicely into the conical hole.

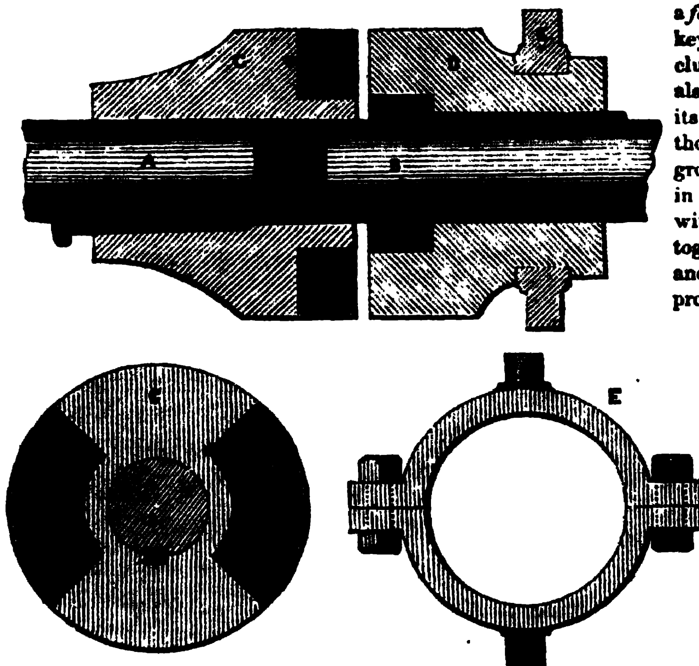


Fig. 217.

When the one is pressed into the other, the friction of the conical surfaces causes them to revolve together; but should an excessive resistance be opposed to the rotation of the driven shaft, the friction of the surfaces proves insufficient to overcome it, and the cone on the driving shaft slips round that on the other without driving it.

When two shafts not lying in the same straight line are to be coupled, recourse is had to a universal-joint. On each of the two shafts A and B (Fig. 218) is keyed a fork, in the

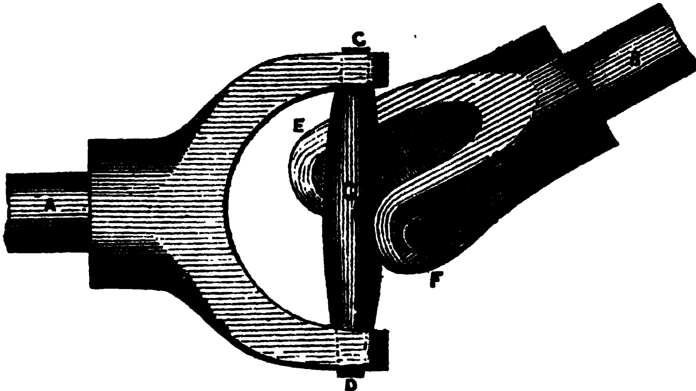
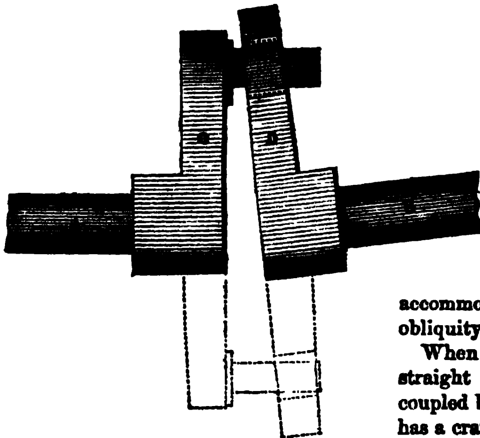


Fig. 218.

ends of which, at C, D, E and F, are pivoted the ends of a cross G, so as to permit its partial rotation on either of its two axes C D or F E. On A being caused to revolve, the cross is carried with it round its centre G, and its ends E F also rotate round the same centre, giving rotary motion to the other fork and shaft B, the pivoting of the cross permitting it to accommodate its position to the varying obliquity of the forks during their rotation.


Fig. 219.

When two shafts are not quite in the same straight line, but nearly so, they may be coupled by cranks. The shaft A (Fig. 219) has a crank and pin C keyed on to it, and the shaft B has also a crank D, with a slot cut in it to receive the crank-pin of C. The pin must be sufficiently long to allow for the departure of the two cranks from each

other by obliquity of position (as indicated by the dotted lines representing them at the opposite point of their revolution); and the length of the slot must be such as to allow for the difference of level of the two shafts.

The bearings in which shafts revolve, are technically called *pillow-blocks* or *plummer-blocks*. They are generally made of two pieces of cast-iron, the *base* A, and the *cap* B, lined with gun-metal *bushes* C, each half a cylinder.

The base is recessed to receive projecting parts of the cap; and two bolts D, with nuts at their upper ends, pass through holes in both, so as to secure the cap firmly on the base when the shaft has been laid in its place. The base has projecting flanges, with holes E through which bolts pass for securing the *plummer-block* to the beam or frame on which it may be fixed. These holes are lengthened to permit a little transverse adjustment of the plummer-block. The cap has generally an oil-cup in its upper part, with a projecting nipple perforated, through which a cotton wick conveys a supply of oil to the surface of the shaft, acting like a siphon by capillary attraction.

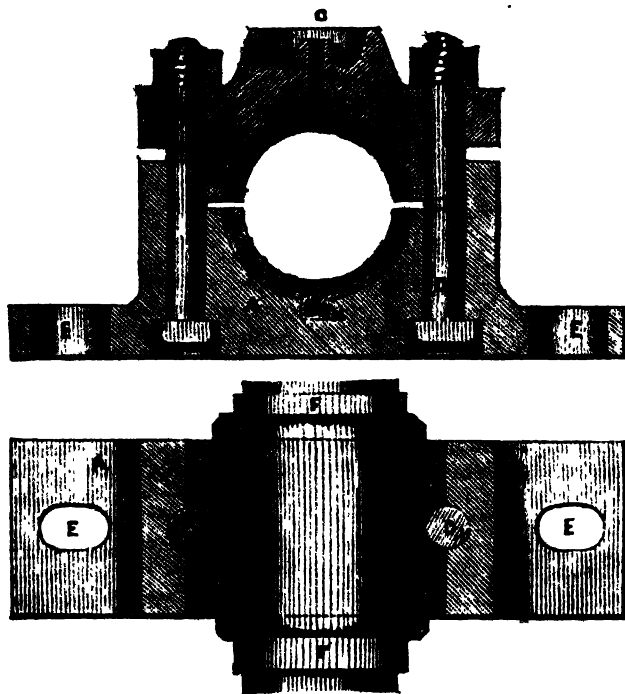


Fig. 220.

The *brasses* C are sometimes made square or octagonal in their outer surfaces to prevent them from turning round within the plummer-block; or, when they are quite cylindrical, small studs or steady-pins H, projecting from them into holes in the cap and base, answer the same purpose. The shaft has collars F or parts of larger diameter outside the brasses, and the brasses have lips or flanges to prevent the shaft from moving longitudinally. As the shaft or brasses wear from the constant friction, the cap and upper brass can be more tightly screwed down by the nuts of the bolts D, which are often made double, so that the one nut tightens the other in its place, and prevents it from becoming loosened by the shaking of the machinery.

Of late years many bearings, instead of being fitted with gun-metal bushes, have been lined with a soft metal, in which tin is the principal ingredient.

Fig. 221 represents a transverse section of a bearing lined with soft metal; A the cast-iron cap, and B the base, cast with a hole larger than the shaft. D D soft metal linings, filling the space between the surface of the shaft and that of the cast-iron. The shaft, or a piece of iron of equal size, being supported in the hole of the bearing, the soft metal melted is poured into the space; and if the iron has been previously heated to nearly the melting temperature of the soft metal, the latter on cooling presents a smooth internal surface, in which the shaft revolves with little friction or wear. A very good soft metal for this purpose consists of tin and lead in equal proportions. When it is inconvenient to form collars on the shaft to prevent longitudinal motion, rings E bored to fit the shaft are put on and kept in their places by tightening-screws pressing against the shaft.

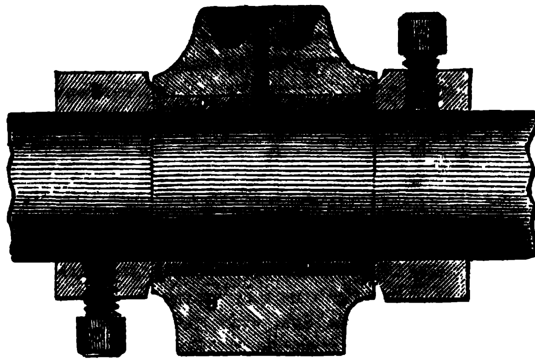


Fig. 221.

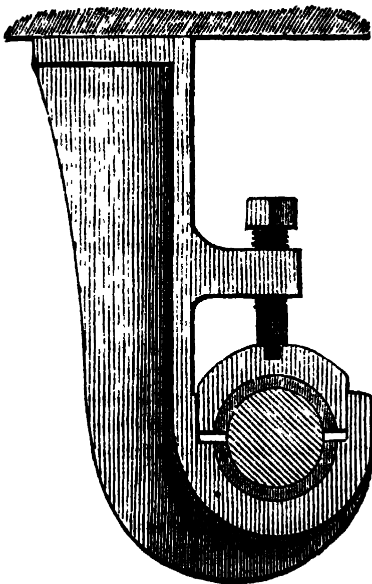


Fig. 222.

When it is not convenient to rest a shaft in plummer-blocks supported on walls or beams, they may be made to revolve in hanging bearings screwed up to a beam above. These bearings may be lined as plummer-blocks with gun-metal or soft metal, and the cap tightened down by a setting-screw, as indicated in Fig. 222.

When it is desired to convey motion from one shaft to another lying parallel to it, each shaft has a *pulley* or *drum* attached to it, and round their circumferences a flexible strap or band is tightly strained. Thus *e* and *f* (Fig. 223) being parallel shafts at some distance apart, a pulley *a* is keyed on the one and a pulley *b* on the other, the flexible strap passing round both. On rotary motion being given to the shaft *a* and its pulley, the friction between the strap and its circumference

puts the former in motion, and it conveys the rotation to *b* and its shaft in the same direction.

When it is desired to reverse the direction of rotation, the strap is crossed (Fig. 224). Also when a different angular velocity of rotation is desired, the pulleys are made of different sizes. If we suppose that the driving pulley has a circumference of 6 feet, and the driven pulley a circumference of 3 feet, half the former; since the strap travels at the same rate with the circumference of the driving pulley, and causes the circumference of the driven pulley to move round with equal velocity, it is clear that the latter must make two revolutions round its axis while the former makes one; or that the angular velocity of the smaller pulley is just double that of the larger, because its circumference is half the length. If we take the circumferences in any other ratio we should find the angular velocities in the inverse ratio. As the circumferences of circles are exactly proportional to their diameters, the angular velocities of pulleys connected by straps are inversely as their diameters, or the diameter of each pulley multiplied by its speed gives a like product. In calculating the speeds due to given dimensions of pulleys, or the dimensions suited for given speeds, we have therefore the following simple rules:—

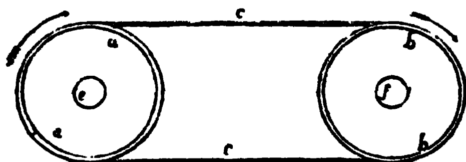


Fig. 223.



Fig. 224.

Given the angular velocity of one of two pulleys (in the number of revolutions per minute) and the diameters of both, to find the speed of the other.

Rule.—Multiply the speed of the first by its diameter, and divide by the diameter of the other.

Example.—A pulley 36 inches in diameter, making 80 revolutions per minute, drives a pulley 24 inches in diameter: required the speed of the latter.

$$\frac{80 \times 36}{24} = 120 \text{ revolutions per minute.}$$

Given the angular velocities of two pulleys and the diameter of one, to find that of the other.

Rule.—Multiply the speed of the first by its diameter, and divide by the speed of the other.

Example.—A pulley 24 inches diameter, making 120 revolutions per minute, drives another at 80 revolutions per minute: required the diameter of the latter.

$$\frac{120 \times 24}{80} = 36 \text{ inches.}$$

Pulleys are often combined with direct or crossed straps, as may be required, in order to vary the velocity and direction of rotation. Thus the pulley A (Fig. 225), 24 inches diameter, revolving at the rate of 10 revolutions per minute, drives B, 12 inches diameter, at 20 revolutions, because $\frac{24 \times 10}{12} = 20$. On the shaft of B is fixed a pulley C, 36 inches diameter, revolving at the same speed with B, and driving D, 12 inches diameter, by a crossed strap at 60 revolutions, because $\frac{20 \times 36}{12} = 60$. Again, on the shaft of D

is another pulley, 27 inches diameter, revolving with a speed of 60, and driving F, 9 inches diameter, at a speed of 180, because

$$\frac{60 \times 27}{9} = 180.$$

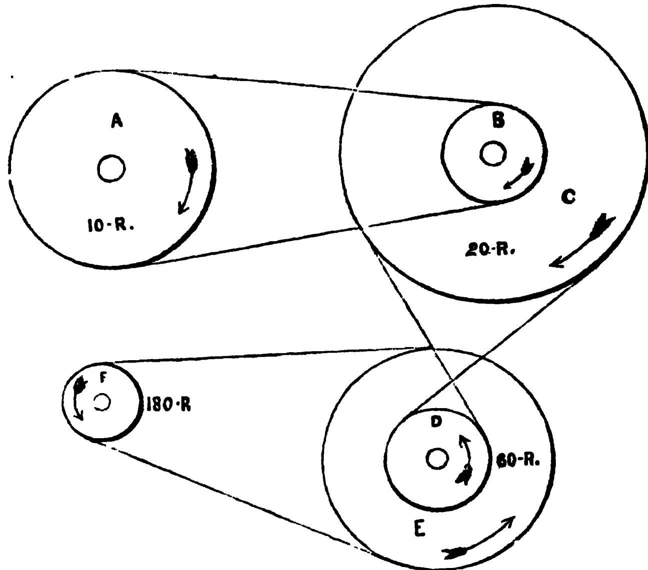


Fig. 225.

In estimating the final speed produced by such a train of pulleys, it is not necessary thus to calculate the velocity of each, as we find the same result by multiplying the initial speed of the driver A, by the product of the diameters of all the drivers A, C, and E, and dividing by the product of the diameters of those driven, B, D, and F; or

$$\frac{10 \text{ revols.} \times 24 \times 36 \times 27}{12 \times 12 \times 9} = 180 \text{ revolutions, the speed of F.}$$

Sometimes, when it is desirable to have the power of varying the speed of machinery driven by a strap, the pulleys are elongated and made conical, and the strap is shifted longitudinally to such a position as shall give the speed required. Thus, if *a a* be the driving-shaft, revolving 24 times per minute, and it be desirable that *b b* shall be driven at any speed from 8 to 72 revolutions per minute, each pulley being made conical with the larger end 3 times the diameter of the smaller, the strap *c* may be shifted from one end to the other to give the desired variation of speed. Nor is the tightness of the strap materially altered; for as it is made to pass round a larger circumference of the

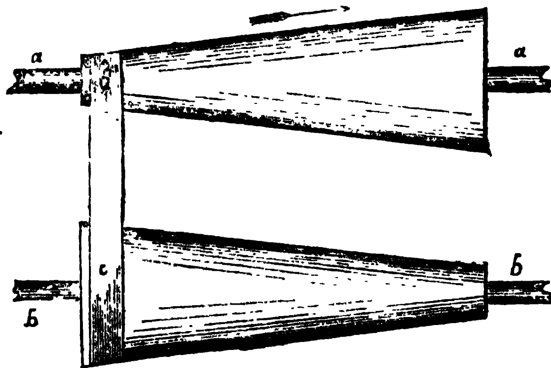


Fig. 226.

one, it passes round a smaller circumference of the other, and gains nearly as much as it loses. Most of the machines that are driven by straps do not require so nice an adjustment of velocity, but act very well at various speeds within certain limits. In such cases the conical pulleys are made with regular steps, or consist each of a set of pulleys placed side by side, having diameters increasing as much in the one as they decrease in the other (Fig. 227). Thus, if the driving-shaft make 48 revolutions per minute, the driven shaft may be made to revolve at the following different speeds: — $48 \times \frac{16}{4} = 192$, $48 \times \frac{12}{8} = 72$, $48 \times \frac{8}{12} = 32$, and $48 \times \frac{4}{16} = 12$ revolutions per minute; the diameters being 16, 12, 8, and 4 inches in each.

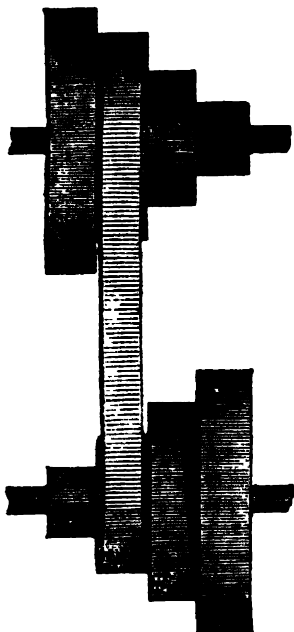


Fig. 227.

Occasionally, a strap may be employed when it is desired that the motion shall be changed transversely, as indicated in Fig. 228. In such a case it is necessary to have very wide pulleys, *a* and *b*, because the two parts of the strap *c* and *d* must pass obliquely over their surfaces. This mode of employing a strap is, however, very disadvantageous, for the strap must be continually slipping longitudinally along the face of each pulley, and thereby be subjected to friction and wear.

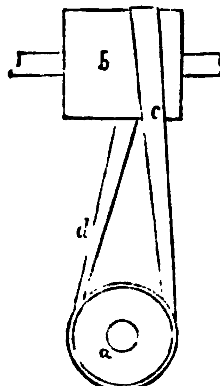


Fig. 228.

In a manufactory where numerous machines are employed, it is essential to have a ready means of throwing any of them into action or out of action at pleasure. When the machine is driven by a strap, this operation is conveniently effected by what are called the *fast* and *loose*, or the *live* and *dead* pulleys; that is to say, two pulleys of equal size placed side by side on the shaft, one being firmly fixed to the shaft by a key so as to revolve with it, while the other revolves freely on the shaft. When it is desired to put the machine in action, the band is put on the *fast* pulley, and thus gives motion to the shaft or receives motion from it. For throwing the machine out of action, the band is shifted on to the circumference of the *loose* pulley, which revolves idly on the shaft. The shifting of the strap from the one pulley to the other is effected by means of the *striking gear*, a forked lever *f e d* (Fig. 230) moving on a pivot *e*, and holding the strap *c* within its forked part *d*. By pulling round the

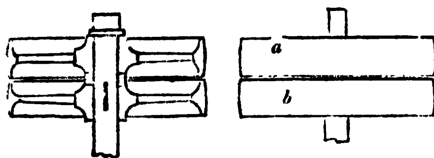


Fig. 229.

handle *f* of the lever, the band, being pressed edgeways by one side of the fork, is pushed gradually on to the fast pulley *a* or the loose pulley *b* at pleasure. It would be difficult thus to shift a tight strap when at rest; but while it is in motion, with the pressure of the fork directed on it, it is gradually edged off from its former position. For this reason, the *fast* and *loose* pulleys should always be placed on the *driven* shaft, because, were they placed on the *driving* shaft, the band when on the *loose* pulley, being at rest, could not easily be shifted.

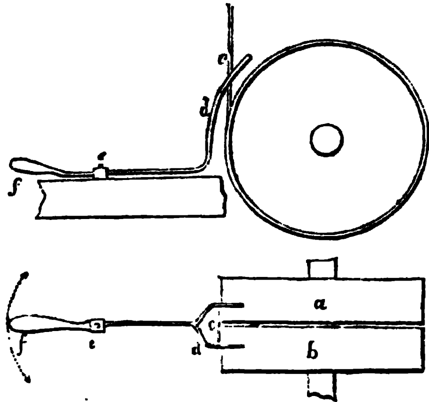


Fig. 230.

In order that straps, which are generally made of leather or *gutta-percha*, may act well without slipping ineffectively along the surfaces of the pulleys, they should be tightly strained into their place. The pulleys should be turned true and smooth on their circumferences, for the smoother the pulley, the more driving friction appears to take place between its surface and that of the strap. When the strap is not sufficiently tight, or does not take sufficient hold of the pulley to drive the machinery with which it is connected, it must be made of greater breadth, and strained more tightly. Sometimes, when a strap slips, powdered resin is strewn upon its inner surface with good effect. The greater the arc of the circumference which a strap embraces, the firmer is its hold, or the less tendency has it to slip; and the greater the width of the strap, the more firm is its hold. The diameter of the pulley does not affect the slip of the strap, except when the diameter is very small, while the strap is rigid, and not easily bent closely round the circumference.

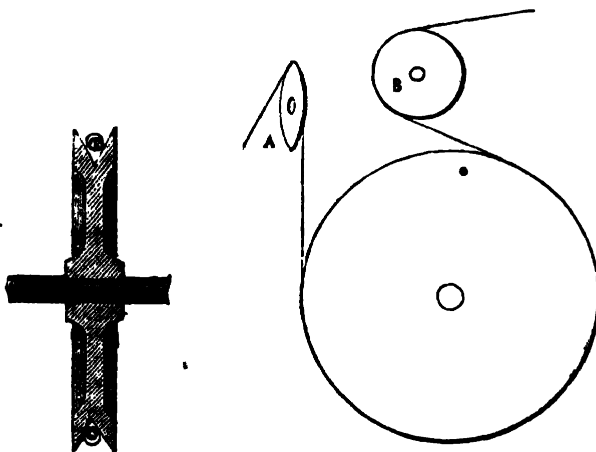


Fig. 231.

Were straps perfectly flexible, it would be found that, with a given breadth and tightness, the driving power would be the same for all diameters of pulleys.

It is often convenient to use cylindrical bands or cords, made of catgut or gutta-percha, instead of flat straps (Fig. 231). In such cases, the pulley is grooved in the circumference, and the band wedges itself into the groove, and thus exerts sufficient friction on the circumference for the communication of power from one pulley to another. The principal advantage of bands of this kind is, that by means of guide pulleys, A and B, grooved in the circumference, the band may be led in almost any direction on the plane of the pulley, or oblique to it.

Straps are best suited for machinery driven at high speeds, with small pressures; they are cheap, convenient, and smooth and noiseless in their action; and when any undue strain comes upon the machinery, they slip, and thus act as a safeguard against damage. But for all machinery intended to communicate heavy strains through their parts, as in cranes, and whenever certainty of connection between one moving part and another is required, as in clockwork, recourse must be had to *gearing*, or toothed wheels.

Toothed Wheels.—Were we to mount two wheels on parallel axes, so that their circumferences touch each other, by causing the one to revolve we should also cause the other to revolve, provided the friction between their surfaces at the point of contact were sufficient to overcome such resistance as might be presented to the rotation of the driven wheel. Practically, this friction is not sufficient in ordinary cases, and it becomes necessary to fit the circumferences with teeth or cogs, or projections and recesses at corresponding intervals, so that each tooth of the one fits successively into each recess of the other. By this arrangement, the direction of rotation is reversed, as indicated by the arrows in Fig. 232. When it is desired that the direction of rotation be retained the same, it is twice reversed by the introduction of an intermediate wheel (Fig. 233). Sometimes wheels are geared internally *b* (Fig. 234), where *a* is a portion of the one, and *b b* part of the circumference of the other. In this case, the direction of rotation is not reversed.

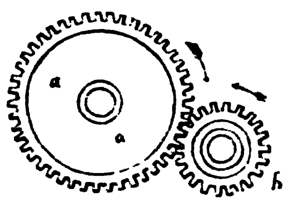


Fig. 232.

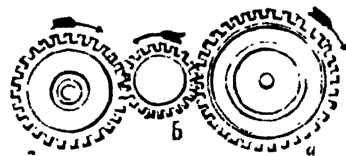


Fig. 233.

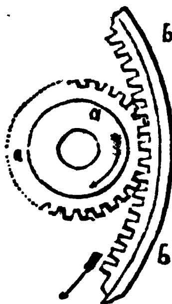


Fig. 234.

By means of toothed gearing, the angular speed of rotation may be altered at pleasure, as will be evident from the following considerations. The wheel *a* (Fig. 232) has 45 teeth, and therefore during each revolution presents 45 successive recesses for the teeth of *b*, which number 22. For every revolution of *a*, therefore, *b* must make 2 revolutions and advance the space of 1 tooth, because $2 \times 22 + 1 = 45$; during 2 revolutions of *a*, *b* makes 4 revolutions and 2 teeth; and so on, until *a* has made 22 revolutions, or caused $22 \times 45 = 990$ of its teeth to pass the point of gear, in which time *b* must have made 45 revolutions, or caused $45 \times 22 = 990$, the same number, to pass the point of gear. And so it would be found with any other number of cogs, that the angular velocities of the geared wheels, or the number of revolutions they make in a given time, are inversely as the numbers of their teeth. When an intermediate wheel

is employed as *b* (Fig. 233), it only affects direction of rotation, not the speeds of the extreme wheels. Thus if *a* (Fig. 233) have 34 teeth, and *b*, the intermediate, have 20 teeth, the angular velocity of *a* is to that of *b* as 20 to 34; or if we take the speed of *a* as 1, the speed of *b* is $\frac{34}{24}$. Again, if *c* have 27 teeth, its speed is to that of *b* as 20 to 27, or it is $\frac{20}{27}$ths of the speed of *b*, that is $\frac{20}{27} \times \frac{34}{20} = \frac{34}{27}$ths of the speed of *a*. But, did we leave the intermediate wheel out of consideration, we find that were *a* with 34 teeth to drive *c* with 27, the speed of *c* would be $\frac{34}{27}$ths of that of *a*, the same result as before, though in the opposite direction. The same principle applies, whatever be the number of intermediate wheels, for the speed of the first and last will always be to one another inversely as their respective numbers of teeth.

If from the centres of two geared wheels A B circles be drawn touching each other at a point midway between the extreme projections of their teeth, these circles are called the *pitch* lines or circles of the teeth, and their circumferences being equally divided, give the intervals from tooth to tooth in each, or what is technically called the *pitch* of the teeth. When we speak of a wheel of 1 inch *pitch* or 2 inches *pitch*, we mean that the distance measured from the centre of one tooth to that of the next (these centre points being taken on the pitch circle) is 1 inch or 2 inches, as the case may be. In Fig. 235, the distance between any two adjoining points, where the dotted radii from A and B cut the pitch circles, is the *pitch* of either wheel, and these *pitches* are necessarily equal in length in order that the wheels may gear. But as this distance is the interval between the central points measured in a straight line, or along the chord of the small arc intercepted between them, it does not accurately correspond to the length of the arc. The larger the circle of which the arc is a portion, and the smaller the pitch or portions into which the circumference is divided, the more nearly do the circular arc and its chord approach to equality. Accordingly, in cases of nearly equal wheels, or whenever the pitch is small in proportion to the dimensions of either wheel, the more nearly do the numbers of teeth express the proportions of the circumferences. But the circumferences of circles being exactly proportional to their diameters, the numbers of teeth are very nearly in the same proportion; and the angular velocities of geared wheels are therefore inversely as the diameters of their pitch circles.

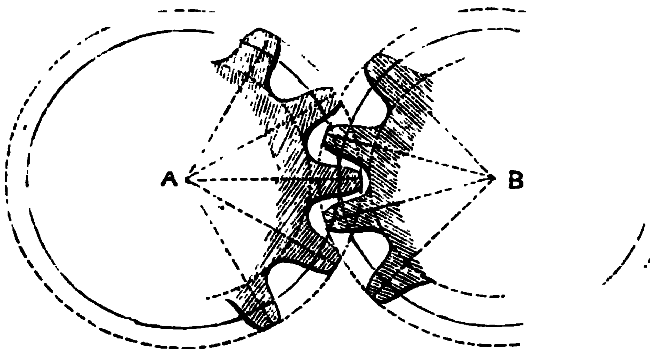


Fig. 235.

When there is a train of geared wheels, so arranged that the first shall drive the second—that the third, fixed on the shaft of the second, shall drive the fourth—the fifth,

fixed on the shaft of the fourth, shall drive the sixth, and so on,—we readily find the angular velocity of any wheel in the train thus.

Rule.—Multiply the angular velocity of the first driver, by its number of teeth or diameter, and by the numbers of teeth or diameters of all the drivers, and divide the result by the product of the numbers of teeth or the diameters of all the driven wheels.

Example.—Wheel 1, 36 inches diameter with 72 teeth, making 120 revolutions per minute, drives wheel 2, 12 ins. diam. with 24 teeth; on the shaft of wheel 2, is fixed wheel 3, 10 ins. diam. with 30 teeth, driving wheel 4, 6 ins. diam. with 18 teeth: required the speed of wheel 4.

$$\frac{120 \text{ rev.} \times 36 \text{ ins.} \times 10 \text{ ins.}}{12 \text{ ins.} \times 6 \text{ ins.}} \text{ or } \frac{120 \text{ rev.} \times 72 \text{ teeth} \times 30 \text{ teeth}}{24 \text{ teeth} \times 18 \text{ teeth}} = 600 \text{ rev. per min.}$$

From the general principle of mechanics, that no power can be gained or lost in any train of mechanism, but that only the two elements of power, pressure and velocity, can be interchanged, irrespective of friction or other useless resistances, it follows that at the end of any train of wheels, the power is the same as at the beginning; but the velocity being different, the pressure or strain on the teeth must be different also. In the example we have given, if we suppose that 1 horse-power has given motion to wheel 1, we should expect to get 1 horse-power from wheel 4; but as the speed of wheel 4 is five times that of wheel 1, so the pressure exerted by it at any point is $\frac{1}{5}$th of that exerted by wheel 1 at a point equally distant from its axis. To show that this is true, not only in general terms but in the particular case, we shall suppose 1 horse-power passing through the teeth of wheel 1, 18 ins. from its axis. The teeth of wheel 2 being in contact with those of wheel 1, receive its full power at 6 ins. from its axis, and convey it through its shaft to the teeth of wheel 3, distant 5 ins. from its axis, and therefore sustaining a pressure of $\frac{6}{5}$ths of the original pressure. Again, this strain being given to the teeth of wheel 4 at 3 ins. from its axis, is equivalent to $\frac{3}{18}$ths of $\frac{6}{5} = \frac{1}{5}$th of the original pressure estimated at 18 ins. from the axis of the last wheel. In any train of wheelwork, then, we may safely diminish the sizes and strengths of the teeth as their velocity increases; and, conversely, we should increase their strengths as the velocity diminishes.

One of the most important matters connected with toothed wheels refers to the forms or outlines of the teeth. It is to be desired that engineers generally should determine on some standard form, so that all wheels of equal pitch should work or gear properly together. Unfortunately this is not the case: the wheel made by one machinist is unsuited to that made by another: it often becomes necessary to make costly patterns of wheels to suit some that may have been already made; these patterns, again, are not suited for other cases; and thus a great amount of time, labour, and material is misapplied in a matter where a little harmony among the views of machinists might do much to avoid these evils. That such a mutual understanding is not impossible, may be proved by the fact, that in a similar case, that of screw-threads, among which there once existed quite as great a variety, almost all engineers now adhere to certain forms and proportions; so that the screw which fits one nut, readily fits any other nut of equal diameter, wherever the screw or the nut may have been manufactured. Perhaps the principal cause of difference in the forms of teeth has been the want of knowledge among practical men as to what the true forms should be. Much has been written on this subject, but a great deal is of too abstruse a character to be generally understood or appreciated; and the practical mechanic desiring information is discouraged by the

difficulties with which the subject appears to be invested. We will endeavour to point out as simply as possible the laws which should govern the forms of teeth, and to lay down a few easy rules for delineating and executing them.

If we suppose A and B (Fig. 236) to be the bosses or central parts of two wheels having straight teeth or flat blades C and D projecting from them respectively. On tracing several positions of these, such as C_1 C_2 and D_1 D_2, we observe that there must be considerable inequalities in their relative movements; or if C_1 and C_2 make equal angles with the line of centres A B, D_1 and D_2 do not make equal angles with the same line. Again we observe that the points of each tooth must rub along the flat surface of the other in the course of the motion; and were this form of tooth practically carried out, the cutting and wear would be considerable. But if, as in Fig. 237, the wheel B have a tooth D of some curved outline, we may perhaps find some particular curve for the face of tooth C, such that an equable movement of the one wheel shall produce an equable movement of the other, and that the surfaces of the two teeth shall rather roll along each other than rub with a pushing or sliding movement. Now, although it may be generally possible to find a proper form for C, whatever be the form of D, yet it is desirable for many reasons that both these curves should be traced according to some fixed type, which shall be constant, whatever the dimensions of the wheels or their numbers of teeth. If we examine somewhat closely the relative motions of two toothed wheels, we may perhaps discover an appropriate form for their teeth.

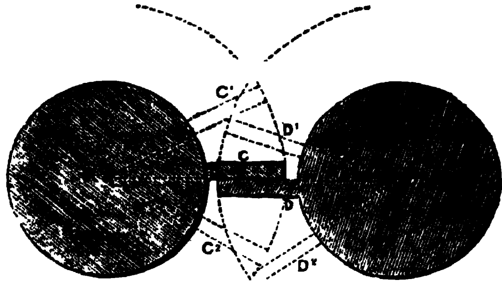
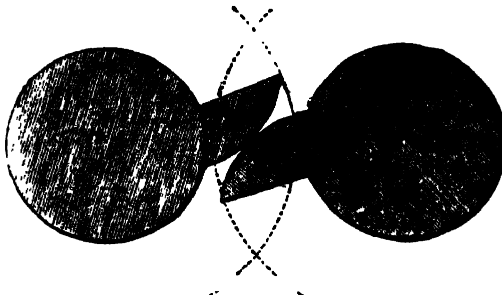


Fig. 236.


Fig. 237.

If A B (Fig. 238) represent a strap passing round the circumferences of two equal pulleys, and therefore touching both, and if through C, the middle point between their centres, two circular arcs be described, these may represent the pitch circles of two equal toothed wheels, whose relative rotation should be precisely the same from the gearing as it is from the motion of the strap, unwinding from the one pulley and winding on to the other. But farther, if we continue the lines A B and D E, and from some other point F in D E describe a circle touching A B in G, and another circle (dotted) through the point C;—since F C bears the same ratio to C E as F G to E B, it appears that the rotation of the large wheel, if geared with the other at C, should be precisely

the same as if it were caused by the strap winding on to its pulley, of which F G is the radius. If we assume the angular velocity of the pulley D to be uniform, the rectilineal velocity of the strap A B G must also be uniform, and likewise the angular velocities of the wheels E and F. Whatever, then, be the relative sizes of the geared wheels, the rectilineal motion of the strap with relation to the rotary motion of any one of them is always constant; and if from this relation we can trace out a form of tooth, that form will apply in all cases, whatever be the diameters of the wheels that are geared together.

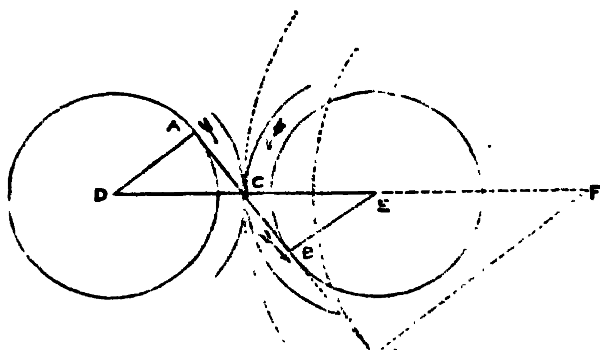


Fig. 238.

In order to avoid complexity, let us first take one wheel, supposing it a circular disc of paper, of which H M K (Fig. 239) is a portion of the circumference, and let us suppose that the strap A B has a pencil projecting from it at some point P, so that as the disc rotates while the strap travels, the pencil shall trace on the disc a line L P M, compounded of these two motions. If, now, we take the other wheel (marked by the dotted lines), and suppose its disc extended to R Q overlapping the former, the same pencil would trace a corresponding line N P Q on it. Now, as the curves thus produced are traced by the same point in the band, and under precisely similar conditions of uniform rotation, we might cut the paper discs to their outlines, and making them rotate in contact, we should obtain that uniform relative motion which is required. The mathematical name of each of the curves so described is the *involute of the circle*, because it is produced by the point of a thread wound on to (*involutum*) a circle, or wound off from a circle. The nature of the curve may be best understood by reference to Fig. 240.

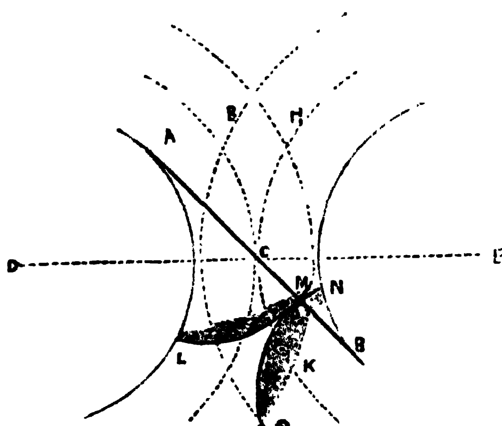


Fig. 239.

If *a* be the centre of a circle or plan of a roller, on which is wound a thread having its end at *e*, a pencil or tracing-point being fixed at the end of the thread will trace

the involute c, l. q, as the thread is unwound; or $b\ q$ being the unwound thread, its point will trace the involute as it is wound on the circle. If we divide the circumference into any number of equal parts at d, e, f, g, &c., and from these points draw tangents or lines touching the circle, at right angles to the radii ad, ae, af, &c., respectively, making the lengths of the tangents equal to the lengths of cir-

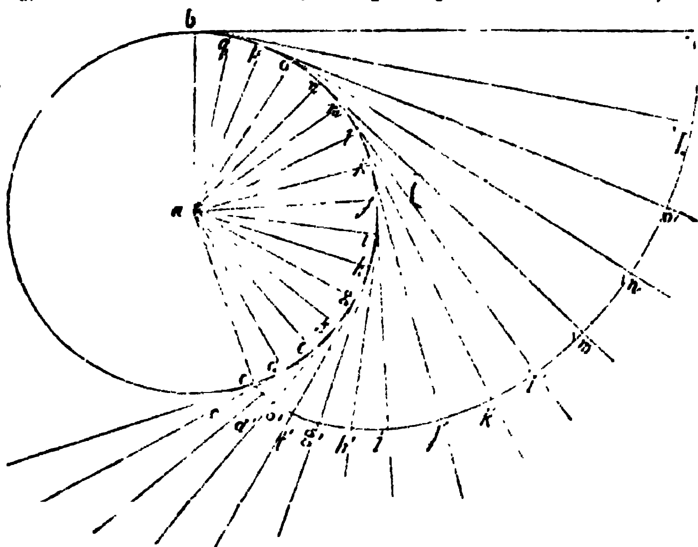


Fig. 240.

cumference measured round from c, the curve joining the extremities of the tangents is the involute—the proper outline for the teeth of wheels, as we have just described. In applying this theory to the practical formation of teeth, we do not find it necessary to describe the actual involute form, because the portion of the curve that belongs to any tooth is so small that we can draw a circular curve so near to the involute as to cause no material error in working. Thus if C (Fig. 241) be the centre of the wheel, and A, B the centres of two adjoining teeth on the pitch circle, small circles being described round those centres to fix the breadth of each tooth D E and F G, and H M N being part of the generating circle of the involutes, the portion H K of any involute H K L very nearly corresponds with a circular arc of which M is the centre, and the radius D M is a tangent to the generating circle H M N at M. In practice, therefore, it is only necessary to determine the circle H M N, and the length of the radius D M, for describing the curved sides of the teeth. From what has preceded, it is clear that we may take any convenient generating circle H M N, but that whatever be the one we may select for any one wheel, that for any other wheel gearing with the former must be proportional to it. It is found practically convenient to make the radius D M with which the side of the tooth is described $\frac{1}{4}$th of the pitch radius C A, and the generating circle or locus of centres H M N should be described within the pitch circle, the interval between them A P being $\frac{1}{8}$th of D M, or $\frac{1}{32}$nd of C A. If this rule be adhered to, all wheels of equal pitch will gear with one another, whatever be their diameters. In drawing or setting out teeth, then, the following process will be found convenient:—

1st. From the centre C, with radius C A, describe the pitch circle and divide it by the compasses into equal parts A B, each equal to the given pitch. The pitch radius may be determined by the following rule:—

Multiply the number of teeth by 7 times their distance apart, or pitch, and divide by 44.

Example.—Required the pitch radius of a wheel having 28 teeth of ¾ inch pitch.

$$\frac{28 \times \frac{3}{4} \times 7}{44} = 3{\cdot}341 \text{ inches the pitch radius.}$$

NOTE.—When the number of teeth is small and their pitch considerable, the pitch radius must be a little increased, as will be found necessary on trying the division of the circumference.

2nd. Round the centres A, B describe circles, each of diameter somewhat less than half the pitch: to determine the breadth of the teeth. The reason for making the breadth of teeth less than half the pitch, is to give a little room or clearance in the spaces between them, so that in the event of slight irregularities occurring in the workmanship, the teeth of two wheels may not become bound or locked into each other. When the teeth are cut by machinery to their exact form, this clearance is not necessary; but when they are merely cast and not shaped afterwards, there should be an allowance of about $\frac{1}{32}$nd of an inch for each inch of pitch—that is to say, if A B be 1 inch, D E should be ½ an inch, wanting $\frac{1}{32}$nd of an inch, or $\frac{15}{32}$nds of an inch, while E F is ½ of an inch and $\frac{1}{32}$nd of an inch, or $\frac{17}{32}$nds of an inch. Were A B 2 inches, then D E would be $\frac{15}{16}$ths of an inch, and E F $\frac{17}{16}$ths of an inch, and so on in like proportion.



Fig. 241.

3rd. Take A Q = ¼th of A C, and A P = ⅛th of A Q, and from the centre C describe through P a circle of centres H M N.

4th. From the points D, E, F, G, with radius A Q in the compass, mark off the points R, S, T, U on the circle H M N, and from these points as centres, with the same radius A Q, describe the curved sides of the teeth as V H D K.

5th. It now only remains to determine the tops of the teeth and the bottoms of the spaces between them. The spaces should have somewhat greater depth below the pitch circle than the height of the teeth beyond it to allow the teeth to clear; and it is convenient both for giving strength of form to the teeth and for providing this clearance of the teeth, especially in case of dirt getting between the gearing, to make the bottom of the spaces semi-circular. It will be found convenient to make A W, the height of the point of the tooth above the pitch circle, half the space E F between two teeth, and A K the depth of the space below greater than A W by ⅛th of an inch for every inch of pitch; and circles described through W and X will determine the tops of teeth and bottoms of spaces.

In the case of internal gearing (as represented in Fig. 242), the form of teeth may be developed on principles similar to those we have adopted for external gearing. If

A B (Fig. 242) be the pitch radius of the wheel, F B G being a portion of the pitch circle, and C B the pitch radius of the pinion working within it, and if on the axes of the wheel and pinion we suppose pulleys to be fixed, having radii A D and C E respectively proportional to the pitch radii, a band E D winding off the one and on to the other pulley would give them the same relative angular velocities as would be produced

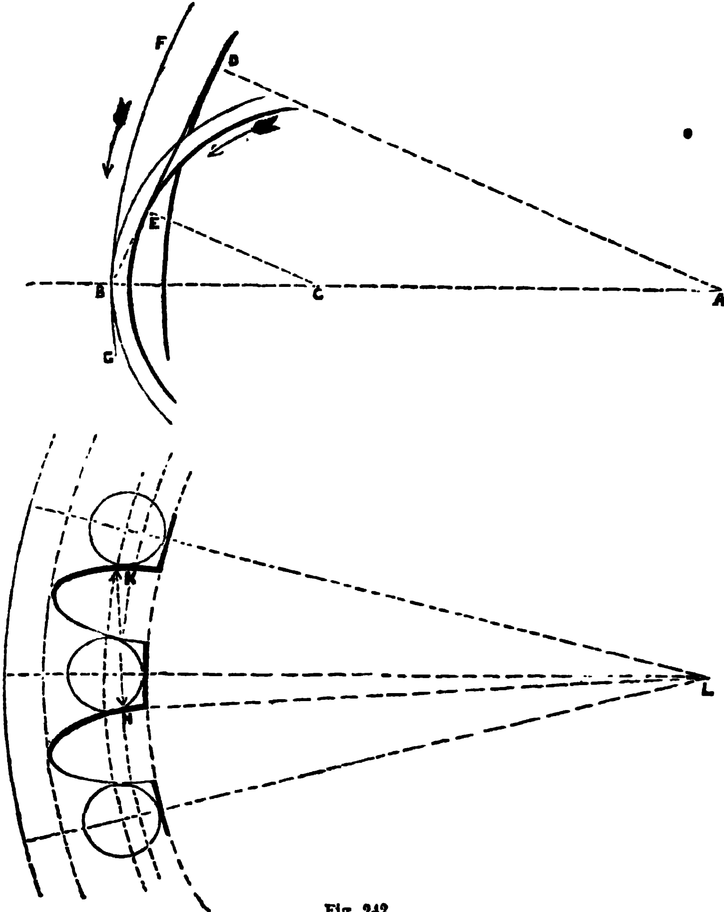


Fig. 242.

by the friction of their circumferences at B. And D E, the common tangent of the two pulleys, which when prolonged must always pass through B, may be supposed to have a pencil fixed to its prolonged part, tracing on the planes of the revolving wheel and pinion the outline of their respective teeth, which would manifestly be involutes of the generating circles D and E.

The inclination of D B being the same as that adapted for external gearing, the teeth of the pinion would evidently be the same, but the teeth of the wheel would be similar in form, not to the teeth of an externally geared wheel, but to the spaces between them. L being the centre of the wheel, L H = $\frac{4}{3}$rds of the pitch radius, and H K = $\frac{1}{4}$th of the pitch radius, the side of a tooth at K is part of a circle described with radius K H. The tops of the teeth in this case project *within* the pitch circle as far as in external gearing they project *beyond* it, and the bottoms of the spaces may be made semi-circular, and of depth similar to that determined for external gearing.

The gearing we have hitherto described applies only in the case of wheels revolving in one plane or on parallel axes.

When the axes are not parallel, it is necessary to employ *bevil gearing*. Let A B and A C (Fig. 243) be the two axes meeting in A, and D E and E F the proper diameters

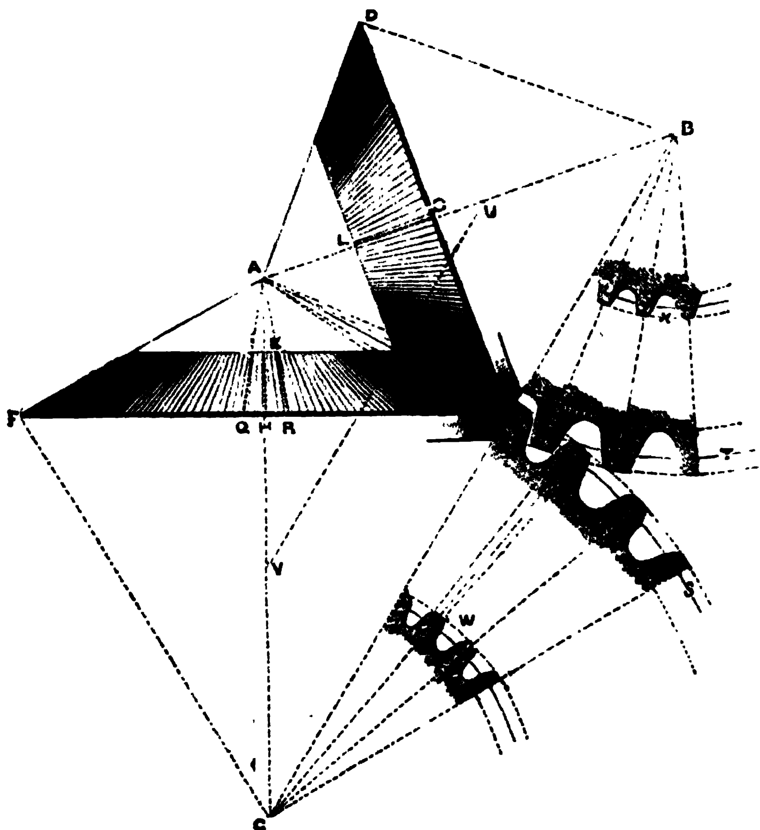


Fig. 243.

of the wheels to give the required speed; if we suppose the cones D A E, F A E to roll upon one another touching along the line A E, then the velocities of their touch-

ing surfaces will be equal at any point, such as M, along the cone, because L M and K M, the radii of rotation at that point, bear the same proportion to each other as G E and H E the radii at any other point E, and therefore the circumferences at these points are in like proportion. Taking a portion of each cone (that shaded), if we conceive their meeting surfaces at M E to have sufficient friction, by causing one to rotate round its axis we should also cause the other to rotate. But as practically the friction is not sufficient, it is necessary to cut the surfaces of the cones into teeth and spaces, as in plain gearing. It is evident that these teeth must tape towards A, both in width and in height. If a line C E B be perpendicular to A E, and N and P the tops of the teeth at E, the converging lines A N and A P will define the tops of the teeth along their whole extent. And again, if Q R be the breadth of a tooth at H, the converging lines A Q and A R will define the tapering breadth. The line P N is part of the boundary of conical surfaces, of which C is the apex for the one wheel, and B the apex for the other; and if from the centres C and B the circles E S and E T be described, they will represent the outlines of the developed surfaces of the cones F C E, D B E respectively, and become the pitch circles on which the outlines of the teeth at E may be described. Were we to cut these teeth in paper, and then wrap them round the cones F C E and D B E, we should have them interlacing and gearing into each other at E. Proceeding in the same manner at the point M, by drawing U V through M perpendicular to A E, and describing from centres C and B the pitch circles W and X, with radii respectively equal to V M and U M, we get the development of the teeth at M, which are precisely similar to those at E, but on a smaller scale, their outlines being defined by the radii converging from those in S and T towards C and B, the centres of development.

Bevil gearing applies when the motion is to be conveyed at any rate of speed from one axis to another, as in Fig. 244, where *a a* is the large *bevil wheel*, and *b b* the smaller

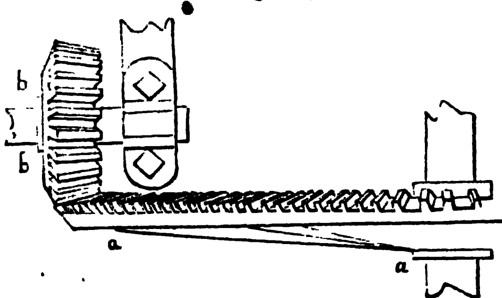


Fig. 244.

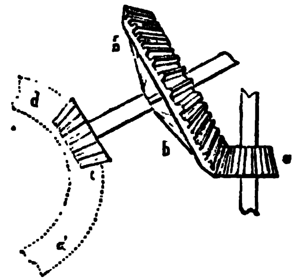


Fig. 245.

one, or the *bevil pinion;* or the motion may be communicated at any other angle, and in various directions, by combinations of bevil wheels and pinions, as in Fig. 245, where the pinion *a* drives the wheel *b b* mounted obliquely to it; and the pinion *c*, fixed on the shaft of *b b*, drives the wheel *d d* at some other angle with it.

When the wheels are equal, and their axes at right angles to one another, as in Fig. 246, the wheels are technically called *mitre wheels.*

The general law as to velocities of rotation and pressure conveyed through bevil gearing is precisely the same as in the case of plain gearing.

Gearing is sometimes used to convert a rotary into a rectilineal motion, by the use of a rack and pinion.

The pinion *a* (Fig. 247) has teeth which fit between the teeth of two racks *b* and *c*; and by giving *a* reciprocating rotary motion round its centre, these two racks are put in reciprocating rectilineal motion.

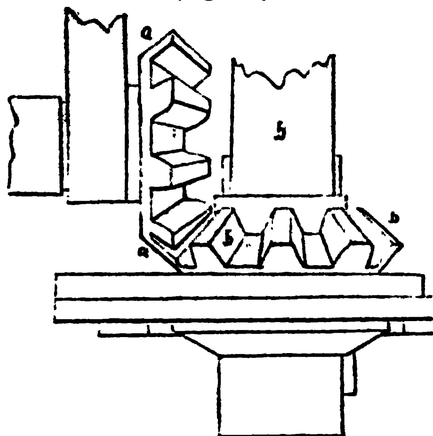


Fig. 246.

The proper form for the teeth of a rack, so as to gear with those of pinions formed as we have described, may be ascertained thus (Fig. 248):—In describing the teeth of a wheel, the radius C F of the generating circle bears a certain proportion to C B the pitch radius; or, as we have taken it, C F is $\frac{4}{11}$nds of C B. Again, F B, the radius of the side of the tooth, is $\frac{1}{4}$th of C B, or bears also a constant ratio to C B. Hence, for every wheel B F makes a constant angle F B C with A C. Applying this to a rack N G, the

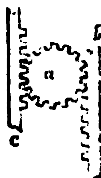


Fig. 247.

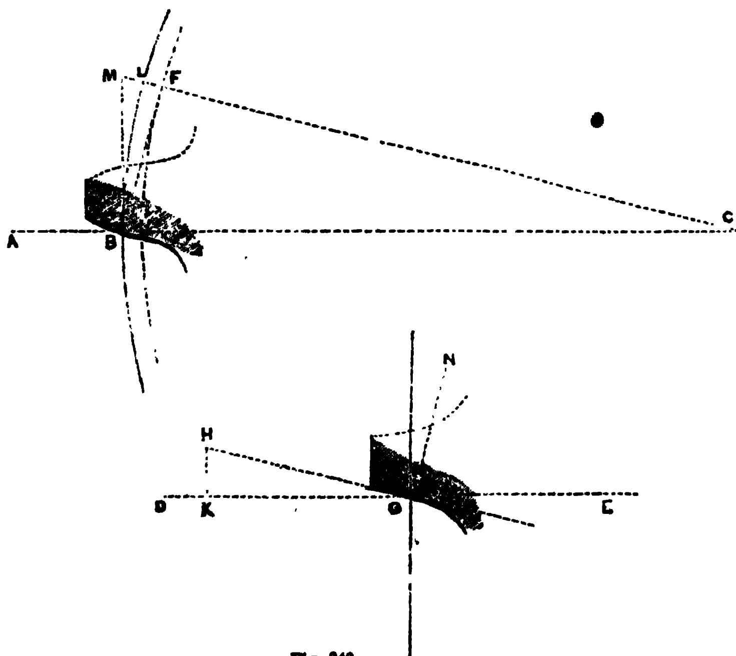


Fig. 248.

radius of the tooth must be at the same angle to D E as that of F B to B C, or the

angle N G E must be equal to the angle F B C; but as the pitch line of the rack is a straight line, or may be supposed a circle of infinitely great radius, so the radius N G of the tooth, being $\frac{1}{4}$th of the pitch radius, must also be infinitely great, and the portion of the circular side of the tooth described with this infinite radius must be a straight line H G perpendicular to N G. To find the amount of obliquity or inclination of H G to D E, we observe that it is the same with the inclination of B F to B M (which is perpendicular to A C). Now M F is very nearly bisected in L; and as F L is $\frac{1}{8}$th of F B, F M is very nearly $\frac{2}{8}$ths or $\frac{1}{4}$th of F B. So, in the rack, H K must be $\frac{1}{4}$th of K G; and hence we have a simple mode of setting out H G the side of the rack-tooth, by taking G K any length, such as 1 inch, along the line G D, and measuring up an offset K H = $\frac{1}{4}$th of K G. The line H G is the side of the tooth, of which the top and bottom may be determined as in wheel gearing.

There still remains a kind of gearing most applicable in cases where it is desired to reduce very greatly the angular velocity, or to increase the moving pressure. We allude to the *perpetual screw*, or *worm and wheel*, as it is usually called (Fig. 249); *b b* the driving shaft has a screw or worm cut on its cylindrical surface, the coils of which fit between teeth on the driven wheel *a a*. These teeth have sides inclined to the plane of their wheel, in such a manner as to suit the obliquity of the screw-thread. When the worm is caused to revolve, one portion of the worm pressing against a tooth of the wheel causes it to advance over a distance equal to the pitch of the thread or of the teeth (which are necessarily alike); and by the time one tooth is disengaged from the worm by the revolution of the wheel, another tooth has come into action, and is caused to advance in like manner. Every revolution of the screw round its axis thus causes the wheel to rotate through a portion of a revolution, equivalent to the distance of the centre of one tooth from that of the next; and if the wheel have, for example, 100 teeth, it thus requires 100 revolutions of the worm to cause one revolution of the wheel. It is evident that, whatever be the size of the worm, the same relation of angular velocities subsists, while the number of teeth in the wheel remains constant; but when we alter the *pitch* of the screw, that is to say, the distance from one of its coils to the next, we must alter to a corresponding extent the *pitch* of the toothed wheel; and, if its diameter be fixed, the number of its teeth must be altered accordingly.

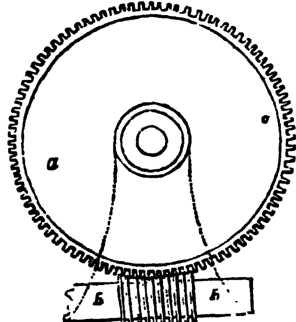


Fig. 249.

In deciding on the form of teeth for screw-gearing, we must be guided by considerations similar to those which we have dealt with in respect of ordinary gearing. If we suppose the screw-wheel B (Fig. 250) to be a thin plate of metal, with teeth of the proper involute form cut in its circumference, and A the section of the screw to be also a thin plate with teeth like those of a rack, by drawing the rack along rectilineally in the direction of the arrow we should cause the wheel to rotate in the direction of the arrow, and the teeth would be of suitable form for their relative movements. So when the wheel is of some thickness, as shown in section at E, with teeth F projecting within the outer circumference of the screw, and clearing the solid central portion, we have only to make the teeth F inclined so as to suit the obliquity of the screw-thread as indicated

by the dotted lines H H on the plan of the screw G; and, instead of moving the screw longitudinally like a rack, we may cause it to rotate round its axis, and while we prevent its longitudinal motion, the teeth of the wheel will be caused to move onwards by the inclined action of the screw-thread. As no part of the screw-thread is a straight line, the teeth of the wheel H H ought theoretically to be curved; but when the diameter of the screw is large in proportion to its pitch, the portions of the thread with which the teeth are in contact approach very nearly to straight lines, and the oblique sides of the teeth may therefore, without much practical error, be made straight. Their proper obliquity may be found thus:—Let I be a cylinder of the same diameter with the screw, on which is wound a triangular piece of paper, so that its edge shall form the outline of the screw. If at any of its convolutions K, instead of continuing to wind the paper on the cylinder, we stretch it out straight as K M L, if K L be half the pitch of the screw, or L half-way between K and the point where the next convolution after that at K would cross the axis, then L M must be equal to half the circumference of the cylinder (about 3½ times its radius), in order that the point M may be brought round to L when the paper is wound on to the cylinder. By taking K L half the pitch of the screw and L M half its circumference, and joining K M, we get a line at the proper

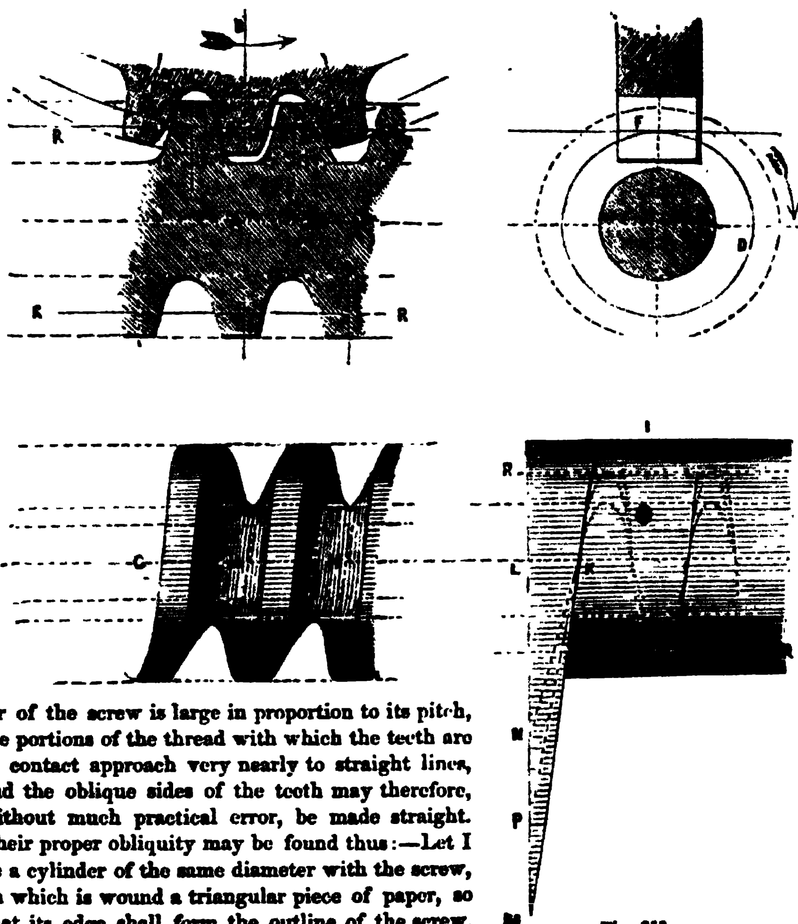


Fig. 250.

obliquity to the axis to suit the thread of the screw. By taking a cylinder equal to the inner part of the screw, or of diameter equal to that of the screw at the bottom of its thread, and proceeding in a similar way (as indicated by the dotted lines), we get the line K N of the proper obliquity to suit the bottom of the thread. Now, as the tops or points of the teeth of the wheel gear with the bottoms or hollows of the screw-threads, while the bottoms of the teeth gear with the tops of the screw-threads, we should make the tops of the teeth less inclined to the plane of the wheel than their bottoms, and we should gradually increase the obliquity from the tops of the teeth downwards. It would be very difficult to effect this in practice; and, indeed, even if it could be done with ease, it would be positively disadvantageous, because some of the teeth are always coming into such a position with respect to the thread as that marked Q, where the top of the tooth is in contact with the top of the thread. Practically, then, it is best to make the obliquity of the teeth like K P a mean between those due to the upper and the lower parts of the screw-thread. In other words, tracing a pitch-line R R for the screw touching the pitch circle of the wheel, and then developing the obliquity K P due to the diameter of the screw measured to that pitch-line, we get an average inclination for the teeth, somewhat in error at points above and below the pitch-line, but correct where the principal contact and communication of power takes place. The greater the diameter of the screw, and the smaller its pitch, the greater is the inclination of the thread to its axis, and the less is the error of obliquity in the contact of the teeth and screw-thread above and below the pitch-line. Therefore, when the screw by rotating drives the wheel, the screw should be made as large in diameter as is consistent with convenience, and the pitch of the thread and teeth should be as small as is consistent with the necessary strength. But when it is intended that the rotation of the wheel shall cause the screw to rotate, the diameter of the screw should be made as small and its pitch as great as possible, so that the thread and the teeth may have small obliquity to the axis of the screw. In either case the screw-thread may be considered as a continuous inclined plane presented to the teeth of the wheel, and the effect of greater or less obliquity be-

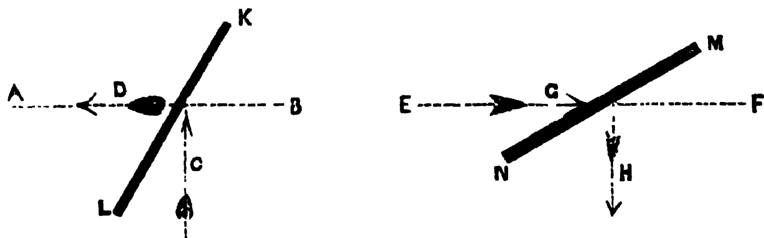


Fig. 251.

comes very evident. In the first case, where the screw drives the wheel, A B being the axis of the screw, K L, part of its inclined surface, travelling in the direction of the arrow C, has to move the tooth of the wheel in the direction of the arrow D; the greater the inclination of K L to the axis, or the more nearly it approaches to a perpendicular to it, the better its effect to move the tooth, or the less is the lateral strain it produces on the tooth. In the second case, where a tooth moving in the direction of the arrow G, and pressing against the inclined side of the screw-thread M N, causes it to travel in the direction of the arrow H, the less the inclination of M N to the axis E F, or the more

nearly it approaches to coincidence with it, the better is the action of the tooth to give it lateral motion, or the less is the longitudinal strain in the direction of the axis.

Although continuous rotary movements are the most convenient for communicating power from one point to another, yet there are many operations to which machinery is adapted demanding motions of another character. These are chiefly of a reciprocating kind; and, whether the reciprocating movement takes place in a straight line or about a centre or axis, it becomes important to inquire into the modes by which continuous rotary motion may be converted into them or the converse. Sometimes it is desirable that these reciprocating movements should be continuous, sometimes that there should be intervals of rest between them, sometimes that they should be equal in velocity, sometimes that the time occupied by them should vary, according to the special character of the work to be done, and the intensity of the force transmitted. To describe all the known modes of effecting these objects would be to compile a list of almost all the mechanical inventions ever made, and, were it possible, would demand space far beyond the limits of a treatise like this. We will, therefore, merely draw attention to some of the modes of converting motion most generally applied in machinery. The arrangement best adapted to any particular case, or the special modification of action that may be most suitable, are matters that must be left to the ingenuity of the designer.

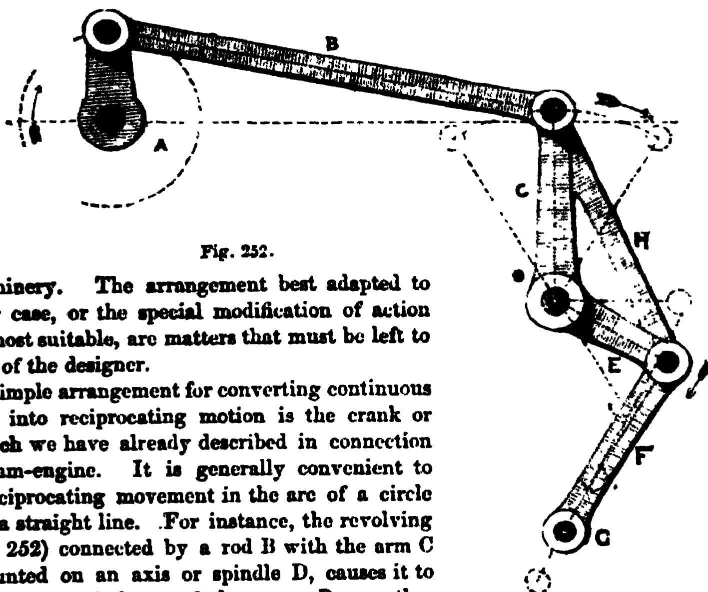


Fig. 252.

The most simple arrangement for converting continuous rotary motion into reciprocating motion is the crank or eccentric, which we have already described in connection with the steam-engine. It is generally convenient to obtain the reciprocating movement in the arc of a circle instead of in a straight line. For instance, the revolving crank A (Fig. 252) connected by a rod B with the arm C of a lever mounted on an axis or spindle D, causes it to vibrate in the arc of a circle round the centre D; another lever E, fixed at any part of the spindle D, can thus be put in reciprocating motion through equal angles, and communicate through a connecting-rod F reciprocating motion to some other body at G. When the arms C and E are in one plane they form a *bell-crank* lever, and are generally connected by the rib H for the sake of strength. The dotted lines on the figure mark the centre lines of the levers at the extreme points of their excursions. The first lever C in its central position is at right angles to a line drawn through the centre of the crank bisecting the vibration of the lever, or cutting the arc in which the pin of the lever vibrates in such a manner that its deviations from the straight line at its middle and extreme points are equal. So also the line in which the point G is required

to reciprocate, when prolonged bisects the vibration of the arm E. By this arrangement the deviation of the pins moving in circular arcs from rectilineal motion is rendered as small as possible, and lateral strain from obliquity of connecting-rods is proportionally reduced. By varying the lengths of the arms C and E, the amount of vibration may be varied at pleasure; for while the angles in which they vibrate remain equal, the lengths of circular arcs in which their pins vibrate are proportional to the lengths of their radii.

By proper arrangements of cranks, levers, and connecting-rods, in respect of lengths and relative positions, it is generally possible to convert a given rotary or reciprocating movement into one of greater or less extent, in a different plane or direction. When the reciprocating movement of a lever is applied to produce the rotary motion of a crank, it is necessary either to fit the spindle of the latter with a heavy fly-wheel to bring the crank over its *dead-centres*, or to adopt some other combination of cranks as in the duplicate marine or locomotive engines, so that while one is on the *dead-centre* the other is receiving motion from its connecting-rod. The crank or eccentric can give reciprocating movement to a lever only in such a manner that for each revolution of the crank the lever makes one complete double stroke, or an excursion from one extreme of its vibration to the other and back. Sometimes it is desirable that each revolution of the rotating shaft shall cause a number of reciprocating movements of a lever or rod connected with it. There are several methods of effecting this object. For moving a stamper *b* (Fig. 253) so as to crush or pulverize materials subjected to it, a wheel *a* is fitted with several curved arms or wipers, which, as the wheel revolves, come successively in contact with a pin or projection *c* on the stamper, lift it, and let it drop. If the wheel have six arms, the stamper makes six strokes during each revolution. Such an apparatus is frequently applied in cases where repeated strokes of falling heavy bodies are required for special operations, as for tilt-hammers used in iron manufacture, seed-crushers, fulling-mills, and washing-machines. For lighter work, such as the movement of pendulums or balance-wheels of time-keepers, there are numerous ingenious arrangements of escapement-wheels having their circumferences cut into teeth of suitable form capable of acting on the pallets presented to them with the regularity and precision required.

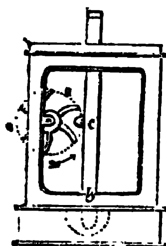


Fig. 253.

Sometimes it is desirable to convert continuous rotary movement into one that shall proceed by fits and starts. A crank or eccentric A (Fig. 254) on the continuously rotating shaft connected by a rod B with a lever C, causes it to vibrate once during each revolution of the crank round a spindle D on which is mounted a *ratchet*-wheel E, having teeth like those of a saw. The lever C is fitted with a pall F hanging freely from a pin, and formed so as to drop into the space between the ratchet teeth. One half-revolution of the crank A causes the pall to move over one or more teeth without putting them in motion, as it can slide freely along their inclined sides; but the other half-revolution bringing back the lever and pall which bears against the abrupt face of a tooth, causes the ratchet-wheel to rotate through the same arc with the point of the pall. In many cases where this arrangement of ratchet and pall is applied, it is necessary to vary the amount of movement of the ratchet-wheel at each stroke. This is effected either by varying the throw of the crank A, or the position of the connecting-rod pin on the lever C, so as to make the back stroke of the pall pass over two or more teeth, as may be required. In machines for boring, turning, planing, and shaping metals, in machinery

or sawing timber and such like operations, this arrangement is applied for moving the material operated on, so as to present a new portion to the action of the tool or saw at each stroke.

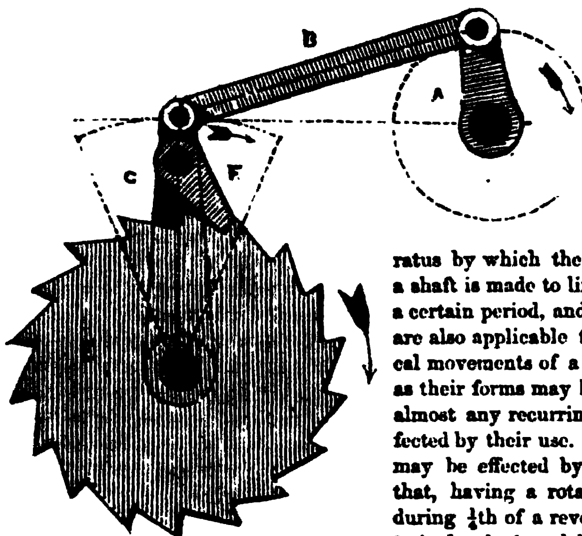


Fig. 254.

In describing the methods of regulating the flow of steam in engines, we alluded to the *cam* as an apparatus by which the continuous rotation of a shaft is made to lift a valve, sustain it for a certain period, and then let it drop. *Cams* are also applicable to many other mechanical movements of a similar character; and as their forms may be varied indefinitely, so almost any recurring movement can be effected by their use. As an example of what may be effected by a cam, let us suppose that, having a rotating shaft, we desire it during $\frac{1}{8}$th of a revolution to raise a weight 1 inch, during $\frac{1}{8}$th to retain it at that height, during the next $\frac{1}{8}$th to raise it 1 inch more, to retain it there during $\frac{1}{8}$th, during the next $\frac{1}{8}$th to drop it the two inches through which it had been raised, and during the remaining $\frac{1}{8}$th to retain it at its lowest position until it is again raised.

Let A (Fig. 255) be the shaft revolving in the direction of the arrow, and let three circles be described round its centre, the inner circle being of any convenient radius, and the others having radii respectively 1 inch and 2 inches greater. Let the circles be divided into eight equal segments, and let H be a roller connected with the weight to be lifted, bearing on the inner circle at B. From B to C, $\frac{1}{8}$th of a revolution, let the circumference be part of a spiral curve touching the inner circle B and the middle circle at C; from C to D, $\frac{1}{8}$th of a revolution, let it follow the middle circle; from D to E, $\frac{1}{8}$th of a revolution, let it be another spiral curve touching the middle circle at D and the outer at E; from E to F, $\frac{1}{8}$th of a revolution, let it follow the outer circle; from F to G, $\frac{1}{8}$th of a revolution, let it again be a spiral touching the outer and inner circles; and from G to B, the remaining $\frac{1}{8}$th of a revolution, let it follow the inner circle. It is obvious that as the shaft rotates and brings the different portions of the cam's circumference successively under the roller, the centre of which we suppose to be capable of vertical movement only, it successively lifts it one inch, retains it, lifts it again one inch, retains it, and permits it to drop, and remain down according to the conditions of the problem. The principal point to be attended to in the construction of a cam is, that the spiral portions leading from the one part of the circumference to the next be not too abrupt, and that they be carefully graduated from the one curvature to the other, so that the roller be not subjected to sudden jerks, but be made to rise from or fall towards the centre as gently and easily as possible. When the rotation of the cam

is rapid, this is of the greatest importance; the amount of eccentric movement given to the roller should be as small as possible; or it should be spread over as large a part of the circumference as possible, so that the inclination of the revolving slope may be gentle.

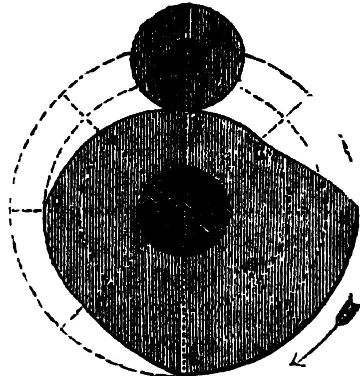


Fig. 255.

In some kinds of printing machinery a peculiar form of screw is applied for the conversion of a continuous rotation into a reciprocating rectilineal motion.

A cylinder A (Fig. 256) rotating in bearings B B has a double groove of a screw form cut in its surface, in which a pin *c*, projecting downwards from the table of the printing machine, is free to slide. The table being guided to slide backwards and forwards in a straight line, is caused to move by the revolution of the cylinder presenting always the inclined face of its screw-groove to the pin. The screw is double, of opposite direction, with a portion at each end not oblique to the axis of the cylinder. One half-revolution of the cylinder causes the pin to travel throughout the length of its screw, another half-revolution retains it at the end of its stroke, the next half-revolution carries it back to the opposite end, the next retains it, and so on successively to suit the alternate rest and motion of the printing-table to the successive impressions and intervals between them.

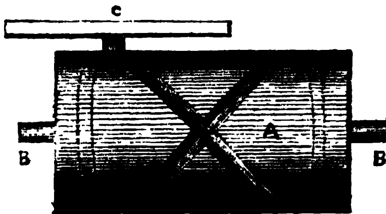


Fig. 256.

Racks and partly-geared pinions are sometimes used for a similar purpose, thus:—A double rack *a a* (Fig. 257) guided at the ends *b* and *c* to move only longitudinally, is acted on by a partly-geared pinion *c* continuously rotating. When the teeth of the pinion gear with those of the upper limb of the rack, it is caused to move from *b* towards *c*, until the pinion having left that limb enters into gear with the other, giving it the contrary movement, and so on successively.

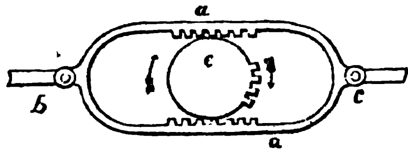


Fig. 257.

Fig. 258 represents an arrangement of a similar character, applied to produce alternating rotary motion. A pinion *c* continuously rotates on an axis fitted with a universal-joint, such as may permit the pinion to gear either with the exterior or interior cogs of the double circular rack *b* fixed on the face of a drum or pulley *a*, and thus to give it rotary motion round its centre *e* in directions alternately opposite.

On referring to our remarks respecting the communication of motion by pulleys and straps, it will be seen that while a pulley driven from another by a direct strap revolves

in the same direction, one driven by a crossed strap revolves in the opposite direction. This principle is frequently applied in machinery where it is desired to reverse the direction of motion. Two pairs of fast and loose pulleys being arranged on a shaft, so that one pair may be connected by a direct and the other pair by a crossed strap, with pulleys on the prime mover; when the direct strap is on its fast pulley, while the crossed strap is on its loose pulley, the machine is driven in the direction of the prime-mover; but when the direct strap is thrown on its loose pulley, while the crossed strap is brought on its fast pulley, the contrary motion is produced. In apparatus where an arrangement of this kind is applied, as in machines for planing iron, the movement of the table of the machine to each extreme of its stroke is made to shift the straps by very simple mechanism, which is capable of being adjusted so as to vary the amount of stroke at pleasure.

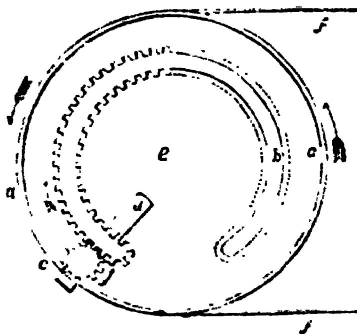


Fig. 258.

Fig. 259 represents another method of reversing rotary motion frequently employed: *a* and *b* are two bevil pinions revolving loosely on a shaft *d e*, and gearing with a bevil wheel *c*. Between the pinions is fitted a clutch, sliding on but revolving with the shaft *d e*. The pinions being geared into opposite sides of the wheel, rotate in opposite directions; and as the clutch is thrown into gear with the one or the other, the shaft *d e* is caused to rotate in the one direction or the other accordingly. Instead of a toothed clutch, a conical friction clutch is occasionally employed with good effect, because, in the first place, a very slight movement of the lever pressing the

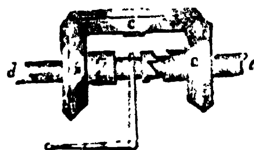


Fig. 259.

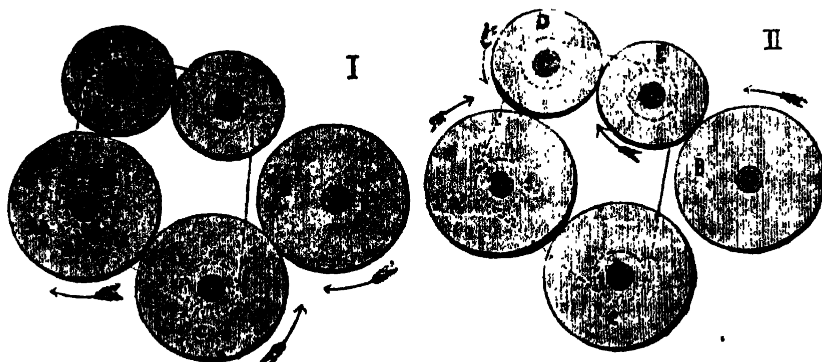


Fig. 260.

cone into its seat on either side is sufficient to couple the shaft with either wheel, and, in the next place, the shaft being driven solely by the friction of the conical surfaces,

cannot be subjected to any strain exceeding the friction in amount, the cones slipping when subjected to extreme strain.

Fig. 260 indicates a mode of reversing the direction of rotation by means of toothed gearing. A being a toothed wheel on the driving shaft, and B one on the shaft to which it is required to convey motion in either direction, C, D and E are intermediate pinions mounted on a frame capable of vibrating on A as an axis. When C is geared with A and B, as in I., both revolve in the same direction; but when C is thrown out of gear with B, and E brought into gear with it as in II., the direction of its rotation is reversed, as indicated by the arrows. By some such arrangements, or modifications of them, suited to the circumstances of any particular case, the direction of rotation may be readily varied.

It is often required to change a rectilineal motion having a certain velocity to one having another velocity, particularly in machinery for cutting screws. A screw, as we have already described, is a line traced on the surface of a cylinder by the motion of a point moving longitudinally along the cylinder parallel to its axis. The pitch of the screw is the distance through which the tracing point moves longitudinally while the cylinder makes one revolution, and in practice it is necessary to form screws of numerous different pitches according to their dimensions and the circumstances under which they are to be used. The pitch of a screw is generally named according to the number of turns or convolutions which the screw makes in a certain length of the cylinder. Thus, when a screw has 8 turns to the inch, we say that it has a pitch of ⅛th of an inch—that is, during one revolution of the cylinder on which the screw is formed, the tracing point advances longitudinally ⅛th of an inch, or while the tracing point advances 1 inch the cylinder revolves 8 times.

A (Fig. 261) is a cylinder revolving in bearings at each end, with a toothed wheel B fixed at one end, and D a screw mounted in bearings parallel to the cylinder, and carrying a nut F with a tracing point G projecting from it to meet the surface of the cylinder, the screw having a toothed wheel E gearing into an intermediate wheel C, which also gears into B. On causing the cylinder A to revolve, the screw D is also caused to revolve, the nut F and tracer G are made to move longitudinally parallel to the axis of A, and the screw-curve traced on A is that due to the velocity with which A revolves as compared to the speed with which G advances. By altering the proportions or numbers of teeth in the wheels B and E, the relative velocities of the cylinder and screw may be changed at pleasure, and the pitch of the screw traced on A from a screw D of constant pitch may be proportionally varied. In lathes or machines for cutting screws, the cylinder A is the piece of metal on which the screw is to be cut, its axis is what is technically called the *mandril* of the lathe, and the tracing-point G is a steel tool cutting into the metal so as to leave the screw-thread projecting. The screw D has generally a pitch which is some simple fraction of an inch, such as ½ or ⅓rd of an inch, and the lathe is furnished with numerous toothed wheels which may be put in the

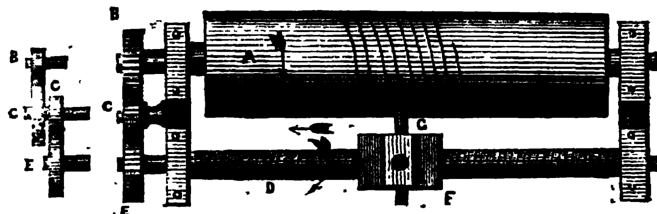


Fig. 261.

place of B and E to vary the pitch of the cut-screw as may be required. If we take the case of a lathe having a screw D of $\frac{1}{2}$ an inch pitch, or making 2 turns per inch, when the wheels B and E have equal numbers of teeth, whatever be the size of the intermediate wheel C, the screw D revolves with the same angular velocity and in the same direction with the work A. The tool G therefore advances $\frac{1}{2}$ an inch along the surface of A during each revolution of A, and cuts a screw of precisely the same pitch and lying in the same direction with that on the screw D. But if in place of two equal wheels B and E we put wheels, B having 20 teeth and E having 40, then the screw D will be caused to revolve at $\frac{20}{40}$ths, or $\frac{1}{2}$ the speed of A, and the tool will advance $\frac{1}{2}$ of $\frac{1}{2}$ an inch, or $\frac{1}{4}$th of an inch, during each revolution of A, and thus to cut a thread of $\frac{1}{4}$th of an inch pitch upon it. Screw-cutting lathes are generally furnished with a table of screw pitches, and the appropriate wheels for producing them, such that the workman by inspecting the table can at once select the proper wheels for giving the desired pitch. The wheels belonging to the lathe have their numbers of teeth stamped upon them to save the trouble of counting them. Sometimes, when the difference of the speed of the screw and of the work is required to be considerable, instead of the simple intermediate wheel C, it is necessary to introduce a wheel and pinion C and *c* fixed together. Thus if it were required that the screw should revolve at $\frac{1}{25}$th of the velocity of the mandril, were the smallest possible wheel B to have 20 teeth, it would be necessary that E should have 500 teeth, because $\frac{20}{500} = \frac{1}{25}$th, were the simple intermediate wheel C employed. This size of wheel for E might be extremely inconvenient, and it would be better to employ the compound intermediate wheel and pinion C and *c*. In this arrangement B having 20 teeth, C 100, *c* 20, and E 100, the speed of E is $\frac{20 \times 20}{100 \times 100} = \frac{1}{25}$th of that of B. In the lathe table the first column gives the pitch of the screw to be cut, the second gives the number of teeth on the mandril wheel B, the third and fourth give the numbers of teeth in the intermediate wheels C and *c* respectively, and the fifth gives the number on the screw-wheel E. When the compound intermediate wheels are not required, the columns of intermediates are left blank, as any simple intermediate may be employed without altering the relative velocities of the mandril and screw-wheels. The following is part of a table for a lathe having a screw of $\frac{1}{2}$ inch pitch :—

TABLE OF SCREW GEARING FOR LATHE.

Pitch : Turns per inch.	Teeth in Mandril Wheel.	Teeth in Intermediate Wheel and Pinion.		Teeth in Screw-wheels.
20	30	120	40	100
18	30	90	30	90
16	30	120	40	80
15	40	100	30	90
	or 20	..	..	150
14	20	80	40	70
12	20	..	..	120
11	20	..	..	110
10	20	..	..	100
9	20	..	..	90
8	30	..	..	120
7	20	..	..	70
6	40	..	..	120
5	40	..	..	100
4	40	..	..	80

If we examine any part of the table, as, for instance, the numbers given for 14 turns to the inch, we find the following rule obtains:—The pitch in col. 1 is equal to twice the product of the numbers in 3 and 5, divided by the product of those in 2 and 4. Thus $14 = 2 \times \frac{80 \times 70}{20 \times 40}$. Again, where no compound intermediate is employed, the pitch is twice the number in 5 divided by that in 2. Thus 6 pitch $=$ 2 pitch $\times \frac{120 \text{ teeth.}}{40 \text{ teeth.}}$

For machinery by which straight lines or the circumferences of circles are divided into a number of equal parts, as for the marking out of scales for measuring or astronomical instruments, or for cutting teeth in racks and wheels, the screw and the worm and wheel are often employed in order to give the means of delicate subdivision. When a screw is used for such a purpose, it is generally called a micrometer (small measurer) screw, and the principle on which it acts may be thus described:—If we suppose an accurately cut screw, having $\frac{1}{10}$th of an inch pitch, to be fitted with a nut, the motion of the screw through one revolution would advance the nut through $\frac{1}{10}$th of an inch. If on one end of the screw there were fitted a wheel having its circumference divided into 100 equal parts, the screw might be turned round any number of revolutions or hundredths of a revolution, as marked by a fixed index pointing to the divisions on the wheel. But for every hundredth part of a revolution of the screw, the nut would be advanced $\frac{1}{100}$th of $\frac{1}{10}$th, that is, $\frac{1}{1000}$th part of an inch; and by using a screw of still finer pitch, and having a wheel mounted upon it divided into still smaller and more numerous parts, the longitudinal advance of the nut through still smaller fractions of an inch could be effected and estimated. It is by such an arrangement that the fine divisions of mathematical instruments are traced, and the fine lines traced upon medallion drawings are engraved. For circular division, the screw, instead of moving a nut longitudinally, acts as a worm on the teeth of a wheel, and causes it to revolve through any required part of its circumference. Thus, if we had a worm-wheel with 360 teeth, and a worm with a wheel fitted on its axis, having 360 divisions, we could move the worm-wheel through $\frac{1}{360}$th of $\frac{1}{360}$th, that is, $\frac{1}{129600}$th of a revolution, or any number of such fractions of a revolution. And farther, if the wheel upon the screw were a toothed wheel, and we had numerous other toothed wheels, with various numbers of teeth like those we have described for a screw-cutting lathe, we might, by the proper selection of wheels gearing with that on the screw, effect the division of the circle into any number of required equal parts. Such apparatus are employed for dividing the circumferences of astronomical instruments, and also for cutting the teeth of wheels. When the number of teeth to be cut is a multiple of some simple numbers, such as 60, which is a multiple of some of the numbers 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, it is generally easy to select wheels which, in connection with the screw, shall give the required number of equal divisions; but when the number of teeth is what is called a prime number, such as 59 or 61, which is not capable of subdivision, we must either provide wheels having such a number of teeth already cut in them, or resort to methods of approximation for their subdivision. Thus, if with the 360 teeth on the worm and 360 divisions on the screw-wheel, we desired to divide a circle into 61 equal parts, we should for each part turn the screw through $\frac{129,600}{61} = 2124\frac{36}{61}$ divisions, or $\frac{2124\frac{36}{61}}{360} =$ 5 revol. $324\frac{36}{61}$ div. nearly. The error at the end of the process would be found thus:—

Since the total number of divisions due to a complete revolution are 129,600,
and since $2124\frac{1}{2} \times 61 = 129{,}594\frac{1}{2}$,

the revolution of the wheel would want $5\frac{1}{2}$ divisions of being complete; that is, $\frac{5\frac{1}{2}}{129{,}600}$, or about $\frac{1}{23564}$th part of its circumference: a quantity quite inappreciable.

In several machines it is necessary to provide a rotary motion more rapid at one portion of a revolution than at another, as, for instance, in machines for slotting and planing iron. The iron to be planed, or the tool which planes it, makes a rectilineal stroke slowly in the one direction while the metal is being cut, but may be drawn back rapidly in the opposite direction when no work is done. For producing motion of this kind, elliptical geared wheels are employed (Fig. 262). The ellipse is a curve, of which the line A B is called the greater axis, and each of the two points C and D in that line is called a focus. If to any point E in the circumference of the ellipse, lines C E and D E be drawn from the foci, the sum of their lengths is equal to that of the axis A B. Farther, if the lines C E and D E be prolonged beyond the curve, and a line F G drawn so as to divide equally either of the angles formed by their intersection, the line F G is a tangent to the curve at E; that is, it touches it, but does not cut it; or every part of it, except merely the point E, lies entirely outside of the curve. If, in the prolonged portions of the lines C E and D E, lengths E K and E H be measured off equal to D E and C E respectively, H and K may be taken as the foci of another ellipse, precisely equal in every respect to the original ellipse, and having the axis L M and the tangent F G touching it at E; and as F G touches both ellipses, they touch each other at the same point E, and the length of the line C K, which is made up of C E and E K, or its equal E D, is equal to A B or L M.

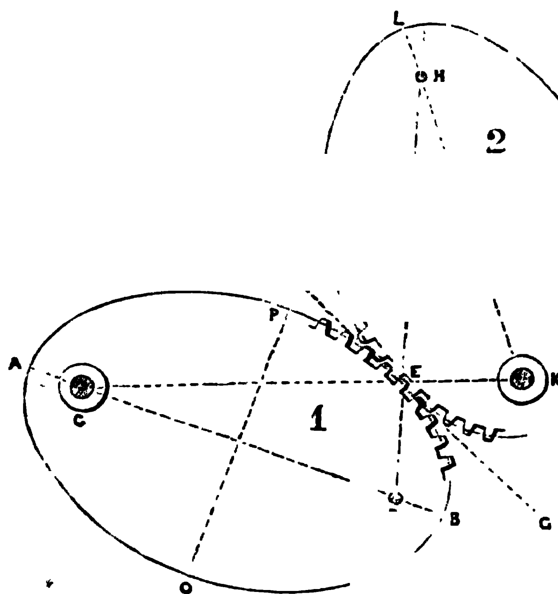


Fig. 262.

These peculiar properties of the ellipse enable us to employ elliptical wheels, each revolving round a focus as a centre, C for the one and K for the other, the two foci maintaining a constant distance apart, whatever be the relative position in which the

wheels lie with respect to each other. The circumferences are cut into teeth and spaces like ordinary circular gearing, the elliptical curves being their pitch lines, on which the equal divisions are set out.

It will be observed that, during a revolution of these wheels, the point M gears with B, and the point L gears with A; but as the radii of these points are C B and K M, and K L and C A respectively, the angular velocities of the wheels at these points vary inversely as the lengths of these radii. If, for instance, C A be ¼th of A B, and therefore C B = ¾ths of C A, K M being equal to C A and K L to C B, when the points B and M are engaged the wheel 2 is revolving with 3 times the angular velocity of 1, because the radius C B is three times the length of K M; but when the points A and L are engaged, the wheel 2 is revolving with ⅓rd of the angular velocity of 1, because A C is ⅓rd of K L. So at all points intermediate to these, except at N, O, P, and Q, where the radii are equal, the relative angular velocities of the wheels vary between the limits we have named.

For some purposes in the manufacture of textile fabrics, gearing like that in Fig. 263 is employed. These vary the angular velocities 4 times in every revolution; and the principal condition of their construction is, that the distance between their centres be a constant quantity.

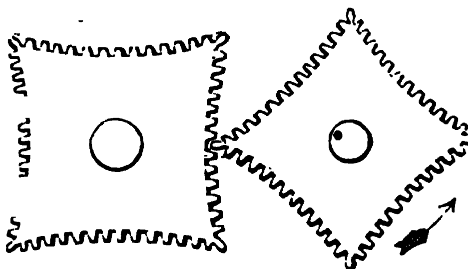


Fig. 263.

The *sun and planet wheel* is a contrivance for converting a reciprocating into a rotary motion. It was employed by Watt instead of the crank in a steam-engine, not because he preferred it to the latter, but because, through the bad faith of a workman who patented the crank as his own invention, he was precluded from employing it. The sun or central wheel A (Fig. 264) gears with the planet-wheel B, which is caused to revolve round A without rotating round its own centre, being fixed to the end of the connecting-rod, and retained in gear with A by means of a link connecting their centres, and revolving freely with B. When the sun and planet-wheels have equal numbers of teeth, every revolution of the latter causes 2 revolutions of the former, as may be understood by watching the relative positions of a tooth in each at different parts of a revolution. When the planet is vertically above the sun-wheel, the tooth B of the one is engaged with the space A of the other; and when the planet has made half a revolution, so as to be vertically below the sun, the sun has made a complete revolution, so as to bring the space A quite round to the point where it was at the

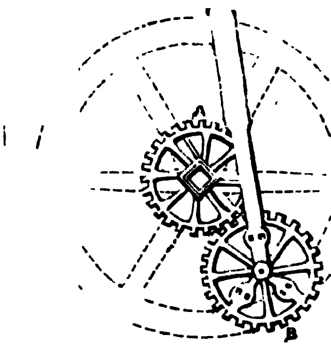


Fig. 264.

commencement. One half-revolution of A is owing to the half-revolution of B propelling it round, while the other half-revolution of A is effected by the roll of the toothed half circumference of B presenting fresh teeth and spaces to A at the successive points of its revolution. When the numbers of teeth in the two wheels are different, the velocity of the sun-wheel varies accordingly, as may be best understood by an example. Let us suppose that the sun-wheel has 40 teeth, and that the planet has 50; then, during one revolution of the planet, it has presented the whole of its 50 teeth to the sun-wheel, and therefore turned it through 50 teeth, as well as its own complete revolution of 40 teeth. The sun-wheel, therefore, during 1 revolution of the planet-wheel, turns round a distance equivalent to 90 of its teeth, or $\frac{90}{40}$ths $= 2\frac{1}{4}$ revolutions.

Friction.—It is often necessary to provide the means of stopping the motion of machinery when the mere cessation of power in the prime mover is not sufficient for the purpose. In a crane, when lowering a heavy weight, it may be desirable to lower it to a certain point and no farther, and therefore to stop the machinery of the crane when the weight has descended sufficiently far. Or, again, in any apparatus provided with a fly-wheel, or parts moving with considerable momentum, such as might continue the movement after the power has been withdrawn, it may be essential to provide the means of stopping the movement more suddenly. The most simple arrangement for this purpose is the *break* or friction-strap (Fig. 265). *a* is a wheel revolving with the rest of the machinery, and *b b* a flexible strap of iron passing round part of its circumference. One end of this strap being fixed by a pin to some motionless part of the machine, and the other attached to a lever *c* pivoted on a fulcrum *d*, on applying force to the long arm of the lever, the strap is drawn tightly round the circumference of the wheel, and the friction caused by the close contact soon brings the wheel to rest. The great advantage of employing friction as a means of arresting motion, consists in the circumstance that it acts not suddenly but gradually. Were some solid obstacle presented to the motion of any part of a train of heavy or rapidly moving machinery, the momentum of all the moving parts would have to be suddenly destroyed, and as no time would be afforded for this operation, the strain would be incalculably great, and inevitable damage would ensue. But when the friction-break is employed, the wheel to which it is applied makes perhaps two or three revolutions before it comes finally to rest, and the time so occupied allows the momentum of all the parts connected with it to expend itself in overcoming the great additional resistance caused by the friction.

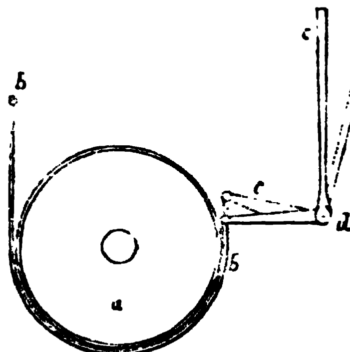
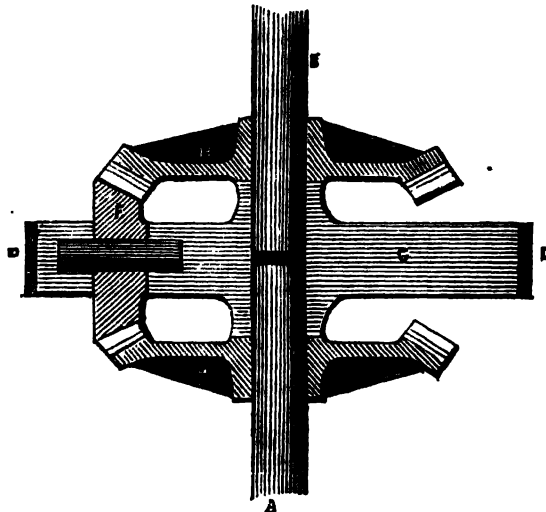


Fig. 265.

In putting an extensive train of machinery in motion, the inertia of all the parts at rest has to be overcome in like manner; and were this done suddenly the strain would be as great as in the opposite case of suddenly arresting their motion. This contingency is generally met by the use of pulleys and straps in communicating the power. A strap,

communicating motion from one pulley to another, acts only by its friction on their circumferences; and when the strain which it has to overcome exceeds the force due to its friction, the strap slips at first to a considerable extent, but gradually less and less, until the proper velocity is attained and the strap and circumference of the pulley move in unison. In cases where straps cannot be conveniently applied for driving a train, a friction coupling is employed. Fig. 266 is a view of one very generally used. A is the driving shaft, and B the driven shaft, the ends of which are free to revolve in the boss of a wheel C, true and smooth on its circumference, to which is applied a friction-strap D worked by a suitable lever. Within the wheel C a bevel-pinion F is mounted in bearings, its axis being at right angles to that of the wheel and shafts; and bevel wheels G and H, one on each shaft, are fitted to gear with the pinion F. If the friction-strap D be loose, so as



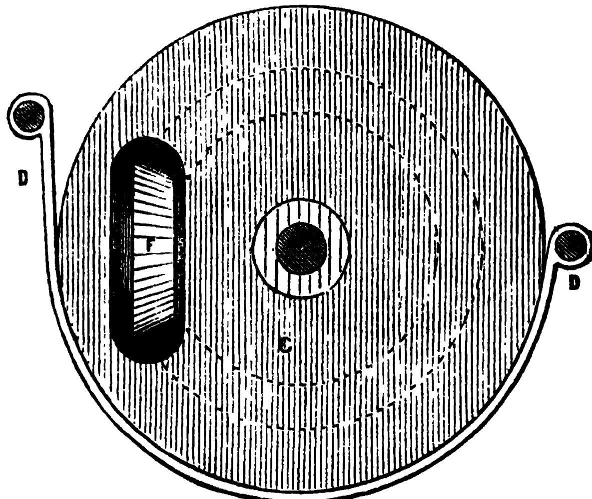


Fig. 266.

to leave the wheel C free to revolve, the rotation of the shaft A and its wheel G gives motion to the pinion F, and causes its teeth to travel in those of the stationary wheel H, and thus to make the wheel C rotate at half the angular velocity of A without putting B in motion. But if C be arrested by the friction of the strap D, the axis of F becomes fixed, and the rotation of A and G is communicated through F to B and H at the same speed, but in the opposite direction. The use of the friction-strap in this apparatus presents the advantages in gradually generating momentum in the driven machinery, similar to those derived from its use in destroying the momentum of machinery in motion. Although, for the sake of simplicity, we have represented only one bevil pinion F mounted in the friction-wheel, it is customary to provide at least two on opposite sides of the centre in order to balance it, and sometimes four are fitted for the sake of equilibrium and strength.

It often becomes important to inquire what amount of power is communicated through a certain train of machinery. When a steam-engine is employed as the prime mover of any machine, the power communicated can be readily ascertained by the indicator. The engine is first worked alone, or with merely the train of wheel-work, in order that the power necessary to overcome friction may be estimated. It is then worked in connection with the machine, and the driving-power required for the machine is ascertained by subtracting the force necessary to overcome friction from the total power, including friction and the resistance of the machine. When machinery is driven by some other power, or when the indicator cannot be conveniently applied, the dynamometer (power-measurer) is employed.

The most simple kind of dynamometer consists of a pulley A (Fig. 267) fixed on the driving-shaft of the machinery whose power is required to be known. This pulley is surrounded by a flexible friction-strap C B D, the ends of which may be drawn closely together by a screw at D, so as to tighten the strap as much as may be necessary on the circumference of the pulley. From a hook C attached to the strap is suspended a scale E, in which sufficient weights may be placed to prevent the strap from being carried round by the pulley in its revolution. A loose cord or chain C G fixed at G is also provided to keep the strap in its place in case of the load E being insufficient to counterbalance the friction of the strap. Without this loose cord, a sudden increase of speed or friction might lift the scale and weights, and, whirling them round the pulley, do serious damage to the machinery. In using this machine, the speed of the shaft is

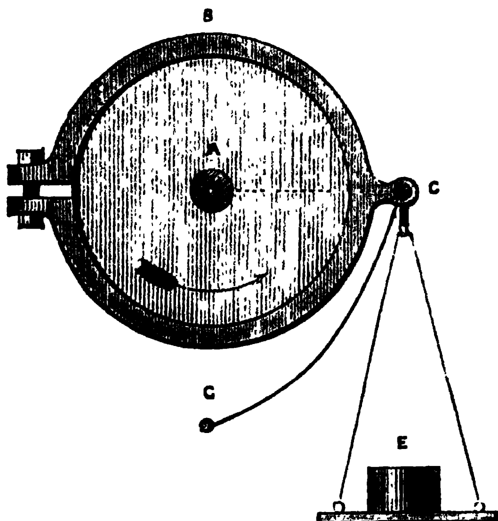


Fig. 267.

carefully ascertained by counting the number of its revolutions per minute, and the screw D is gradually tightened until the scale and its load are just kept up by the friction, the imaginary line A C being horizontal. If the tightening of the screw causes the scale to be lifted without reducing the speed of the pulley, more weight has to be added to the scale; but if the scale with its load cannot be lifted without retarding the pulley, the weight must be reduced. Having found the weight that is just supported when the velocity is correct, the power may be ascertained as follows:—The length of A C being the leverage at which the weight E acts to retard the rotation of the pulley, the power passing through the pulley must be such as would lift the given weight at the distance A C from its centre, or that would during each revolution move the weight E through a space equivalent to the circumference of a circle having A C for its radius. Since twice A C is the diameter of this imaginary circle, and the circumference is $3\frac{1}{7}$ times the diameter, $2 \times 3\frac{1}{7}$ or $6\frac{2}{7}$ times A C is the space through which the weight is driven during each revolution, and this quantity multiplied by the number of revolutions per minute is the total space through which the resistance is moved during each minute. The power is the weight multiplied by its velocity or the distance through which it is moved per minute; and as 33,000 lbs. moved through 1 foot per minute is the standard horse-power, we have the following rule for estimating the horse-power as indicated by the dynamometer.

Rule.—Multiply weight E (in lbs.) by the length of A C, the lever at which the weight acts (in feet) by $6\frac{2}{7}$ and by the number of revolutions of the pulley per minute, and divide by 33,000 for the horse-power.

Example.—The length of A C being 2 feet 4 inches, or $2\frac{1}{3}$ feet, the load E (including the weight of the scale) being 78 lbs., and the velocity of the pulley 120 revolutions per minute, required the power.

$$\frac{78 \text{ lbs.} \times 2\frac{1}{3} \times 6\frac{2}{7} \times 120}{33000} = 4{\cdot}16 \text{ horse-power.}$$

In dynamometers of this kind the friction-strap is plentifully supplied with oil, and it is found better to face the interior of the friction-strap with blocks of wood bearing on the surface of the pulley; because the friction of wood on iron is of a more constant character than that of iron on iron. When iron rubs on iron without the presence of oil or grease, great heat is produced, and the metal surfaces cut into each other and become roughened. But when wood bears on iron, if the surfaces are not oiled, the heat produced by the friction will only char the wood without damaging the iron, and will not effect any great variation in the amount of friction.

Friction is also employed as a means of arresting motion in the case of carriages on declivities and railway trains. By the use of wheels to carriages the friction is transferred from the surface of the road or rail to that of the axle.* As the radius of the wheel is the length of lever at which any obstacle opposed to its progress acts, while the very much smaller radius of the axle is the lever at which its friction acts to oppose the rotation of the wheel, the larger the wheel and the smaller the axle, the less is the resistance from friction. If we were to suppose the axle extended to almost the whole dimensions of the wheel, that, in fact, the wheel were only the thin iron tyre revolving round a solid central part, the resistance to the motion of the carriage would be almost as great as if it were dragged along like a sledge. So if the wheel be prevented from revolving, it has to be dragged along the road, and the opposition which the friction thereby created presents, acts as a great retarding force. In ordinary carriages the wheel is generally prevented from revolving by placing under it a skid or plate of iron attached to the car-

riage by a chain, which has to be dragged along like a sledge. In railway carriages the rotation of the wheels is arrested by means of blocks of wood pressed against their circumferences. These blocks are connected by levers and rods with screws conveniently situated, so that the engine-driver and guards can force them against the wheels or remove the pressure at pleasure.

Except in these few instances, and that of pulleys and straps, in which friction is employed as a means of communicating power or arresting motion, it acts as a resistance in all mechanical arrangements. Although this resistance cannot be totally overcome, yet by carefully designing the arrangements of machinery with regard to the simplicity and proper formation of its parts, by good execution of the work, by the selection of suitable materials, and due provision of lubricating materials wherever surfaces move in contact with each other, it may be diminished to a very great extent.

In order that we may form a clear estimate of friction as a retarding force, we may suppose A (Fig. 268) to be a piece of material, such as iron, wood, brass, or the like, having a smooth lower surface in contact with a smooth table B of the same or any other material. If A be pressed down by a perpendicular force C, it will be found that in order to move it laterally along the table, some force D will have to be impressed upon it, and this force will be greater the greater the perpendicular pressure C. There is, therefore, under these circumstances, a force acting in the direction of the arrow E, not tending to put A in motion, but resisting its motion in obedience to the force D. The resisting force E is the friction of the surfaces A and B. In order to measure the amount of this resistance, let us suppose a force F A acting obliquely at such an angle as just to cause A to slide along the table; then, on completing the parallelogram F G A H, while F H

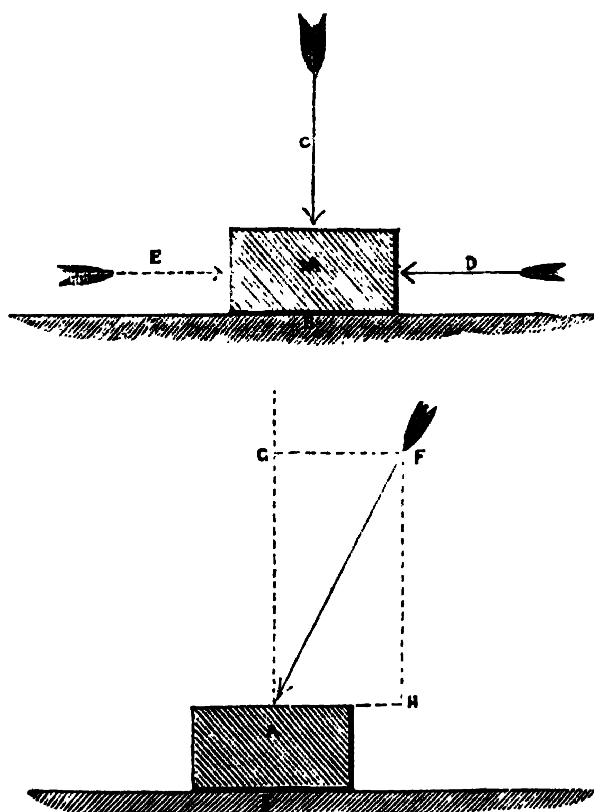


Fig. 268.

or G A measures the amount of force acting perpendicularly to the table, F G or H A measures the force acting parallel to the table and causing A to slide along it.

Now it is found practically that whatever be the absolute force F A, and whatever be the extent of A's surface in contact with B, the obliquity of F A or the angle F A G which it makes with the perpendicular, when it just causes A to slide, is very nearly constant for any given material.

The ratio which A H bears to A G, or the fraction expressing the division of A H by A G, is called the *coefficient of friction*, and the angle F A G is called the *limiting angle of resistance*. Some very careful experiments have been made to determine the values of these for different materials; their results are embodied in the accompanying table. The coefficient of friction is the tangent of the limiting angle of resistance; and if we know the one we can easily find the other from a trigonometrical table. We have, however, given an approximate value of both to save the trouble of reference. As an example of the practical application of these numbers, we may take the case of brass and iron, for which the coefficient of friction is ·143 and the limiting angle 8°. If, then, a piece of smooth iron weighing 1 cwt. rested on a brass plate, it would require a lateral force of 16 lbs. to cause it to slide, for 112 lbs. (the vertical pressure) $\times$ ·143 = 16 lbs. Or, if the brass plate were inclined 8° to the horizon, the iron would slide along it from its own weight. Or, again, if we suppose a smooth round shaft of iron, weighing 1 ton, revolving in a brass bearing, the force necessary to be applied at the circumference of the shaft to overcome its friction would be 2240 $\times$ ·143 = 320 lbs. Assuming the shaft to be 6 inches diameter, or to have a radius of 3 inches at the bearing, the length of the radius 3 inches is the leverage at which the friction acts to resist its rotation, and the resistance 320 lbs. at this leverage is equivalent to $320 \times \frac{12 \text{ in.}}{3 \text{ in.}} = 80$ lbs. at a radius of 1 foot. By a similar method of calculation the resistance due to the sliding friction either of plane or of cylindrical surfaces of these or other materials may be readily estimated.

TABLE OF FRICTION.

Materials of which the Rubbing Surfaces Consist.	Coefficient of Friction, Pressure being 1.	Limiting Angle of Resistance.
Steel and Ice	0·014	$\frac{3}{4}°$
Ice and Ice	0·130	$7\frac{1}{8}°$
Hard Wood and Hard Wood	0·135	$7\frac{3}{4}°$
Brass and Steel or Iron	0·142	8°
Soft Steel and Soft Steel	0·147	$8\frac{1}{4}°$
Cast Iron and Steel	0·150	$8\frac{1}{2}°$
Wrought Iron and Wrought Iron	0·160	9°
Cast Iron and Cast Iron	0·163	$9\frac{1}{4}°$
Hard Brass and Cast Iron	0·167	$9\frac{1}{2}°$
Cast Iron and Wrought Iron	0·170	$9\frac{3}{4}°$
Brass and Brass	0·175	10°
Tin and Iron	0·180	$10\frac{1}{4}°$
Soft Steel and Wrought Iron	0·190	$10\frac{3}{4}°$
Leather and Iron	0·250	14°
Tin and Tin	0·265	$14\frac{3}{4}°$
Granite and Granite	0·300	$16\frac{3}{4}°$
Yellow Deal and Yellow Deal	0·347	$19\frac{1}{4}°$
Sandstone and Sandstone	0·364	20°
Woollen Cloth and Woollen Cloth	0·435	$23\frac{1}{2}°$

The numbers in the preceding table apply in the case of smooth surfaces, such as are employed in well-constructed machinery. When the surfaces are rough, the resistance may be increased indefinitely, and there can be no means of calculating its amount. When the surfaces are oiled, the friction is considerably reduced; but as, in all machinery, the rubbing surfaces are liable to become dry, or roughened by wear or the presence of grit, we think that in estimating the loss by friction, the numbers in the table should be employed without any allowance for lubrication.

The best experiments made to fix these numbers are those by Rennie, who tested the materials under a pressure of 36 lbs. per square inch of rubbing surface. For greater pressures the friction is rather less in proportion, but the numbers given are sufficiently near for practical use. In general it appears that when both surfaces are of the same material, the friction is greater than when they are of different materials. This is believed to be owing to the presence of a certain amount of that cohesive force which holds the molecules of any material together; and it is found to be greater the smoother the surfaces, and therefore the more intimate their contact. Independently of this circumstance, however, it is generally found inexpedient to make the rubbing surfaces in machinery of the same material, especially when there is any risk of their becoming heated by great pressure or rapidity of motion. In such cases, the particles of the one appear to blend with those of the other, the surfaces become cut into ridges and hollows, and sometimes cohesion takes place with such force that the materials themselves give way rather than separate from each other at their surfaces. This result is particularly observed in cases where iron and iron rub upon each other, especially when the iron is soft. When it is desired that the rubbing surfaces should be both of iron, it is better to case-harden the rubbing surfaces, as is usually done in the case of carriage axles. The process of case-hardening is effected by exposing the smooth surface to a red heat for several hours in a furnace in contact with substances capable of furnishing it with carbon, such as prussiate of potash, leather shavings, and the like. The outer skin of the iron is penetrated by the carbon, and becomes a species of steel, which is rendered very hard by plunging it while still red-hot into cold water. After hardening, the surfaces have to be carefully ground smooth and true with oil and emery, or such like polishing substances; and, before use, the emery must be carefully cleaned off, as its presence would otherwise cause the surfaces to cut, to become hot, and to cohere. In the case of shafts revolving in bearings, the shafts being generally of iron are made to rest in bushes made of brass, gun-metal, tin, or some soft alloy. The rubbing surfaces are made to fit each other accurately, being turned, bored, filed or scraped where necessary until the contact is made nearly perfect. When the contact is very imperfect, or only takes place at a few points of the surface, these become rapidly worn and cut, and the heat produced by their wear expands unequally other portions of the surfaces, which become abraded in their turn. In cases where great pressure is sustained on a bearing, the surfaces of contact are extended as much as possible, so that the intensity of pressure on any portion of the surface may be as small as possible. For shafts lying in cylindrical bearings, the extension of surface is best effected by lengthening the bearing, not by increasing its diameter. By increasing the length, the surface is proportionally increased, the pressure per square inch is proportionally diminished, the amount of friction is not altered, but the tendency to cutting or wear is reduced. But were the diameter increased, although the surface would be also increased, and the wear reduced, yet the friction would act at an increased leverage, and have greater effect as a resisting force. When the pressure is directed longitudinally along the

shaft, the bearing surface is increased by forming on the shaft numerous collars or projecting rings, which are made to press against fixed ring-bearings fitted between them. The total pressure of the shaft is thus subdivided over the combined surface of all the collars, and the ring-bearings being made capable of adjustment by regulating screws, no one of them is unduly pressed upon. Such a form of bearing is particularly useful in steam vessels fitted with screw-propellers. The propeller in revolving through the water tends to throw it backwards from the vessel, and the reaction of the water is the force which propels the vessel. But this force, sometimes amounting to several tons, is communicated through the shaft of the propeller to some fixed point in the vessel; and at this point is situated the bearing such as we have described, technically called the *pushing bearing*.

The friction of straps upon pulleys depends upon the extent to which they are tightened, the extent of circumference with which they are in contact, and their breadth. It is commonly believed, that the greater the diameter of pulley, the more surely does the strap cause it to revolve without slipping. Theoretically, however, and we believe practically, it will be found that, with equal degrees of tightness, equal breadths of strap, and equal circumstances as to perfection of contact, the friction of a strap on the circumference of a pulley is the same, whatever be its diameter. The only circumstance that can affect the constancy of the result is, that straps not being perfectly flexible lie more closely to surfaces curved to a large radius than to those of smaller radius. When a certain amount of power has to be communicated through a strap, the speed at which the strap moves has to be taken into account, because power being pressure multiplied by velocity, the greater the velocity with which the power is transmitted the less the pressure that has to be communicated at that speed. In this sense, then, it appears that the larger the pulley the less is the slip of the strap, because the greater the circumference of the pulley revolving at a given angular velocity, the greater is its absolute velocity through space, and therefore the less the pressure required to communicate a given power. It is found practically that a leather strap 8 inches wide, embracing half the circumference of a smoothly-turned iron pulley, and travelling at the rate of 100 feet per minute, can communicate 1 horse-power. For communicating any given power at any given velocity, the breadth of the strap may be found thus:—

Rule.—Multiply the power (horse) by 800, and divide by the speed (in feet per minute); the quotient is the breadth of strap in inches.

Example 1.—Required the breadth of strap, travelling 600 feet per minute, to communicate 12 horse-power.

$$\text{Width, } \frac{12 \times 800}{600} = 16 \text{ inches.}$$

Note.—When the diameter of pulley (in feet) and the number of revolutions per minute are given, the speed of the strap is found by multiplying the given number of revolutions by the diameter (in feet), and by $3\frac{1}{7}$.

Example 2.—Required the breadth of strap for communicating 10 horse-power to a pulley 3 ft. 6 ins. diameter, revolving 150 times per minute.

$$\text{Speed of strap, } 150 \times 3\tfrac{1}{2} \text{ ft.} \times 3\tfrac{1}{7} = 1650 \text{ feet per minute.}$$

$$\text{Breadth, } \frac{10 \times 800}{1650} = 5 \text{ inches nearly.}$$

When less than half the circumference of the pulley is embraced, the strap must be

proportionally wider; and when more than half the circumference is embraced, its width may be less. Thus, if the strap be in contact with only $\frac{1}{4}$th of the circumference, its breadth must be doubled, because $\frac{1}{2}$ is double of $\frac{1}{4}$th. Again, if it be in contact with $\frac{3}{4}$ths of the circumference, its breadth may be $\frac{2}{3}$rds, because $\frac{1}{2}$ is $\frac{2}{3}$rds of $\frac{3}{4}$ths; and so in other cases, the breadth being inversely proportional to the amount of circumference embraced.

In all cases, however, much depends on the tightness of the strap, the limits to the force with which it is strained being, first, the tensive strength of the strap itself, and, secondly, the amount of pressure that it may be convenient to throw upon the shaft and its bearings. New straps become extended by use, and it is therefore frequently necessary to take them up or shorten them. Before use, they should be strained for some time by weights suspended from them, so as to leave less room for extension while in use. Wherever straps are employed, they should be of the greatest breadth, and travel at the greatest speed consistent with convenience, as it is most important to have the requisite strength in the form most suited for flexure, and the least possible strain on the shafts and bearings.

When ropes or chains are employed, as in cranes, capstans, windlasses, or the like, for raising heavy weights or resisting great strains, the requisite amount of friction is obtained by coiling them more than once round the barrel of the apparatus. It is found that one complete coil of a rope, as in Fig. 269, produces a friction equivalent to 9 times the tension on the rope, the barrel being fixed; that is to say, 1 lb. or 1 cwt. of tension on the end of a rope at A, can support 9 lbs. or 9 cwt. of tension at B. Were the end B of the rope coiled again round a barrel, it would support 9 times its tension, that is, 9×9, or 81 times the tension of A, and so on, coil after coil increasing the friction in a very high ratio.

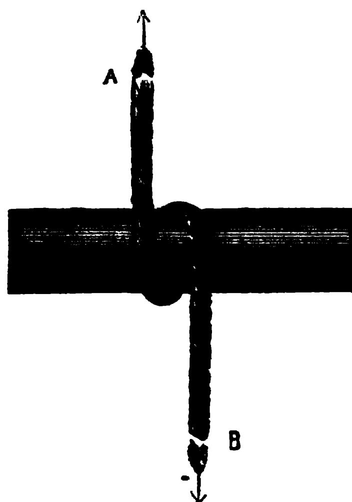


Fig. 269.

The rule by which this may be calculated is very simple, viz.:—Multiply 9 by itself as many times as there are coils, and the product will be the number of times the tension at one end that will be supported at the other. For example, 1 cwt. at one end of a rope coiled three times round a barrel would support $9 \times 9 \times 9 = 729$ cwt., or 36½ tons at the other end of the rope. The diameter of the barrel does not affect the result. Having regard to these facts, we may readily understand the force with which a knot on a cord or rope resists the slip of the coils of which it consists, for the several parts of the cord act as small barrels, round which the other parts are coiled; and the yielding nature of the material of which the barrels are composed, permits the coils to become impressed into their substance on the application of force, and prevents them from slipping more effectually than if they were coiled on a hard and resisting barrel.

Before concluding this part of the subject, we may briefly allude to suggestions that have from time to time been made for communicating power by means of the move-

ment of fluids. Some of these have been practically carried out with very good effect, and we believe that much may yet be done in this direction.

Manufactories, containing numerous machines, are generally arranged in such a manner that the power of a great prime-mover, whether a steam-engine or water-wheel, is communicated by shafts, pulleys, straps, and wheelwork, to the different machines. However excellent may be the arrangements or workmanship of the connecting machinery, still there must be considerable waste of power in overcoming the mere friction attending its motion; and this waste is very much increased when the accuracy of arrangement is affected by wear, or by the unequal settlement of any part of the building supporting the bearings of shafting. Of late years, engineers have devoted attention to the communication of power to some extent without the intervention of wheelwork and shafting, but by the supply of steam from a central boiler to numerous small engines, arranged throughout the building in the immediate neighbourhood of the machinery to be put in motion. The pipes that convey the steam are cheaper in first cost than shafting of equivalent length, their efficacy is not dependent on accuracy of levels, nor is it impaired by wear; and the friction attending the passage of steam through them is much less than the friction of shafting. It is true that the force of steam passing through great lengths of pipes is considerably diminished by cooling; but much of this loss may be saved by covering the pipes with non-conducting materials, such as felt or sawdust. The objection generally made to this mode of communicating power is, that the first cost of the numerous small engines generally exceeds that of one large engine, and that each of the small engines requires separate attendance. We believe, however, that, in an extensive work, the cost of steam-pipes and numerous small engines will not be found to exceed that of a large engine, with all the shafting and connecting machinery; and that, with proper arrangements, the attendance given by workmen to the separate machines will be sufficient for the engines that put them in motion. A number of machines are now made to act directly by steam power, without the intervention of machinery. Among them we may cite, as examples, the steam-hammer and steam rivetting-machine. In both these cases, the direct pressure of steam upon a piston takes the place of the former complicated arrangements of wheelwork, eccentrics, cams, and levers, which were necessary for converting the rotary motion of shafting into the reciprocating movements required for hammering and rivetting. To many other manufacturing operations we think the same principle of movement might be applied with good effect; and we believe that the simplicity, directness of action, and facility of control attending such arrangements, would soon cause them to take the place of the more complicated methods of deriving power from rotary motions.

The atmospheric railway is an example of an arrangement for communicating power on a large scale by means of the motion of fluids. A pipe, several miles in length, with a longitudinal slit or opening along its upper side, is laid down between the rails. The pipe is fitted with a piston, and the slit is covered by a flexible valve, which can be lifted so as to permit an upright rod attached to the piston to pass through the upper part of the pipe to a carriage mounted on the rails. When the carriage and piston are at one end of the pipe, the valve being closed tightly over the slit, and luted by grease melted over it, the air is extracted from the pipe by pumps worked by a steam-engine; and the pressure of air on one side of the piston being thus in a great measure removed, the atmospheric pressure acting on the other side forces it along the pipe with great velocity, and thus gives motion to the carriage which is connected with the piston and the train of carriages attached to it. The piston and connecting-rod are fitted with

suitable rollers for lifting the valve in front of the upright rod, and for closing it and sealing it up behind it, so that after the passage of the train in one direction, the pipe may be ready for exhaustion to draw a train in the opposite direction. In the trials made with this system of propelling trains, it was found possible to secure great velocities, and to move very heavy trains over severe inclinations when the pipe and its valve were in good order. But the great practical difficulties attending the construction and tight-closing of the valve proved an almost insuperable objection to the use of the atmospheric railway, as even a small leakage of air into the vacuum-pipe necessitated the expenditure of great power in the pumping engine to sustain the required vacuum.

It was at one time proposed that in the case of a city like London, with numerous small steam-engines and boilers scattered through it, power should be communicated to different establishments from some central source, by means of compressed air forced through pipes laid down like the gas and water-pipes in the streets. One large engine continually sustaining a supply of air condensed to a pressure of several atmospheres, would thus take the place of the numerous separate boilers now supplying steam, and the air thus supplied would work the present engines as effectually as the steam. There is little doubt that, were such an arrangement carried out, the first cost of the air-pipes being met, there would be considerable advantages attending it, in respect of economy and the diminution of danger from boiler explosions, as well as the removal of numerous sources of offence from the chimneys of manufacturing establishments.

Arrangements of a similar character might at present be carried out in cities like London without any great outlay, where the supply of water is adequate. The pumping engines of the water companies might be made the means of supplying power to numerous establishments of machinery at a very cheap rate. It is found practically that in pumping water, 1 ton of fuel is capable of raising 25,000 gallons of water 100 feet high; and as water from a cistern 100 feet high would press with a force of 46 lbs. on every square inch of a piston exposed to its action, this power might be readily made available for raising weights, giving pressure, or even driving machinery by the intervention of properly constructed water-pressure engines.

RULES AND TABLES.

THE practical mechanic has frequent occasion to calculate the lengths of lines, areas of figures, capacities of solids, and weights of masses of material. We think, therefore, that we cannot better conclude the general view we have endeavoured to give of mechanics practically applied, than by furnishing a few of those simple rules and methods of calculation, which most commonly occur in practice.

We must presume that the mechanic is tolerably intimate with the ordinary operations of arithmetic—Addition, Subtraction, Multiplication, and Division; and that he will bear in mind the following symbols, as a kind of arithmetical short-hand, often useful in expressing rules and modes of operation.

$+$ placed before a quantity, means that it is to be added to the quantity preceding it.

$-$ placed before a quantity, means that it is to be substracted from the quantity preceding it.

$\times$ placed between two quantities, means that the first is to be multiplied by the other.

$\div$ placed between two quantities, means that the first is to be divided by the last.

When instead of the two dots in this symbol of division, one quantity is written above the line and the other below it, the upper is to be divided by the lower. Thus $12 \div 4$ may be written $\frac{12}{4}$, which means that 12 is to be divided by 4.

$=$ placed between two quantities, means that the one is equal to the other.

Example 1.—*Power* = *weight* × *velocity*, means that the power of any machine is equal to the weight raised by it, multiplied by the velocity with which it is raised. Thus, if an engine raise 330 lbs. at the rate or velocity of 100 feet per minute, we should say its power is equivalent to 330 lbs. × 100 feet = 33,000 lbs. at the rate of 1 foot per minute.

2. $\textit{Horse-power} = \frac{\textit{weight (in lbs.)} \times \textit{velocity (in feet per minute)}}{33000}$ means that the power of a machine reduced to the standard of horse-power, is equivalent to the weight in lbs. multiplied by the velocity in feet per minute, and divided by 33000. Thus, if an engine raise 330 lbs. 100 feet high per minute, its horse-power is $\frac{330 \times 100}{33000} = 1$, that is to say, one-horse power.

The tables of weights and measures most necessary in computations connected with Practical Mechanics, are those of Avoirdupois Weight, Lineal, Superficial, and Solid Measure, Time, Temperature, and the Division of the Circle. Unfortunately for ease of recollection and computation, the English tables of measures have no regular system, and therefore require to be remembered separately, or to be constantly referred to. The French have adopted a regular and simple system, both in the names and in the relations of their different denominations. As their measures are frequently referred to in scientific works, we subjoin tables of them along with the English tables; and short rules for the reduction of quantities given in the one, to their corresponding values in the other.

Troy Weight.

Used for the precious metals and for chemical analysis.

			Contracted.
24 Grains	= 1 Pennyweight	. .	24 gr. = 1 dwt.
20 Pennyweights	= 1 Ounce	. . . .	20 dwt. = 1 oz.
12 Ounces	= 1 Pound	. . . .	12 oz. = 1 lb.

1 pound troy therefore contains 5760 grains.

1 pound avoirdupois contains 7000 troy grains.

Avoirdupois Weight.

Used for weighing all materials except those to which troy weight is confined.

			Contracted.
16 Drams	= 1 Ounce	. . .	16 dr. = 1 oz.
16 Ounces	= 1 Pound	. . .	16 oz. = 1 lb.
28 Pounds	= 1 Quarter	. . .	28 lbs. = 1 qr.
4 Quarters	= 1 Hundredweight		4 qrs. = 1 cwt.
20 Hundredweights	= 1 Ton	. . . .	20 cwt. = 1 ton.

French Decimal System of Weight.

10 Milligrammes	= 1 Centigramme	
10 Centigrammes	= 1 Decigramme.	
10 Decigrammes	= 1 Gramme	= 15·434 troy grains.

10 Grammes	= 1 Decagramme.		
10 Decagrammes	= 1 Hectogramme.		
10 Hectogrammes	= 1 Kilogramme	= 2·20486 lbs. avoirdupois.	
10 Kilogrammes	= 1 Myriagramme.		
10 Myriagrammes	= 1 Quintal	= 1 cwt. 3 qrs. 25 lbs. nearly.	
100 Quintals	= 1 Millier or Bar	= 9 tons 16 cwt. 3 qrs. 12 lbs.	

The general principle adopted in the French system, is that of the decimal scale. They settle on some unit of weight or measure—as the gramme for weight, and the mètre for measure. For the names of all fractions of that unit, proceeding by tenths, hundredths, and thousandths downwards, they prefix the Latin numerals deci for tenth, centi for hundredth, milli for thousandth, to the name of the unit. Thus, a centigramme is the hundredth part of a gramme, and would be written in figures 0·01 gramme; a millimètre is a thousandth part of a mètre, and would be written 0·001 mètre. Again, for the names of all multiples of the unit, proceeding by tens, hundreds, thousands, and ten thousands upwards, they use the Greek numerals—deca, ten; hecto, hundred; kilo, thousand; myria, ten thousand, prefixed to the unit. Thus, for a thousand mètres they say a kilomètre, written 1000 mètres; for ten thousand grammes they say myriagramme, written 10,000 grammes. There are a few exceptions for the larger denominations in the scale of weights which we have given in full. According to this system, each denomination finds its place in the ordinary decimal scale of notation, and arithmetical operations are reduced to the simple rules, without the necessity of complicated reductions.

For example, if we wished to ascertain the cost of 7 myriagrammes, 3 kilogrammes, 4 hectogrammes, 6 decagrammes, 5 grammes, 3 centigrammes, of a material at 2 francs 20 cents per kilogramme,

we should write the quantity . . . 73465·03 grammes
which is equivalent to . . . . . . 73·46503 kilogrammes, pointing off 3 figures.
Multiply by the price per kilogr. . . 2·2

155 623066 francs.

Or, 155 francs, 62 cents.

In the English system, on the other hand, a similar question—as for instance, the cost of 17 tons 4 cwt. 3qrs. and 18 lbs., at 12*s.* 9*d.* per cwt.—would involve the necessity of reducing the several denominations, or of artifices for employing fractional parts as in the ordinary arithmetical rule of Practice.

To reduce kilogrammes to lbs. avoirdupois.

Rule.—Multiply by 2·20486.

Example 1.—To reduce 17 kilogrammes to lbs.

Multiply by 2·20486

37·48262 lbs.

For general practical computations, the decimal fraction, after its first figure, may be neglected, and the multiplier may be taken as 2·2 simply.

Example 2.—To reduce 23 kilogrammes to lbs.

$$23 \times 2{\cdot}2 = 50{\cdot}6 \text{ lbs.}$$

For closer approximation, after multiplying the number of kilogrammes by 2·2, add to the result the 200th part of the number, or the half with two decimal places pointed off.

Thus, 23 × 2·2 = 50·6
add $\frac{1}{200}$th of 23 ·115

50·715 lbs.
The accurate number 50·71178

To reduce lbs. avoirdupois to kilogrammes.

Rule.—Divide by 2·20486, or multiply by its reciprocal ·45355.

Example 1.—To reduce 37·48262 lbs. to kilogrammes.

Dividing by 2·20686)————
17 kilogrammes.

The reduction may be effected with sufficient accuracy by the following artifice:—

Add together one-third of the given number of lbs., one-third of that third, and one-twelfth of the last third.*

Example 2.—To reduce 37·48262 lbs. to kilogrammes.

one-third	12·49421
one-third of one-third	4·16477
one-twelfth of the last	·34706
	17·00604

Example 3.—To reduce 5 cwts. 2 qrs. 12 lbs. to French weight.

4
—
22 qrs.
28
—
3)628 lbs.
—
3)209·33
12) 69·77
5·81
—
284·91 kilogrammes.

LINEAL MEASURES.

12 Inches	=	1 Foot	6 Feet	=	1 Fathom
3 Feet	=	1 Yard			
5½ Yards	=	1 Pole or Rod	7·92 Inches	=	1 Link
40 Poles	=	1 Furlong	100 Links	=	1 Chain
8 Furlongs	=	1 Mile	80 Chains	=	1 Mile

The measures employed for the smaller dimensions of mechanical work, are the foot, its twelfth part the inch, and the fractional divisions of the inch, dividing successively by 2; viz.—the half, the quarter, the eighth, the sixteenth, the thirty-second, &c.

---

* The reason of this rule is the following:—

One-third of 1, expressed decimally, is	·3333 &c.
One-third of ·333, &c. expressed decimally, is . .	·1111 ,,
One-twelfth of ·111, &c. expressed decimally, is . .	·0092 ,,
Their sum	·4536
is a near approximation to	·45355

For larger works, as cuttings, embankments, shafts of mines, and the like, the yard of 3 feet, or the fathom of 6 feet, are the ordinary units. For surveying purposes, the chain of 11 fathoms, or 22 yards, or 66 feet, or 100 links, is the unit. And for greater dimensions, the mile, or 1760 yards, or 5280 feet, and its fractional divisions by 2, the half-mile, the quarter mile, the eighth of a mile or furlong, are employed.

The unit of French measure is the mètre, which is equal in length to 39·371 English inches, or about 3·281 English feet, or 3 feet $3\frac{3}{8}$ inches very nearly.

To reduce French measure to English.

Rule.—Multiply the number of French mètres by 3, and take the product as so many English feet, so many inches, and so many eighths of inches; or, add together the product, its twelfth part, and the eighth part of the twelfth, for the value in English feet.

Example 1.—To reduce 53 mètres to English measure.

	ft.	in.
53 × 3 = ...	159	
159 inches = ...	13	3
159 eighths of inches =	1	$7\frac{7}{8}$
	173	$10\frac{7}{8}$ nearly.

Example 2.—To reduce 42 kilomètres to English miles. The kilomètre is 1000 mètres, and 42 kilomètres are therefore equivalent to 42000 mètres.

Multiply by	3
	126000
One-twelfth	10500
One-eighth	1312·5

One mile contains 5280)137812·5 feet

26·1 miles nearly.

1 kilomètre is exactly 1093·6389 yards.
1 mile is exactly 1760 yards.

Therefore, to reduce kilomètres to miles, multiply by $\frac{1093·6389}{1760}$ or ·621386.

For rough calculations, take $\frac{5}{8}$ths decimally, ·625.

To reduce miles to kilomètres, multiply by $\frac{1760}{1093·6389}$, or 1·60931 nearly.

For rough calculations, take $1\frac{3}{5}$th decimally, 1·6.

The French pied, or foot, is equal to 1·09 English foot.

The French pouce, or inch, is equal to 1·09 English inch.

Superficial Measure.

144 square inches	=	1 square foot.
9 square feet	=	1 square yard.
$30\frac{1}{4}$ square yards	=	1 square pole.
40 square poles	=	1 rood.
4 roods	=	1 acre.

The acre, therefore, contains 4840 square yards, or 10 square chains, each of 484 square yards.

The French superficial measure is, for small areas, reckoned by the squares of the mètre and its parts, the mètre carré, or square mètre, being 13·765 square feet, or

1·196 square yard. For larger areas, the French unit is the Are, which is a square decamètre, or 100 square mètres, equivalent to 119·605 square yards.

Solid Measure.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
42 cubic feet = 1 ton of shipping.
40 „ = 1 load of rough timber.
50 „ = 1 load of squared timber.

The French solid measure is reckoned by the cube of the mètre and its parts.

1 cubic foot = ·0283 cubic mètre.
1 cubic yard = ·7645 „
1 cubic mètre = 35·32 cubic feet.

Liquid Measure.

There are several varieties of English liquid measure, and provincial variations in their scales; but the principal unit of liquid measure is the imperial gallon, which contains 277·274 cubic inches, and of which a cubic foot contains 6·2321, or nearly $6\frac{1}{4}$. The unit of French liquid measure is the litre, or cubic decimètre, the thousandth part of a cubic mètre, equivalent to 61·028 cubic inches.

Time Measure.

60 seconds = 1 minute.   24 hours = 1 day.
60 minutes = 1 hour.   7 days = 1 week.

The French measurement of time is the same.

Temperature.

There are three scales of temperature in use :—

1. Fahrenheit's, used in England, in which the point of water freezing is 32°, and the point of water boiling is 212°.

2. The Centigrade, used in France, in which the freezing-point of water is the zero, or 0°, and its boiling-point is 100°.

3. Reaumur's, used among some Continental nations, in which the freezing-point of water is zero, or 0°, and its boiling-point 80°.

To reduce temperature by Fahrenheit to temperature by Centigrade.

Subtract 32°, multiply by 5, and divide by 9.

Example 1.—To reduce 180° Fahrenheit to Centigrade.

$$\begin{array}{r} 180 \\ \text{Subtract} \;\; . \;\; . \quad 32 \\ \hline \end{array}$$

$$\frac{148 \times 5}{9} = 82°\cdot222 \text{ Centigrade.}$$

Example 2.—To reduce 15° Fahrenheit to Centigrade.

$$\begin{array}{r} 15 \\ \text{Subtract} \;\; . \;\; . \quad 32 \\ \hline \end{array}$$

$$\frac{-17 \times 5}{9} = -9°\cdot44, \text{ \&c., or } 9°\cdot44, \text{ \&c. below zero.}$$

To reduce temperature by Centigrade to temperature by Fahrenheit.

Multiply by 9, divide by 5, and add 32°.

Example 3.

To reduce 82°·2 Centigr. to Fah.
Multiply by 9
Divide by 5)740
148
Add . . . 32
180° Fah.

Example 4.

To reduce — 9°·4 Centigr. to Fah.
9
5)— 85
— 17
Add . . . 32
15° Fah.

Note.—When the temperature has the mark — minus prefixed, it means below zero, and is to be subtracted in cases where it would be added if it had not the mark —.

Example 5.—To reduce — 100° Centigrade to Fahrenheit.

9
5)— 900
— 180
32
— 148° Fahrenheit, or 148° below zero.

To reduce temperature by Fahrenheit to temperature by Reaumur.
Subtract 32, multiply by 4, and divide by 9.

Example 6.

To reduce 62° Fah. to Reaum.
32
30
4
9)120
13°·3 Reaum.

Example 7.

To reduce 15° Fah. to Reaum.
32
— 17
4
9)— 68
— 7°·5 Reau., or 7°·5 below zero.

To reduce temperature by Reaumur to temperature by Fahrenheit.
Multiply by 9, divide by 4, and add 32.

Example 8.

To reduce 13° 3′ Reaum. to Fah.
9
4)120
30
32
62° Fah.

Example 9.

To reduce — 7° 5′ Reaum. to Fah.
9
4) — 68
— 17
32
15° Fah.

To reduce Centigrade to Reaumur. Subtract ⅕th.

Example 10.—To reduce 85° Centigrrade. to Reaumur.
One-fifth . . . 17
68° Reaum.

To reduce Reaumur to Centigrade. Add ¼th.

Example 11.—To reduce 68° Reaumur to Centigrade.
One-fourth . . . 17
85° Centigr.

Division of the Circle.

60 Seconds	= 1 Minute	60″ = 1′
60 Minutes	= 1 Degree	60′ = 1°
90 Degrees	= 1 Quadrant or right angle	
360 Degrees or 4 Quadrants	= 1 Circumference.	

The French Division is

100 Seconds	= 1 Minute.
100 Minutes	= 1 Degree.
100 Degrees	= 1 Quadrant.
4 Quadrants	= 1 Circumference.

The quadrant, right angle, or quarter of the circumference, is the same for the French and English systems, and 100° French are therefore equivalent to 90° English.

To reduce French degrees to English. Subtract $\frac{1}{10}$th.

Example 1.—To express 73° French in English measure.

Subtract one-tenth . 7·3

65·7 — 65° 42′ English.
60

42·0

Example 2.—To express 26° 7′ 35″ French in English measure.

The French number would be written 26°·0735
Subtract one-tenth . . . . . 2 ·60735

23 ·46615
60

27 ·969
60

58·14

An. 23° 27′ 58″·14 English.

To reduce English degrees to French. Add $\frac{1}{9}$th.

Example 3.—To express 65° 42′ English in French measure. $42' = \frac{42}{60}$ or 0°·7, therefore the English, expressed decimally, is 65°·7
Add one-ninth . . . . . . 7 ·3

73°· French.

Example 4.—To express 23° 27′ 58″·14 English in French. The quantity, decimally expressed, is 23°·46615 English.
Add one-ninth . 2 ·60735

26°·0735 French.

The circumference of every circle is 3·1416 times its diameter, or 6·2832 times its radius or half diameter.

Hence, to find the circumference of a circle whose diameter or whose radius is given,

Rule.—Multiply the diameter by 3·1416, or the radius by 6·2832.

Example 1.—To find the circumference of a pulley whose diameter is 4 feet 6 ins.

4 feet 6 ins., or 54 ins. × 3·1416 = 169·6464 ins., or 14 feet $1\frac{5}{8}$ in. nearly.

Example 2.—To find the circumference of the pitch circle of a wheel, the pitch radius being 11 inches.

11 ins. × 6·2832 = 69·1152 ins., or nearly 69⅛ ins.

Since the decimal ·1416 is nearly equal to ⅐th (which is accurately ·142587̇), we have the approximate rule for finding the circumference of a circle.

Rule.—Multiply the diameter by 3⅐, or (since 3⅐ = ²²⁄₇) multiply the diameter by 22 and divide by 7, or multiply the radius by 44 and divide by 7.

Example 3.—To find the circumference of a pulley 4 feet 6 inches diameter.

4 feet 6 ins. × 3	=	13 feet 6 ins.
4 feet 6 ins. × ⅐ (or ÷ 7) =		7⅝ ins. nearly.
Circumference		14 feet 1⅝ ins. nearly.

Example 4.—To find the circumference of a wheel whose radius is 11 inches.

$$\frac{11 \times 44}{7} = 69\tfrac{1}{7} \text{ ins. nearly.}$$

The circumference of an ellipse is that of a circle whose diameter is a mean between the two axes of the ellipse.

Example 5.—To find the circumference of an ellipse having a longer axis 18 ins., and a shorter axis 12 ins. Mean $\frac{18 + 12}{2} = 15$ ins. × 3·1416 = 47·124 ins. or 3 feet 11⅛ ins.

The converse operation of finding the diameter of a circle having a given circumference is,

Rule.—Divide the circumference by 3·1416, or multiply it by ·31831, the reciprocal of 3·1416.

Example 6.—To find the diameter of a pulley whose circumference is 14 feet 1⅝ in. or 169·625 ins.

$$\frac{169{\cdot}625}{3{\cdot}1416} = 54 \text{ ins. or 4 feet 6 ins. very nearly.}$$

or 169·625 × ·31831 = 54 ins. nearly.

The following approximation is generally sufficiently seen for practical purposes.

Rule.—Multiply the circumference by 7 and divide by 22, the quotient is the diameter.

Example 7.—To find the diameter of a wheel having 23 teeth, each of 3 ins. pitch.

23 × 3 ins. = 69 ins. the circumference of the pitch circle, and $\frac{69 \times 7}{22} = 22$ ins. nearly.

The pitch radius is ½ of 22 ins. = 11 ins.

Mensuration of Superficies.

The object of Mensuration of Superficies is to discover the number of square units in a figure, the form and dimensions of whose boundary are known. The simplest kind of figure is that bounded by three straight lines, the triangle.

To find the area of a triangle.

Rule.—Multiply the length of any one of the sides by that of the perpendicular let fall upon it from the opposite angle (or, briefly, multiply the base by the height), and halve the result.

Example 1.—To find the area of a triangle having base 2 feet 3 ins., and height 1 foot 8 ins.

2 feet 3 ins. = 27 ins. and 1 foot 8 ins. = 20 ins. Area, $\frac{27 \times 20}{2} = 270$ sq. ins.

Example 2.—To find the area of a triangular field, having base 23 chains 8 links, and height 14 chains 73 links.

$$\frac{23·08 \text{ ch.} \times 14·73 \text{ ch.}}{2} = 339·9684 \text{ sq. chains; or } 33·99684 \text{ acres, very nearly } 34 \text{ acres.}$$

The parallelogram is a figure bounded by four straight lines, the opposite sides being equal and parallel to one another. When the angles of the parallelogram are right angles, or, as it is said in ordinary language, when the sides are square to one another, the figure is called a rectangle or oblong.

To find the area of a parallelogram.

Rule.—Multiply the base by the perpendicular height.—When the figure is an oblong, we say, multiply the length by the breadth.

Example 3.—To find the area of a parallelogram, base 2 feet 3 ins., height 1 foot 8 ins.

27 ins. × 20 ins. = 540 sq. ins.

Example 4.—To find the surface of a cylinder, diameter 4 feet, length 14 feet.

The surface of a cylinder, if developed or unrolled, would form a rectangle whose length is the length of the cylinder, and breadth the circumference. Its area is therefore found by multiplying the length by the diameter, and by $3\frac{1}{7}$.

14 feet × 4 feet × $3\frac{1}{7}$ = 176 sq. feet.

The trapezoid is a figure bounded by four straight lines, two of which are parallel. To find the area of a trapezoid.

Rule.—Multiply half the sum of the two parallel sides by their perpendicular distance apart.

Example 5.—Required the area of a trapezoid having parallel sides, respectively 5 feet and 7 feet, 3 feet apart. $\frac{5 + 7}{2} \times 3 = 18$ sq. ft.

Any figure bounded by straight lines may be divided into triangles; and the sum of the areas of all the triangles into which it is divided, is the area of the whole figure.

Of surfaces bounded by curved lines, the most regular and frequent is the circle.

To find the area of a circle.

Rule.—Multiply the square of the diameter by ·7854. Or, multiply the circumference by the radius, and halve the product.

Example 6.—Required the area of a circle 4 ft. diam. 4 × 4 × ·7854 = 12·5664 sq. [illegible].

Otherwise, the radius is 2 feet, and the circumference is 4 × 3·1416 = 12·5664, and the area is $\frac{12·5664 \text{ circumf.} \times 2 \text{ rad.}}{2} = 12·5664$ sq. ft.

The following method furnishes a near approximation to the area.

Rule.—Multiply the diameter by itself.

Take half the product, to which add its half, and the seventh part of that half.*

Example 7.—Required the area of a circle 4 feet diameter.

4 × 4 = 16 and $\frac{1}{2}$ of 16 = 8
$\frac{1}{2}$ of 8 = 4
$\frac{1}{7}$ of 4 = 0·57

Area 12·57 sq. ft. nearly.

---

* This approximate rule is thus derived:—

Since $\frac{1}{2}$ decimally is ·5000
$\frac{1}{2}$ of ·5000 = ·2500
$\frac{1}{7}$ of ·2500 = ·0357

Their sum = ·7857

which is nearly equal to ·7854, the proper multiplier of the square of the diameter.

Example 8.—Required the area of a circle $37\frac{5}{8}$ ins. diameter.

$37\frac{5}{8}$ or $37{\cdot}625 \times 37{\cdot}625 = 1415{\cdot}64$ sq. ins.
$\frac{1}{2}$ of $1415{\cdot}64 = 707{\cdot}82$ nearly.
$\frac{1}{2}$ of $707{\cdot}82 = 353{\cdot}91$
$\frac{1}{7}$ of $353{\cdot}91 = 50{\cdot}56$

Area 1112·29 nearly.
The correct area is 1111·84

Error in the approximate method 0·45 sq. ins.

The converse operation of finding the diameter of a circle when its area is given, is the following:—

Rule.—Divide the area by ·7854, or multiply the area by 1·273 (the reciprocal of ·7854), and extract the square root of the result.

Example 9.—Required the diameter of a circle having 12·5664 sq. ft. area.

$\frac{12{\cdot}5664}{{\cdot}7854}$ or $12{\cdot}5664 \times 1{\cdot}273 = 16$; and the square root of 16 is 4 feet.

Approximate method.

Rule.—To the given area, add its fourth part, and $\frac{1}{10}$th of that fourth, or the fourth with one decimal place pointed off, and extract the square root.*

Example 10.—Required the diameter of a circle whose area is 1111·84 sq. ins.

Area $= 1111{\cdot}84$
Add $\frac{1}{4}$ of $1111{\cdot}84 = 277{\cdot}96$
$\frac{1}{10}$ of $277{\cdot}96 = 27{\cdot}796$

1417·596

The square root of which is 37·65, or $37\frac{5}{8}$ inches nearly.

The area of an ellipse is ·7854 times the product of its two axes.

Example 11.—Required the area of an ellipse whose axes are 2 feet 3 inches and 1 foot 8 inches respectively.

27 ins. $\times$ 20 ins. $\times$ ·7854 = 424·116 sq. in.

or, $\frac{424{\cdot}116}{144} = 2{\cdot}9452$ sq. ft.

The area of a sector of a circle C A B is the same part of the area of the whole circle, as the arc of the sector A B is of the whole circumference.

Example 12.—Required the area of a sector; radius 12 ins., and arc 67° 30′.

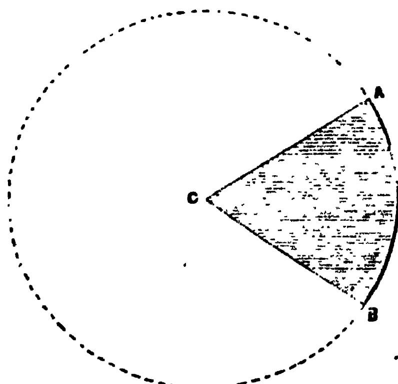


Fig. 272.

* This rule like the former is thus derived,—

$1 = 1{\cdot}000$
$\frac{1}{4}$ of $1 = 0{\cdot}250$
$\frac{1}{10}$ of $0{\cdot}25 = 0{\cdot}025$

1·275, which is nearly 1·273.

The area of the whole circle is $12 \times 12 \times 3{\cdot}1416 = 452{\cdot}39$ sq. ins.

$$67^\circ\ 30' \text{ is } \frac{67\frac{1}{2}}{360}, \text{ or } \frac{67\frac{1}{2} \times 2}{360 \times 2} = \frac{135}{720}, \text{ or } \frac{135 \div 45}{720 \div 45} = \tfrac{3}{16}\text{ths}$$

of the whole circumference; therefore

Area of sector is $\frac{3}{16}$ths of $452{\cdot}39$ sq. ins. $= 84{\cdot}825$ sq. ins.

In measuring the area of figures bounded by lines of irregular curvature, as A L B S, the most convenient method is to draw through the figure a straight line A B, the longest possible, and to divide it into numerous equal parts at the points 1, 2, 3, 4, 5. If we bisect each of those parts at C, D, E, F, G, H, and draw perpendiculars

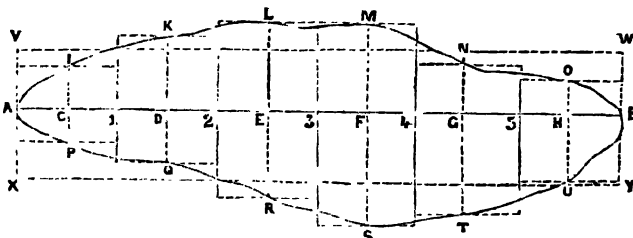


Fig. 271.

to the line A B, meeting the curve in I, P, K, Q, &c., by drawing parallels through these points meeting perpendiculars through the points A, C, D, &c., we form a number of rectangles, each of which is very nearly equal in area to the part of the curve contained between its two perpendicular sides. The area of the whole curved figure is, therefore, very nearly equal to the sum of the areas of all those rectangles. Now, as the area of each rectangle is equal to its base multiplied by its perpendicular height, and as the bases of all are equal, the area of all the rectangles is equal to the length of one of the bases, such as C D multiplied by the sum of all the heights I P, K Q, L R, &c. It is to be observed that the distances A C and B H of the first and last perpendiculars from the extremities of the line, are each half the distance C D or D E between any two perpendiculars. Or, we may view the question in another way, thus:—Having divided the line A B as before, and set off the perpendiculars I P, K Q, &c. (technically called offsets), we may find a mean or average V X or W Y, by adding together the lengths of all the perpendiculars, and dividing their sum by their number, and then the whole rectangle V W Y X has very nearly the same area with the curved figure. Hence the following—

Rule I.—Multiply the sum of all the offsets by the distance between two adjoining offsets; or,

Rule II.—Add together the lengths of all the offsets, divide by their number, and multiply by the whole length of the base line.

Example 13.—The length of A B is 24, and the offsets are IP = 3, KQ = 5, LR = 7, MS = 8, NT = 6, OU = 4: required the area.

By *Rule* I.—There are six offsets; therefore the distance between any two adjoining is $\frac{24}{6} = 4$ (note, A C and B H are each 2), and the area is

$$(3 + 5 + 7 + 8 + 6 + 4) \times 4 = 132.$$

By *Rule* II.—The mean of all the offsets is $\frac{3 + 5 + 7 + 8 + 6 + 4}{6} = 5{\cdot}5$, and the area is $5{\cdot}5 \times 24 = 132$.

The surface of a cone may be developed, or unrolled, into the form of a sector of a circle, and its area found by the rule. Or thus,

Rule.—Multiply the diameter of the base by the length of the side (from the apex to the edge of the base), and by 1·5708.

Example 14.—Required the surface of a cone, having a base 12 inches in diameter, and 18 inches length of side.

$$12 \times 18 \times 1{\cdot}5708 = 339{\cdot}2928.$$

The surface of a sphere is 4 times that of any of its circular sections passing through the centre. Hence, to compute the surface of a sphere—

Rule.—Multiply the square of the diameter by 3·1416.

Example 15.—Required the surface of a sphere 18 inches diameter.

$$18 \times 18 \times 3{\cdot}1416 = 1017{\cdot}8784 \text{ square inches.}$$

MENSURATION OF SOLIDS.

By mensuration of solids we discover the number of cubic units in a body of given form and dimensions. A parallelopiped, or prism, has two opposite sides equal and parallel to one another. Either of these is called a base, and their perpendicular distance apart is called the height or altitude of the prism. A cylinder has a circular base.

For finding the solid contents of a prism or cylinder.

Rule.—Multiply the area of the base by the height.

Example 1.—Required the capacity of a cistern, whose base is a rectangle 4 feet long by 3 feet wide, and whose height is 2 feet 6 inches.

Area of base $4 \times 3 \times 2\frac{1}{2}$ height $= 30$ cub. feet, or $30 \times 6{\cdot}2321 = 187$ gals. nearly.

Example 2.—Required the solid contents of a roller 22 ins. diameter and 42 ins. long.

Area of base $22 \times 22 \times {\cdot}7854 \times 42 = 15965{\cdot}6$ cubic inches.

When the prism or the cylinder is hollow,

Rule.—Subtract the area of the internal part of the base from that of the whole base, and multiply by the height.

Example 3.—Required the weight of a cast-iron hollow cylinder 5 feet high and 30 inches diameter internally, having metal 1 inch thick (reckoning 3·8 cubic inches of cast-iron equivalent to 1 lb.). The external diameter is the internal diameter increased by twice the thickness of metal; it is therefore 32 inches, and its area would be $32 \times 32 \times {\cdot}7854$. The area of the internal would be $30 \times 30 \times {\cdot}7854$; and, subtracting, we have $(32 \times 32 - 30 \times 30) \times {\cdot}7854 = 97{\cdot}436$ sq. inches for the area of the section of metal. The solid contents are $97{\cdot}436 \times 60$ ins. $= 5846{\cdot}16$ cub. ins., and $\frac{5846{\cdot}16}{3{\cdot}8} = 1538{\cdot}46$ lbs., or 13 cwt. 2 qrs. $26\frac{1}{2}$ lbs. When the diameter of a hollow cylinder is large in proportion to the thickness of its material, as in the case of a boiler, which may be several feet in diameter, while the thickness of its metal is less than half an inch, we may multiply the area of its surface by the thickness of material to ascertain the quantity of material.

Boilers are generally made of wrought-iron plate; and it happens that a square foot of iron plate $\frac{1}{8}$th of an inch thick weighs 5 lbs. In order to ascertain the weight in pounds of a wrought-iron boiler $\frac{1}{8}$th of an inch thick, we should then have to multiply its surface by 5; and were the thickness $\frac{2}{8}$ths or $\frac{1}{4}$th, we should multiply by 2×5 or 10; for thickness $\frac{3}{8}$ths, multiply by 3×5 or 15, and so on. But in constructing boilers, there are many plates used, which overlap each other at their joints, and at the corners there are angle-irons, and numerous rivets are used to make the joints tight. Making allow-

ance for all these additions to the weight, we may estimate the weight per superficial foot at about 6 lbs. for every $\frac{1}{8}$th of an inch in thickness, or 3 lbs. for every $\frac{1}{16}$ of an inch. Again, as $\frac{6}{112}$ is nearly $\frac{1}{20}$th, we may roughly approximate the weight in cwts. by the following rule:—

Rule.—Multiply the surface (in square feet) by the thickness (in $\frac{1}{8}$ths of an inch), and divide by 20 for the weight (in cwts.).

Example 4.—Required the weight of a Cornish boiler, external diameter 4 feet, diameter of flue-tube 2 feet, length 12 feet, thickness $\frac{3}{8}$ $\frac{1}{16}$ of an inch (or $3\frac{1}{2}$ eighths of an inch).

External circumference . $4 \times 3\frac{1}{7}$
Circumference of flue $2 \times 3\frac{1}{7}$

		nearly		length		surface of casing and flue.
Both	$6 \times 3\frac{1}{7} =$	18·9	$\times$	12	=	226·8

Area of end of boiler $4 \times 4 \times \cdot 7854$
Area of flue . . $2 \times 2 \times \cdot 7854$

			surface of ends.
Difference	$12 \times \cdot 7854 = 9{\cdot}4 \times 2$ (ends)	=	18·8

245·6 total surface.

$$\text{And } \frac{245{\cdot}6 \times 3\frac{1}{2}}{20} = 43 \text{ cwt. nearly, or 2 tons 3 cwt.}$$

The cubic contents of a sphere is $\frac{2}{3}$rds of that of a cylinder of the same diameter and altitude. But the altitude being equal to the diameter, and $\frac{2}{3}$rds of ·7854 being ·5236, the contents of the sphere may be found by the following method:—

Rule.—Multiply the cube of the diameter by ·5236.

Example 5.—Required the number of cubic inches in a ball 12 inches diameter.

$$12 \times 12 \times 12 \times \cdot 5236 = 904{\cdot}7808 \text{ cubic inches.}$$

When the sphere is hollow,

Rule.—Subtract the cube of the internal diameter from the cube of the external diameter, and multiply by ·5236.

Example 6.—Required the weight of a hollow ball of iron, 30 inches diameter externally, metal $1\frac{1}{2}$ inch thick, reckoning 3·8 cubic inches to the pound.

External diameter $30 \times 30 \times 30 = 27000$ cubed
Internal diameter $27 \times 27 \times 27 = 19683$

7317

$$\text{And } \frac{7317 \times \cdot 5236}{3{\cdot}8} = 1008 \text{ lbs. or 9 cwt.}$$

The cubic contents of a cone is $\frac{1}{3}$rd of that of a cylinder of equal base and altitude. Hence, to find the content of a cone:—

Rule.—Multiply the square of the base by the height and by ·2618.

Example 7.—Required the solid contents of a cone, having a base 18 inches diameter, and a height of 22 inches.

$$18 \times 18 \times 22 \times \cdot 2618 = 1866{\cdot}11 \text{ cubic inches.}$$

The solids with which the practical mechanic has to deal are chiefly those called solids of revolution, or such as may be conceived to be produced by the revolution of some figure round an axis. Any work that is formed in a turning-lathe is a solid of revolution. An imaginary line extending between the centres of the lathe is the axis; and if the solid were cut across by a plane passing through this line, the section or surface exposed on the removal of the half cut off is symmetrical on each side of the axis or

centre line. One-half of this symmetrical figure, or the part lying on either side of the axis, is called the generating figure. Thus, a cylinder is generated by the revolution of an oblong or rectangle round one side. A cone is generated by the revolution of a right-angled triangle round its perpendicular. A sphere is generated by the revolution of a semicircle round its diameter. A spindle, by the revolution of a segment of a circle, or other curve, round its chord. In the same way, other mathematical figures, such as the ellipse, or the parabola, are capable of producing solids of revolution by causing them to revolve round their axes. When the ellipse is supposed to revolve round its shorter axis, the solid produced is called an oblate or shortened spheroid; when it revolves round its longer axis, the solid is an oblong spheroid. The parabola, by revolution, produces the paraboloid.

There are singular relations among some of these solids of revolution, one of which it is useful to remember. If A B E D represent the section of a cylinder, whose base has a diameter A B double its height, or axis, C F,—A F B the section of a cone on the same base, and of the same height with the cylinder—A G F H B the section of half a sphere,—and A K F L B the section of a paraboloid; then

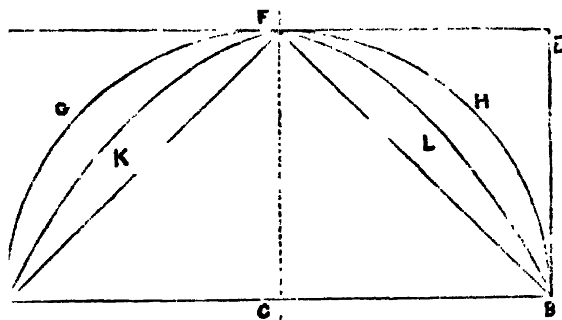


Fig. !

The solid contents of the cone is $\frac{1}{3}$rd of that of the cylinder.
" " hemisphere is $\frac{2}{3}$rds " "
" " paraboloid is $\frac{1}{2}$ " "

As the paraboloid comes midway between the cone and the hemisphere in form of section, so its capacity comes midway between them in amount. In many cases of mensuration of solids of revolution, when the form of section is something between that of a cone and of a hemisphere, it may be taken as a paraboloid, and its capacity may be found thus:—

Rule.—Square the diameter of the base, multiply by the height and by ·3927 (half of ·7854). Or, multiply the area of the base by half the height.

Example 8.—Required the solid content of a paraboloid, base 2 feet 6 inches diameter, height 1 foot 8 inches diameter.

Area of a circle 2 feet 6 inches, or 30 inches diameter, 707 square inches, nearly.
Half of 1 foot 8 inches, or 20 inches 10

Solid content 7070 cubic inches.

In general, the capacity of any solid of revolution may be found by multiplying the area of its generating figure by the circumference described by the centre of gravity of that figure. In a ring, for instance, generated by the revolution of a circle round some centre without it, if we know the diameter of its generating circle or section, and the distance of its centre from that round which it revolves, we can compute its solid contents, as we may show by an example.

Example 9.—A ring of circular section measures 10 ins. external diameter, and 6 ins. internal: required its solid contents. Subtracting 6 from 10, there are left 4 ins., which must be double the diameter of the ring's section, giving 2 ins. as the actual diameter of the section, which has therefore an area 2 × 2 × ·7854 = 3·1416 sq. ins. Again, adding 6 to 10, and taking half, we get 8 ins. as the distance between A and B (Fig. 273), the centres of the circular sections, and the circumference of a circle 8 ins. diam. is 8 × 3·1416 = 25·1328 ins. The solid content of the ring is therefore 3·1416 sq. ins. × 25·1328 ins. = 78·9572 cub. ins. In this particular case, it happens that the centre of gravity of the section coincides with the centre of figure. When this is not the case, the position of the centre of gravity may readily be found by the following mechanical process:—

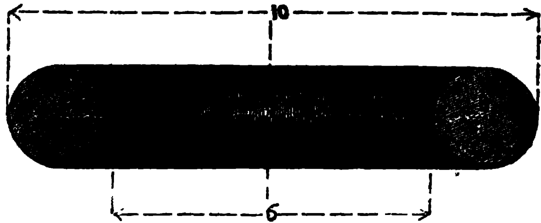


Fig. 273.

Draw upon a card or piece of thin wood, or plate of metal such as zinc, the figure, either of its full size or to any convenient scale; and having cut it out to the shape, make two small holes in it, such as A and C (Fig. 274). Suspend it by a thread from one of the holes, and along the face of it let a plumb-line hang and mark B, the point where the plumb-line crosses its edge, tracing a line A B coinciding with the plumb-line. Suspend it now by the other hole C, with the plumb-line crossing the line formerly traced at D. This point is the centre of gravity of the figure, or the point round which all parts are balanced. The position of the centre of gravity thus ascertained may be laid down upon the drawing of the work to be estimated, and the solid content may be found by the rule we have given.

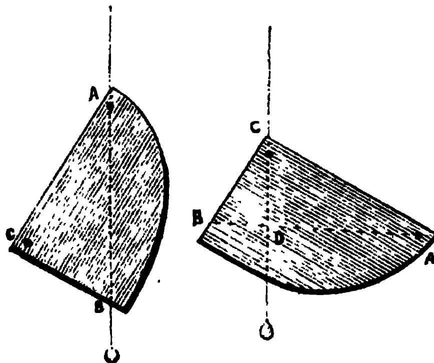


Fig. 274.

For the measurement of a solid of irregular form, the most convenient method is to suppose it divided by numerous parallel planes into sections of equal thickness,—to measure the area of each of those sections, and find the mean or average of them by summing them all and dividing by their number. This average area multiplied by the total thickness, measured perpendicularly across the planes of section, will give the solid content. In many cases of the measurement of cylindrical bodies, such as timber or round bars of metal, it is more convenient to use the girth or circumference as a known dimension for estimating the cubic contents. A near approximation to the capacity may be made thus:—

Rule.—Multiply the square of the girth by the length, and by 8, and point off two decimal places.

Example 10.—Required the cubic contents of a round log, having an average girth of 6 ft. 3 ins., and 12 ft. long. 6 ft. 3 ins. decimally expressed, is 6·25, and 6·25 × 6·25 × 12 × 8 = 3750. Pointing off two decimal places, the solid content is 37·5 cub. ft. The ordinary rule for timber gives a considerably smaller result, and is intended not to furnish the accurate contents, but the contents estimated according to trade custom. It is,

Rule.—Multiply the square of ¼th the girth by the length.

According to this rule, the contents of the log in *Example* 10 would be (¼th of 6·25 being 1·5625) 1·5625 × 1·5625 × 12 = 29·3 cubic feet.

Duodecimals.

In the mensuration of surfaces and solids, a system of computation is frequently adopted, called the duodecimal system, because the notation is reckoned by twelves (Latin *duodecim*), instead of the ordinary decimal scale of tens (Latin *decem*).

According to the duodecimal system, a square foot is supposed to be divided into 12 equal parts, each containing 12 square inches: and each of those parts again into twelfths, each 1 square inch; and these again into twelfths. So, also, a cubic foot is divided into 12 equal parts, each 144 cubic inches; each of those into 12 parts, each 12 cubic inches; and each of those into 12 parts, each 1 cubic inch. To multiply a certain number of feet and inches by some other number of feet and inches, the one quantity is written below the other, as in ordinary multiplication.

1. The inches are multiplied by inches, the product divided by twelve, the remainder written one place back from the inches, and the quotient carried to the next operation.

2. The feet are multiplied by inches, and the number carried from the former operation added, the result divided by twelve, the remainder written in the place of inches, and the quotient written in the place of feet.

3. The inches are multiplied by feet, the result divided by twelve, the remainder written in the place of inches, and the quotient carried.

4. The feet are multiplied by feet and the carried number added, and the whole written in the place of feet.

5. The results of the two multiplications are added, carrying by twelves.

For cubing, the same process is repeated, remembering that in the cubed result, or the result of multiplying three quantities of feet and inches, there are four places,—*viz.*, cubic feet; twelfths, each 144 cubic inches; twelfths of twelfths, each 12 cubic inches; and cubic inches.

Example.—Required the cubic contents of a cistern, the base of which is 4 ft. 8 ins. long by 3 ft. 10 ins. wide, and the height 5 ft. 7 ins.

We first find the area of the base.

4 ft.	8 in.	
3	10	
3*c*.	10*b*.	8*a*.
14*e*.	0*d*.	
17 ft.	10 pts.	8 ins.

Operation. 8 ins. × 10 ins. = 80 sq. ins. and $\frac{80}{12}$ = 6 pts. 8 ins. *a*.
carry 6 and write 8, *a*.

4 ft. × 10 ins. = 40, add 6 = 46, and $\frac{46}{12}$ = 3 ft. 10 pts. *b*, *c*.

8 ins. × 3 ft. = 24, and $\frac{24}{12}$ = 2 ft. 0 pts.
carry 2, and write 0, *d*.

4 ft. × 3 ft. = 12 ft., add 2, 14 ft., *e*.
Sum the two lines.

We now multiply the area of base by the height.

	ft.	pts.	ins.	
Area	17	10	8	
	5	7		
	10		2	8
	89		4	
	99	10	6	8

Operation. $8 \times 7 = 56$, and $\frac{56}{12} = 4$ ft. 8 ins., carry 4.

$10 \times 7 + 4 = 74$, and $\frac{74}{12} = 6$ ft. 2 ins., carry 6.

$17 \times 7 + 6 = 125$, and $\frac{125}{12} = 10$ ft. 5 ins.

$8 \times 5 = 40$, and $\frac{40}{12} = 3$ ft 4 ins., carry 3.

$10 \times 5 + 3 = 53$, and $\frac{53}{12} = 4$ ft. 5 ins., carry 4.

$17 \times 5 + 4 = 89$. Sum the lines.

The cubic contents are therefore

99 cub. ft. 10 twelfths, 6 hundred and forty-fourths, 8 cub. ins., or 99 ft. 1520 cub. ins.,

because 10 twelfths, each of 144 cubic inches . . .	=	1440 cubic inches.
6 hundred and forty-fourths, each 12 cub. ins.	=	72 "
8 cubic inches		8 "
		1520

Water is generally taken as the standard of specific gravity, and it fortunately happens that 1 cubic foot of water weighs very nearly 1000 ounces. By reference to any table of specific gravities, the weight of a cubic foot of each material is given in ounces; and from this may be readily derived—

1. The weight of 1 cubic foot in lbs., viz. $\frac{1}{16}$th of the specific gravity or weight in oz., because 1 oz. = $\frac{1}{16}$th of 1 lb.

2. The weight of 1 cubic inch in ounces—viz., the specific gravity divided by 1728, because there are 1728 cubic inches in 1 cubic foot—and the weight of 1 cubic inch in lbs., viz. the specific gravity divided by 1728 and the quotient by 16, or the specific gravity divided by 27648.

3. The number of cubic inches in 1 lb., viz. 16 times 1728, or 27648 divided by the specific gravity.

4. The number of cubic feet in 1 ton, viz. 2240 lbs. $\times$ 16 oz., or 35840 oz. divided by the specific gravity.

Example 1.—Required the weight in lbs. of 7 cubic feet of English oak.

$$\text{Spec. grav. } \frac{900}{16} \times 7 = 393\tfrac{3}{4} \text{ lbs., or 3 cwt. 2 qr. } 1\tfrac{3}{4} \text{ lbs.}$$

Example 2.—Required the weight of 178 cubic inches of cast-iron.

$$\text{Spec. grav. } \frac{7200}{1728} \times 178 = 742 \text{ oz.}$$

$$\text{Or} \quad \frac{7200}{27648} \times 178 = 46\tfrac{1}{3} \text{ lbs.}$$

Example 3.—Required the number of cubic inches in 2 cwt. (or 224 lbs.) of gun-metal.

$$\frac{27648}{\text{Spec. grav. } 8800} \times 224 = 703\tfrac{3}{4} \text{ cubic inches.}$$

Example 4.—Required the number of cubic feet in 20 tons of granite.

$$\frac{35840}{\text{Spec. grav. } 2700} \times 20 = 265\tfrac{1}{2} \text{ cubic feet.}$$

In estimating the weight of wrought-iron cylindrical bars, the following method is a very near approximation:—

Rule.—Square the diameter reduced to eighths of an inch, multiply by the length in feet and by $4\frac{1}{7}$, and point off two decimal places for the weight in lbs.

Example 1.—Required the weight of a bar of wrought-iron $5\frac{7}{8}$ ins. diameter and 7 ft. 7 ins. long.

$$5\tfrac{7}{8} \text{ ins.} = 47 \text{ eighths, and } 7 \text{ ft. } 7 \text{ ins.} = 7\tfrac{7}{12} \text{ ft.}$$
$$47 \times 47 \times 7\tfrac{7}{12} \times 4\tfrac{1}{7} = 69401.$$

The weight is therefore 694 lbs., or 6 cwt. 0 qr. 22 lbs.

For wrought-iron bars of rectangular section:—The widths of these advance by quarters, and the thickness by eighths of an inch.

Rule.—Multiply the breadth in quarters of an inch by the thickness in eighths, by the length in feet, and by $10\frac{1}{2}$, and point off two decimal places for the weight in lbs.

Example 2.—Required the weight of a bar of wrought-iron $2\frac{1}{4}$ ins. wide, $\frac{3}{8}$ in. thick, and 3 feet long.

Width 9 qrs. $\times$ thickness 3 eighths $\times$ length 3 ft. $\times$ $10\frac{1}{2} = 850$; therefore the weight is 8·5 lbs.

For wrought-iron plates.

Rule.—Multiply the area in square feet by the thickness in eighths of an inch, and by 5 for the weight in lbs.

Example 3.—Required the weight of 123 square feet of boiler-plate $\frac{1}{4}$ $\frac{1}{16}$ in. thick.

Thickness $\frac{1}{4}$ $\frac{1}{16}$, or $2\frac{1}{2}$ eighths of an inch,

$$123 \times 2\tfrac{1}{2} \times 5 = 1538 \text{ lbs., or } 13 \text{ cwt. } 1 \text{ qr. } 24 \text{ lbs.}$$

For cast-iron balls.

Rule.—To the cube of half the diameter (in inches) add its eleventh part for the weight in lbs.

Example 4.—Required the weight of a cast-iron ball 6 ins. diameter. Half the diameter is 3 ins.

And $3 \times 3 \times 3$ . . =	27
Add one-eleventh of 27 =	2·5
Weight	29·5 lbs.

The practical mechanic who has frequent occasion for calculations as to weights, volumes, and strengths of materials, would do well to provide himself with some of the published tables of squares, cubes, areas, and circumferences of circles, weights, and the like. These tables are useful in abridging the labour of calculation, and in diminishing the chances of error; but as it often happens that such books of reference are not accessible at the time a computation may be required, it is well that he should be prepared with some of the simple rules given above, and that he should bear in his memory a few of the numerical values of the gravity and other properties of some of the materials most generally used, such as iron, gun-metal, and timber.

INDEX.

A.

B.

N.

O.

Q.

R.